Ch-2: Linear time-invariant systems

P2.1. Determine and sketch the convolution of the following signals:

\[ f(t) = \begin{cases} 
  t+1; & 0 \leq t \leq 1 \\
  2-t; & 1 < t \leq 2 \\
  0; & \text{elsewhere} 
\end{cases} \]

\[ h(t) = \delta(t+2) + 2\delta(t+1) \]

P2.2. Suppose that

\[ f(t) = \begin{cases} 
  1; & 0 \leq t \leq 1 \\
  0; & \text{elsewhere} 
\end{cases} \]

and

\[ h(t) = f\left(\frac{t}{\alpha}\right) \text{ where } 0 < \alpha \leq 1 \]

a) Determine and sketch \( y(t) = f(t) * h(t) \)?

b) If \( dy(t)/dt \) contains only three discontinuities, what is the value of \( \alpha \)?

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P2.3. Let:

\[ f(t) = u(t-3) - u(t-5) \]

and

\[ h(t) = e^{-3t}u(t) \]

a) Compute \( y(t) = f(t) * h(t) \)

b) Compute \( g(t) = [df(t)/dt] * h(t) \)

c) How is \( g(t) \) related to \( y(t) \)

P2.4. Let:

\[ h(t) = \Delta(t) \] and

\[ f(t) = \sum_{k=-\infty}^{+\infty} \delta(t-kT) \]

Determine and sketch \( y(t) = f(t) * h(t) \) for the following values of \( T \):

(a) \( T = 4 \); (b) \( T = 2 \); (c) \( T = 3/2 \); and (d) \( T = 1 \)

P2.5. Which of the following impulse response correspond(s) to stable LTI systems?

(a) \( h_1(t) = e^{-(t-j2)}u(t) \)

(b) \( h_2(t) = e^{-i\cos(2t)}u(t) \)
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**P2.6.** The following are the impulse response of continuous-time LTI systems. Determine whether each systems is causal and/or stable. Justify your answer.

(a) \( h(t) = e^{-4t}u(t - 2) \)  
(b) \( h(t) = e^{6t}u(3 - t) \)  
(c) \( h(t) = e^{2t}u(t + 50) \)

(d) \( h(t) = e^{3u}u(-1 - t) \)  
(e) \( h(t) = e^{6t}u(t) \)  
(f) \( h(t) = te^{-u(t)} \)

(g) \( h(t) = [2e^{-t} - e^{-(t-100)/100}]u(t) \)

**P2.7.** Consider a system whose input \( f(t) \) and output \( y(t) \) satisfy the first-order differential equation

\[
\frac{dy(t)}{dt} + 2y(t) = f(t)
\]

The system also satisfies the condition of initial rest.

(a) Determine the system output \( y_1(t) \) when the input is \( f_1(t) = e^{-3t}u(t) \)

(b) Determine the system output \( y_2(t) \) when the input is \( f_2(t) = e^{-2t}u(t) \)

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**P2.8.** Consider an LTI system with input and output related through the equation

\[
y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} f(\tau - 2) d\tau
\]

(a) What is the impulse response \( h(t) \) for this system?

(b) Determine the response of the system when the input \( f(t) \) is as shown in Figure P2.8

![Figure P2.8](image-url)
P2.9. (a) Show that if the response of an LTI system to \( f(t) \) is the output \( y(t) \), then the response of the system to \( f'(t) = \frac{df(t)}{dt} \) is \( y'(t) \)?

(b) An LTI system has response \( y(t) = \sin \omega_0 t \) to input \( f(t) = e^{-5t}u(t) \). Use the result of part (a) to aid in determining the impulse response of this system.

P2.10. Consider an LTI system and a signal \( f(t) = 2e^{-3t}u(t-1) \). If

\[
f(t) \rightarrow y(t)
\]

and

\[
\frac{df(t)}{dt} \rightarrow -3y(t) + e^{-2t}u(t)
\]

Determine the impulse response of this system.

P2.11. We are given a certain linear time-invariant system with impulse response \( h_0(t) \). We are told that when the input is \( f_0(t) \) the output is \( y_0(t) \), which is sketched in Figure P2.11. We are then given the following set of inputs \( f(t) \) to linear time-invariant systems with the indicated impulse response \( h(t) \):

(a) \( f(t) = 2f_0(t) \); \( h(t) = h_0(t) \)
(b) \( f(t) = f_0(t) - f_0(t-2) \); \( h(t) = h_0(t) \)
(d) \( f(t) = f_0(-t) \); \( h(t) = h_0(t) \)
(c) \( f(t) = f_0(t+2) \); \( h(t) = h_0(t+1) \)
(e) \( f(t) = f_0(-t) \); \( h(t) = h_0(-t) \)
(f) \( f(t) = f_0(t) \); \( h(t) = h_0(t) \)

In each of these cases, determine whether or not we have enough information to determine the output \( y(t) \) when the input is \( f(t) \) and the system has impulse response \( h(t) \). If it is possible to determine \( y(t) \), provide an accurate sketch of it with numerical values clearly indicated on the graph.
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P2.12. The input signal of the LTI system shown in Figure P2.12 is the following:

\[ f(t) = u(t) - u(t-2) + \delta(t+1) \]

The impulse responses of the subsystems are \( h_1(t) = e^{-t}u(t) \), and \( h_2(t) = e^{-2t}u(t) \).

a) Compute the impulse response \( h(t) \) of overall system
b) Find an equivalent system (same impulse response) configured as a parallel interconnection of two LTI systems.
c) Sketch the input signal \( f(t) \). Compute the output signal \( y(t) \)

Figure P2.12

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P2.13. Consider the following second-order, causal differential system initially at rest:

\[ \frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = f(t) \]

Calculate the impulse response \( h(t) \) of this system

P2.14. Consider the following second-order, causal differential system initially at rest:

\[ \frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + y(t) = f(t) \]

Calculate the impulse response \( h(t) \) of this system
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**P2.15.** Compute and sketch the impulse response $h(t)$ of the following causal LTI, first-order differential system initially at rest:

$$2 \frac{dy(t)}{dt} + 4y(t) = 3 \frac{df(t)}{dt} + 2f(t)$$

**P2.16.** Find the impulse response $h(t)$ of the following causal LTI, second-order differential system initially at rest:

$$\frac{d^2y(t)}{dt^2} + \sqrt{2} \frac{dy(t)}{dt} + y(t) = \frac{df(t)}{dt} + f(t)$$