Ch-4: Fourier transform representation of signal

P4.1. Use the Fourier transform analysis equation to calculate the Fourier transform of the following signals:

a) \( f(t) = e^{-2(t-1)}u(t-1) \)

b) \( f(t) = e^{-2t-1} \)

c) \( f(t) = \delta(t+1) + \delta(t-1) \)

d) \( f(t) = \frac{d}{dt}[u(-2-t)+u(t-2)] \)

Sketch and label the magnitude of each Fourier transform.

P4.2. Determine the Fourier transform of each of the following periodic signals:

a) \( f(t) = \sin(2\pi t + \frac{\pi}{4}) \)

b) \( f(t) = 1 + \cos(6\pi t + \frac{\pi}{6}) \)

P4.3. Use the Fourier transform synthesis equation to determine the inverse Fourier transform of:

a) \( F(\omega) = 2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi) - \omega^2 \)

b) \( F(\omega) = 2\text{rect}(\frac{\omega - 1}{2}) - 2\text{rect}(\frac{\omega + 1}{2}) \)

P4.4. Given that \( f(t) \) has the Fourier transform \( F(\omega) \), express the Fourier transform of the signals listed below in terms of \( F(\omega) \). You may use the Fourier transform properties.

a) \( f_1(t) = f(1-t) + f(-1-t) \)

b) \( f_2(t) = f(3t - 6) \)

c) \( f_3(t) = \frac{d^2}{dt^2}f(t-1) \)

P4.5. For each of the following Fourier transforms, use Fourier properties to determine whether the corresponding time-domain signal is (i) real, imaginary, or neither and (ii) even, odd, or neither. Do this without evaluating the inverse of any of the given transform.

a) \( F_1(\omega) = \text{rect}(\frac{\omega - 1}{2}) \)

b) \( F_2(\omega) = \cos(2\omega)\sin(\frac{\pi}{4}) \)

b) \( F_3(\omega) = A(\omega)e^{iB(\omega)} \); where \( A(\omega) = (\sin 2\omega)/\omega, B(\omega) = 2\omega + \frac{\pi}{2} \)

c) \( F_4(\omega) = \sum_{n=-\infty}^{\infty} (-i)^n \delta(\omega - n\frac{\pi}{2}) \)
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P4.8. Determine the Fourier transform of the following signal:

\[ f(t) = \frac{1}{\pi t} \sin^2 t \]

Use the Parseval’s relation and the result of the previous part to determine the numerical value of \( E_f \).

P4.9. Given the relationships \( y(t) = f(t) * h(t) \) and \( g(t) = f(3t) * h(3t) \), and given that \( f(t) \) has Fourier transform \( F(\omega) \) and \( h(t) \) has Fourier transform \( H(\omega) \), use the Fourier transform properties to show that \( g(t) \) has the form \( g(t) = Ay(Bt) \). Determine the values of \( A \) and \( B \).

P4.10. Consider the Fourier transform pair: \( e^{-\alpha t} \leftrightarrow \frac{2}{1 + \omega^2} \)

a) Determine the Fourier transform of \( te^{-\alpha t} \)

b) Determine the Fourier transform of \( 4t/(1 + t^2)^2 \)

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P4.6. Determine the Fourier transform of the signal depicted in Figure P4.6

Figure P4.6

P4.7. Determine the Fourier transform of the signal depicted in Figure P4.7

Figure P4.7
**Ch-4: Fourier transform representation of signal**

**P4.11.** Consider the signal \( f(t) = \sum_{n=-\infty}^{\infty} \text{sinc} \left( \frac{n\pi}{4} \right) \delta(t - \frac{n\pi}{4}) \)

a) Determine \( g(t) \) such that \( f(t) = g(t) \int_{-\infty}^{\infty} \frac{\sin(t)}{\pi t} \)

b) Use the multiplication property of the Fourier transform to argue that \( F(\omega) \) is periodic. Specify \( F(\omega) \) over one period.

**P4.12.** Determine the continuous-time signal corresponding to each of the following transform.

a) \( F(\omega) = 2\sin[3(\omega - 2\pi)]/\omega - 2\pi) \)

b) \( F(\omega) = \cos(4\omega + \pi/3) \)

c) \( F(\omega) = 2[\delta(\omega - 1) - \delta(\omega + 1)] + 3[\delta(\omega - 2\pi) - \delta(\omega + 2\pi)] \)

d) \( F(\omega) \) as given by the magnitude and phase plots of Figure P4.12a

e) \( F(\omega) \) as in Figure P4.12b

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**P4.13.** Let \( F(\omega) \) denote the Fourier transform of the signal \( f(t) \) depicted in Figure P4.13.

a) Find \( \angle F(\omega) \)

b) Find \( F(0) \)

c) Find \( \int_{-\infty}^{\infty} F(\omega) d\omega \)

d) Evaluate \( \int_{-\infty}^{\infty} F(\omega) \frac{2 \sin \omega}{\omega} e^{i2\omega} d\omega \)

e) Evaluate \( \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega \)

f) Sketch the inverse Fourier transform of \( \text{Re}\{F(\omega)\} \)

**Note:** you should perform all these calculations without explicitly evaluating \( F(\omega) \)
P4.14. Find the impulse response of a system with the frequency response

\[ H(\omega) = \frac{(\sin^2(3\omega)) \cos \omega}{\omega^2} \]

P4.15. Consider a causal LTI system with frequency response 

\[ H(\omega) = \frac{1}{3 + j\omega} \]

For a particular input \( f(t) \) this system is observed to produce the output \( y(t) = e^{-3t}u(t) - e^{-4t}u(t) \). Determine \( f(t) \).

P4.16. Consider an LTI system \( S \) with impulse response

\[ h(t) = \frac{\sin[4(t-1)]}{\pi(t-1)} \]

Determine the output of this system for each of the following inputs:

a) \( f(t) = \cos(6t + \frac{\pi}{2}) \)

b) \( f(t) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \sin(3nt) \)

c) \( f(t) = \frac{\sin[4(t+1)]}{\pi(t+1)} \)

d) \( f(t) = \left(\frac{\sin 2t}{\pi t}\right)^2 \)

P4.17. The input and the output of a causal LTI system are related by the differential equation \( (D^2 + 6D + 8)y(t) = 2f(t) \)

a) Find the impulse response of this system.

b) What is the response of this system if \( f(t) = te^{-2t}u(t) \)?

c) Repeat part a) for the causal LTI system described by the equation \( (D^2 + \sqrt{2}D + 1)y(t) = (2D - 2)f(t) \)
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P4.18. Shown in Figure P4.18 is the frequency response \( H(\omega) \) of a continuous-time filter referred to as a low-pass differentiator. For each of the input signals \( f(t) \) below, determine the filtered output signal \( y(t) \).

a) \( f(t) = \cos(2\pi t + \theta) \)

b) \( f(t) = \cos(4\pi t + \theta) \)

c) \( f(t) = |\sin(2\pi t)| \)

![Figure P4.18](image)

Ch-4: Fourier transform representation of signal

P4.19. Shown in Figure P4.19 is \( |H(\omega)| \) for a low-pass filter. Determine and sketch the impulse response of the filter for each of the following phase characteristics:

a) \( \angle H(\omega) = 0 \)

b) \( \angle H(\omega) = \omega T \), where \( T \) is a constant

c) \( \angle H(\omega) = \begin{cases} \pi/2 & \omega > 0 \\ -\pi/2 & \omega < 0 \end{cases} \)

![Figure P4.19](image)

P4.20. Consider an ideal high-pass filter whose frequency response is specified as:

\[
H(\omega) = \begin{cases} 1 & |\omega| > \omega_c \\ 0 & \text{otherwise} \end{cases}
\]

a) Determine the impulse response \( h(t) \) for this filter

b) As \( \omega_c \) is increased, does \( h(t) \) get more or less concentrated about the origin?

c) Determine \( s(0) \) & \( s(\infty) \), where \( s(t) \) is the step response of the filter
Ch-4: Fourier transform representation of signal

P4.21. Figure P4.21 shows a system commonly used to obtain a high-pass filter from a low-pass filter and vice versa.

a) Show that, if $H(\omega)$ is a low-pass filter with cutoff frequency $\omega_{LP}$, the overall system corresponds to an ideal high-pass filter. Determine the system’s cutoff frequency and sketch its impulse response.

b) Show that, if $H(\omega)$ is a high-pass filter with cutoff frequency $\omega_{HP}$, the overall system corresponds to an ideal low-pass filter. Determine the system’s cutoff frequency and sketch its impulse response.

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Ch-4: Fourier transform representation of signal

P4.22. Let $f(t)$ be a real-valued signal for which $F(\omega)=0$ when $|\omega|>2000\pi$. Amplitude modulation is performed to produce the signal $g(t)=f(t)\sin(2000\pi t)$. A proposed demodulation technique is illustrated in Figure P4.22 where $g(t)$ is the input, $y(t)$ is the output, and the ideal lowpass filter has cutoff frequency $2000\pi$ and passband gain of 2. Determine $y(t)$. 

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P4.23. Suppose \( f(t) = \sin 200\pi t + 2\sin 400\pi t \) and \( g(t) = f(t)\sin 400\pi t \). If the product \( g(t)\sin 400\pi t \) is passed through an ideal low-pass filter with cutoff frequency \( 400\pi \) and pass-band gain of 2, determine the signal obtained at the output of the low-pass filter.

P4.24. Suppose we wish to transmit the signal

\[
f(t) = \frac{\sin 1000\pi t}{\pi t}
\]

using a modulator that creates the signal \( w(t) = [f(t) + A]\cos(10000\pi t) \). Determine the largest permissible value of the modulation index \( m \) that would allow asynchronous demodulation to be used to recover \( f(t) \) from \( w(t) \). For this problem, you should assume that the maximum magnitude taken on by a side lobe of a sinc function occurs at the instant of time that is exactly halfway between the two zero-crossings enclosing the side lobe.

P4.25. An AM-SSB/SC system is applied to a signal \( f(t) \) whose Fourier transform \( F(\omega) \) is zero for \( |\omega| > \omega_M \), the carrier frequency \( \omega_c \) used in the system is greater than \( \omega_M \). Let \( g(t) \) denote the output of the system, assuming that only the upper sidebands are retained. Let \( q(t) \) denote the output of the system, assuming that only the lower sidebands are retained. The system in Figure P4.25 is proposed for converting \( g(t) \) into \( q(t) \). How should the parameter \( \omega_0 \) in the figure be related to \( \omega_c \)? What should be the value of pass-band gain \( A \)?
Ch-4: Fourier transform representation of signal

**P4.26.** In Figure P4.26, a system is shown with input signal \( f(t) \) and output signal \( y(t) \). The input signal has the Fourier transform \( F(\omega) = \Delta(\omega/4\omega_0) \). Determine and sketch \( Y(\omega) \), the spectrum of \( y(t) \).

\[
\begin{align*}
\text{f(t)} & \rightarrow \times H_1(\omega) \rightarrow \times H_2(\omega) \rightarrow \text{y(t)} \\
& \uparrow \cos(5\omega_0 t) \uparrow \cos(3\omega_0 t)
\end{align*}
\]

Figure P4.26

Assume \( H_1(\omega) = \text{rect} \left( \frac{\omega - 4\omega_0}{2\omega_0} \right) + \text{rect} \left( \frac{\omega + 4\omega_0}{2\omega_0} \right) \)

and \( H_2(\omega) = \text{rect} \left( \frac{\omega}{6\omega_0} \right) \)

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Ch-4: Fourier transform representation of signal

**P4.27.** A commonly used system to maintain privacy in voice communication is a speech scrambler. As illustrated in Figure P4.27(a), the input to the system is a normal speech signal \( f(t) \) and the output is the scrambler version \( y(t) \). The signal \( y(t) \) is transmitted and then un-scrambler at the receiver.

We assume that all inputs to the scrambler are real and band limited to the frequency \( \omega_0 \); that is \( F(\omega) = 0 \) for \( |\omega| > \omega_0 \). Given any such input, our proposed scrambler permutes different bands of the spectrum of the input signal. In addition, the output signal is real and band limited to the same frequency band; that is \( Y(\omega) = 0 \) for \( |\omega| > \omega_0 \). The specific algorithm for the scrambler is

\[
Y(\omega) = \begin{cases} 
X(\omega - \omega_0); & \omega > 0 \\
X(\omega + \omega_0); & \omega < 0 
\end{cases}
\]
a) If \( F(\omega) \) is given by the spectrum shown in Figure P4.27(b), sketch the spectrum of the scrambled signal \( y(t) \).

b) Using amplifiers, multipliers, adders, oscillators, and whatever ideal filters you find necessary, draw the block diagram for such an ideal scrambler.

c) Again using amplifiers, multipliers, adders, oscillators, and ideal filters, draw a block diagram for the associated unscrambler.

P4.28. Figure P4.28(a) the system that to perform single-sideband modulation. With \( F(\omega) \) illustrated in Figure P4.28(b), sketch \( Y_1(\omega) \), \( Y_2(\omega) \), and \( Y(\omega) \) for the system in Figure P4.28(a), and demonstrate that only the upper-sidebands are retained.