Ch-5: Sampling

P5.1. A real-valued signal f(t) is known to be uniquely determined by its samples when the sampling frequency is $\omega_s = 10^4 \pi$. For what values of $\omega$ is $F(\omega)$ guaranteed to be zero?

P5.2. A continuous-time signal f(t) is obtained at the output of an ideal low-pass filter with cutoff frequency $\omega_c = 1000 \pi$. If impulse-train sampling is performed on f(t), which of the following sampling period would guarantee the f(t) can be recovered from its sampled version using an appropriate low-pass filter?
   a) $T=5.10^{-4}$; b) $T=2.10^{-3}$; c) $T=10^{-4}$

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P5.3. The frequency which, under the sampling theorem, must exceeded by the sampling frequency is called the Nyquist rate. Determine the Nyquist rate corresponding to each the following signals:
   a) $f(t)=1+\cos(2000 \pi t)+\sin(4000 \pi t)$
   b) $f(t)=\frac{\sin(4000 \pi t)}{\pi t}$
   c) $f(t)=\left[\frac{\sin(4000 \pi t)}{\pi t}\right]^2$

P5.4. Let $f(t)$ be a signal with Nyquist rate $\omega_0$. Also let $y(t)=f(t)p(t-1)$
   Where $p(t)=\sum_{n=0}^{\infty} \delta(t-nT)$ and $T<\frac{\pi}{\omega_0}$

Specify the constraints on the magnitude and phase of the frequency response of a filter that gives f(t) as its output when y(t) is the input.
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P5.5. A signal \( f(t) \) undergoes a zero-order hold operation with an effective sampling period \( T \) to produce a signal \( f_0(t) \). Let \( f_1(t) \) denote the result of a first-order hold operation on the samples of \( f(t) \); i.e.,

\[
f_1(t) = \sum_{n=-\infty}^{\infty} f(nT)h_1(t-nT)
\]

Where \( h_1(t) \) is the function shown in Fig.P5.5. Specify the frequency response of a filter that produce \( f_1(t) \) as its output when \( f_0(t) \) is the input.

![h1(t)](image)

Fig.P5.5

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P5.6. Shown in Fig.P5.6 ia a system in which the sampling signal is impulse train with alternating sign. The Fourier transform of the input signal is indicated in the figure.

a) For \( \Delta < \pi/(2\omega_M) \), sketch the Fourier transform of \( f_p(t) \) and \( y(t) \)

b) For \( \Delta < \pi/(2\omega_M) \), determine a system that will recover \( f(t) \) from \( f_p(t) \)

c) For \( \Delta < \pi/(2\omega_M) \), determine a system that will recover \( f(t) \) from \( y(t) \)

d) What is the maximum value of \( \Delta \) in relation to \( \omega_M \) for which \( f(t) \) can be recovered from either \( f_p(t) \) or \( y(t) \)
**P5.7.** The sampling theorem, as we have derived it, states that a signal $f(t)$ must be sampled at a rate greater than its bandwidth (or equivalently, a rate greater than twice its highest frequency). This implies that if $f(t)$ has a spectrum as indicated in Fig. P5.7a then $f(t)$ must be sampled at rate greater than $2\omega_2$. However, since the signal has most of it energy concentrated in a narrow band, it would seem reasonable to expect that a sampling rate lower than twice the highest frequency could be used. A signal whose energy is concentrated in a frequency band is often referred to as a band pass signal. There are a variety of techniques for sampling such signals, generally referred to as bandpass-sampling techniques.

To examine the possibility of sampling a bandpass signal as a rate less than the total bandwidth, consider the system shown in Fig. P5.7b. Assuming that $\omega_i > \omega_2 - \omega_1$, find the maximum value of $T$ and the value of the constants $A$, $\omega_a$, and $\omega_b$ such that $f_r(t) = f(t)$. 
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In P5.7, we considered on procedure for bandpass sampling and reconstruction. Another procedure, used when \( f(t) \) is real, consists of multiplying \( f(t) \) by a complex-exponential and then sampling the product. The sampling system is shown in Fig.P.5.8a. With \( f(t) \) real and \( F(\omega) \) nonzero only for \( \omega_1 < |\omega| < \omega_2 \), the frequency is chosen to be \( \omega_0 = 1/2(\omega_1 + \omega_2) \), and the lowpass filter \( H_1(\omega) \) has cutoff frequency \( 1/2(\omega_2 - \omega_1) \).

a) For \( F(\omega) \) as shown in Figure P.5.8b, sketch \( F_p(\omega) \)

b) Determine the maximum sampling period \( T \) such that \( f(t) \) is recoverable from \( f_p(t) \)

c) Determine a system to recover \( f(t) \) from \( f_p(t) \)

\[
p(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)
\]

\[
F(\omega)
\]

Fig.P.5.8b

Fig.P.5.8
**Ch-5: Sampling**

**P5.9.** A signal \( f(t) = \text{sinc}(200\pi t) \) is sampled by a periodic pulse train \( p_T(t) \) represented in Fig.P5.9. Find and sketch the spectrum of the sampled signal. Explain if you will be able to reconstruct \( f(t) \) from these samples. If the sampled signal is passed through an ideal lowpass filter of bandwidth 100Hz and unit gain, find the filter output. What is the filter output if its bandwidth is \( B \) Hz, where 100\(<B<150? What will happen if the bandwidth exceeds 150Hz?

![Fig.P5.9](image)

**Ch-5: Sampling**

**P5.10.** Fig.P5.10 shown the so called flat top sampling of the signal \( f(t) = 5\text{sinc}^2(5\pi t) \).

a) Show that the signal \( f(t) \) can be recovered from flat top samples if the sampling rate is no less than the Nyquist rate.

b) Explain how you would recovered from flat top samples

c) Find the expression for the sampled signal spectrum \( \tilde{F}(\omega) \) and sketch it roughly

![Fig.P5.10](image)