### Ch-1: Elementary of signal and system

#### P1.1. A continuous-time signal $f(t)$ shown in Figure P1.1. Sketch and label carefully each of the following signals:

(a) $f(t-1)$  
(b) $f(2-t)$  
(c) $f(2t+1)$  
(d) $f(4-\frac{1}{2})$  
(e) $[f(t)+f(-t)]u(t)$  
(f) $f(t)[\delta(t+\frac{1}{2})-\delta(t-\frac{1}{2})]$  

![Figure P1.1](image1.png)

#### P1.2. The continuous-time signal $f(t)$ depicted in Figure P1.2. Sketch and label carefully each of the following signals:

(a) $f(t+3)$  
(b) $f(\frac{1}{2}-2)$  
(c) $f(1-2t)$  
(d) $4f(\frac{1}{2})$  
(e) $\frac{1}{2}f(t)u(t)+f(-t)u(t)$  
(f) $f(\frac{1}{2})\delta(t+1)$  
(g) $f(t)[u(t+1)-u(t-1)]$  

![Figure P1.2](image2.png)
P1.3. Consider the continuous-time signals $f(t)$ and $h(t)$ shown in Figure P1.3. Sketch an label carefully each of the following signals:

(a) $f(t)h(t+1)$
(b) $f(t)h(-t)$

![Figure P1.3](image)

P1.4. Find the energies of the signals illustrated in Figure P1.4. Comment on the effect on energy of sign change, time shifting, or doubling of the signal.

![Figure P1.4](image)
P1.5. (a) Find the energies of the pair signals \( x(t) \) and \( y(t) \) illustrated in Figure P1.5a and b. Sketch and find the energies of signals \( x(t)+y(t) \) and \( x(t)-y(t) \)? Can you make any conclusion from these results? (b) Repeat part (a) for signal pair illustrated in Figure P1.5c? Is the conclusion in part (a) still valid? Can you generalize condition of \( x(t) \) and \( y(t) \) that conclusion in part (a) always right?

![Figure P1.5](image)

P1.6. Determine power of the following signals:
\[ f_1(t)=C_1\cos(\omega_1 t+\theta_1), \quad f_2(t)=C_2\cos(\omega_2 t+\theta_2), \] and \( f_1(t)+f_2(t) \) in two cases: (a) \( \omega_1=\omega_2 \), and (b) \( \omega_1 \neq \omega_2 \)?

P1.7. Consider signal \( f(t) \) depicted in Figure P1.7. Find power of the following signals: \( f(t) \), \(-f(t)\), and \( 2f(t) \)?

![Figure P1.7](image)
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P1.8. Simplify the following expressions:
(a) \( \left( \frac{\sin t}{t^2+2} \right) \delta(t) \)
(b) \( \left( \frac{j\omega+2}{\omega^2+9} \right) \delta(\omega) \)
(c) \( e^{-t}\cos(3t-60^\circ) \delta(t) \)
(d) \( \left( \frac{\sin \left( \frac{\pi}{2} (t-2) \right)}{t^2+4} \right) \delta(t-1) \)
(e) \( \left( \frac{1}{j\omega+2} \right) \delta(\omega+3) \)
(f) \( \left( \frac{\sin(\omega)}{\omega} \right) \delta(\omega) \)

P1.9. Evaluate the following integrals:
(a) \( \int_{-\infty}^{\infty} \delta(t) f(t-\tau) d\tau \)
(b) \( \int_{-\infty}^{\infty} f(\tau) \delta(t-\tau) d\tau \)
(c) \( \int_{-\infty}^{\infty} \delta(t) e^{j\omega t} dt \)
(d) \( \int_{-\infty}^{\infty} \delta(t-2) \sin(\pi t) dt \)
(e) \( \int_{-\infty}^{\infty} \delta(t+3) e^{t} dt \)
(f) \( \int_{-\infty}^{\infty} \left( t^2+4 \right) \delta(1-t) dt \)
(g) \( \int_{-\infty}^{\infty} f(2-t) \delta(3-t) dt \)
(h) \( \int_{-\infty}^{\infty} e^{(t-1)} \cos \left[ \frac{\pi}{2} (x-5) \right] \delta(x-3) dx \)

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P1.10. Determine and sketch the even and odd parts of the signals depicted in Figure P1.10. Label your sketches carefully.

![Figure P1.10](image-url)
P1.11. Determine the even and odd parts of the following signals:
(a) \( u(t) \); (b) \( tu(t) \); (c) \( \sin(\omega_0 t)u(t) \); (d) \( \cos(\omega_0 t)u(t) \); (e) \( \sin\omega_0 t \); and
(f) \( \cos\omega_0 t \)?

P1.12. For the systems described by the equations below, with the input \( f(t) \) and output \( y(t) \), determine which of the system are linear and which are nonlinear.

(a) \( \frac{dy(t)}{dt} + 2y(t) = f^2(t) \)
(b) \( \frac{dy(t)}{dt} + 3ty(t) = t^2f(t) \)
(c) \( \left[ \frac{dy(t)}{dt} \right]^2 + 2y(t) = f(t) \)
(d) \( \frac{dy(t)}{dt} + y^2(t) = f(t) \)
(e) \( 3y(t) + 2 = f(t) \)
(f) \( y(t) = \int_{-\infty}^{t} f(\tau)d\tau \)
(g) \( \frac{dy(t)}{dt} + (\sin t)y(t) = \frac{df(t)}{dt} + 2f(t) \)
(h) \( \frac{dy(t)}{dt} + 2y(t) = f(t) \frac{df(t)}{dt} = f(t) \)

P1.13. For the systems described by the equations below, with the input \( f(t) \) and output \( y(t) \), determine which of the system are time-invariant parameter systems and which are time-varying parameter systems.

(a) \( y(t) = f(t-2) \)
(b) \( y(t) = f(-t) \)
(c) \( y(t) = f(at) \)
(d) \( y(t) = tf(t-2) \)
(e) \( y(t) = \int_{-\infty}^{t} f(\tau)d\tau \)
(f) \( y(t) = \left[ \frac{df(t)}{dt} \right]^2 \)

P1.14. For the systems described by the equations below, with the input \( f(t) \) and output \( y(t) \), determine which of the system are causal and which are noncausal.

(a) \( y(t) = f(t-2) \)
(b) \( y(t) = f(-t) \)
(c) \( y(t) = f(at); a > 1 \)
(d) \( y(t) = f(at); a < 1 \)
P1.15. Consider a continuous-time system with input \( f(t) \) and output \( y(t) \) related by: \( y(t) = f(t) \sin(t) \). (a) Is the system causal? (b) Is the system linear?

P1.16. In particular, a system may or may not be (1) Memoryless, (2) Time-invariant, (3) Linear, (4) Causal, (5) Stable. Determine which of these properties hold and which do not hold for each of the following continuous-time systems. Justify your answers. In each example, \( y(t) \) denotes the system output and \( f(t) \) is the system input.

(a) \( y(t) = f(t - 2) + f(2 - t) \)  
(b) \( y(t) = [\cos(3t)]f(t) \)  
(c) \( y(t) = \int_{-\infty}^{3t} f(\tau) d\tau \)  
(d) \( y(t) = \begin{cases} 0 & (t < 0) \\ f(t) + f(t - 2) & (t \geq 0) \end{cases} \)  
(e) \( y(t) = \begin{cases} 0 & [f(t) < 0] \\ f(t) + f(t - 2) & [f(t) \geq 0] \end{cases} \)  
(f) \( y(t) = f(\frac{t}{2}) \)  
(g) \( y(t) = \frac{df(t)}{dt} \)

P1.17. Consider the system shown in Figure P1.17a. Here the square root produces the positive square root. (a) Find an explicit relationship between \( y(t) \) and \( f(t) \)? (b) Is the system linear? (c) Is it time-invariant? (d) What is the response \( y(t) \) when the input is shown in Figure P1.17b?
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P1.18. Suppose that a LTI system has the following output \( y(t) \) when the input is the unit step \( f(t) = u(t) \): \( y(t) = e^{-t}u(t) + u(-1-t) \). Determine and sketch the response of the system to the input \( f(t) \) shown in Figure P1.18.

Figure P1.18

P1.19. A particular linear system has the property that the response to \( t^k \) is \( \cos(kt) \). What is the response of this system to the input:

\[
f(t) = \pi + 6t^2 - 47t^5 + \sqrt{e}t^6
\]

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P1.20. A continuous-time linear system with input \( f(t) \) and output \( y(t) \) yields the following input-output pairs:

\[
f(t) = e^{2t} \rightarrow y(t) = e^{j2t} \quad \text{and} \quad f(t) = e^{-j3t} \rightarrow y(t) = e^{-j3t}
\]

a) If \( f_1(t) = \cos(2t) \), determine the corresponding output \( y_1(t) \) for the system?
b) If \( f_2(t) = \cos(2(t-1/2)) \), determine the corresponding output \( y_2(t) \) for the system?