PROBLEM SOLUTIONS: Chapter 4

Problem 4.1

part (a): $\omega_m = 1200 \times \pi/30 = 40\pi \text{ rad/sec}$

part (b): 60 Hz; $120\pi \text{ rad/sec}$

part (c): $1200 \times 5/6 = 1000 \text{ r/min}$

Problem 4.2

The voltages in the remaining two phases can be expressed as $V_0 \cos (\omega t - 2\pi/3)$ and $V_0 \cos (\omega t + 2\pi/3)$.

Problem 4.3

part (a): It is an induction motor.

parts (b) and (c): It sounds like an 8-pole motor supplied by 60 Hz.

Problem 4.4

part (a):

part (b):
part (c):

![Diagram](image1)

part (d):

![Diagram](image2)

**Problem 4.5**
Under this condition, the mmf wave is equivalent to that of a single-phase motor and hence the positive- and negative-traveling mmf waves will be of equal magnitude.

**Problem 4.6**
The mmf and flux waves will reverse direction. Reversing two phases is the procedure for reversing the direction of a three-phase induction motor.

**Problem 4.7**

\[
\mathcal{F}_1 = F_{\text{max}} \cos \theta ae \cos \omega_e t = \frac{F_{\text{max}}}{2} (\cos (\theta ae - \omega t) + \cos (\theta ae + \omega t))
\]

\[
\mathcal{F}_2 = F_{\text{max}} \sin \theta ae \sin \omega_e t = \frac{F_{\text{max}}}{2} (\cos (\theta ae - \omega t) - \cos (\theta ae + \omega t))
\]

and thus

\[
\mathcal{F}_{\text{total}} = \mathcal{F}_1 + \mathcal{F}_2 = F_{\text{max}} \cos (\theta ae - \omega t)
\]
Problem 4.8
For \( n \) odd
\[
\left| \frac{\int_{-\beta/2}^{\beta/2} \cos (n\theta) d\theta}{\int_{-\pi/2}^{\pi/2} \cos (n\theta) d\theta} \right| = \sin \left( \frac{n\theta}{2} \right)
\]
For \( \beta = 5\pi/6 \),
\[
\sin \left( \frac{n\theta}{2} \right) = \begin{cases} 
0.97 & n = 1 \\
0 & n = 3 \\
0.26 & n = 5 
\end{cases}
\]

Problem 4.9
part (a): Rated speed = 1200 r/min
part (b):
\[
I_r = \frac{\pi g B_{ag1.\text{peak}}(\text{poles})}{4\mu_0 k_r N_r} = 113 \text{ A}
\]
part (c):
\[
\Phi_p = \left( \frac{2}{3} \right) l RB_{ag1.\text{peak}} = 0.937 \text{ Wb}
\]

Problem 4.10
From the solution to Problem 4.9, \( \Phi_p = 0.937 \text{ Wb.} \)
\[
V_{\text{rms}} = \frac{\omega N \Phi}{\sqrt{2}} = 8.24 \text{ kV}
\]

Problem 4.11
From the solution to Problem 4.9, \( \Phi_p = 0.937 \text{ Wb.} \)
\[
V_{\text{rms}} = \frac{\omega k_w N_s \Phi}{\sqrt{2}} = 10.4 \text{ kV}
\]

Problem 4.12
The required rms line-to-line voltage is \( V_{\text{rms}} = 13.0/\sqrt{3} = 7.51 \text{ kV.} \) Thus
\[
N_a = \frac{\sqrt{2} V_{\text{rms}}}{\omega k_w \Phi} = 39 \text{ turns}
\]

Problem 4.13
part (a): The flux per pole is
\[
\Phi = 2l RB_{ag1.\text{peak}} = 0.0159 \text{ Wb}
\]
The electrical frequency of the generated voltage will be 50 Hz. The peak voltage will be
\[ V_{\text{peak}} = \omega N \Phi = 388 \ \text{V} \]

Because the space-fundamental winding flux linkage is at its peak at time \( t = 0 \) and because the voltage is equal to the time derivative of the flux linkage, we can write

\[ v(t) = \pm V_{\text{peak}} \sin \omega t \]

where the sign of the voltage depends upon the polarities defined for the flux and the stator coil and \( \omega = 120\pi \ \text{rad/sec} \).

part (b): In this case, \( \Phi \) will be of the form

\[ \Phi(t) = \Phi_0 \cos^2 \omega t \]

where \( \Phi_0 = 0.0159 \ \text{Wb} \) as found in part (a). The stator coil flux linkages will thus be

\[ \lambda(t) = \pm N \Phi(t) = N\Phi_0 \cos^2 \omega t = \pm \frac{1}{2} N\Phi_0(1 + \cos 2\omega t) \]

and the generated voltage will be

\[ v(t) = \mp \omega \Phi_0 \sin 2\omega t \]

This scheme will not work since the dc-component of the coil flux will produce no voltage.

**Problem 4.14**

\[
\mathcal{F}_a = i_a[A_1 \cos \theta_a + A_3 \cos 3\theta_a + A_5 \cos 5\theta_a] \\
= I_a \cos \omega t[A_1 \cos \theta_a + A_3 \cos 3\theta_a + A_5 \cos 5\theta_a]
\]

Similarly, we can write

\[
\mathcal{F}_b = i_b[A_1 \cos (\theta_a - 120^\circ) + A_3 \cos 3(\theta_a - 120^\circ) + A_5 \cos 5(\theta_a - 120^\circ)] \\
= I_a \cos (\omega t - 120^\circ)[A_1 \cos (\theta_a - 120^\circ) + A_3 \cos 3\theta_a + A_5 \cos (5\theta_a + 120^\circ)]
\]

and

\[
\mathcal{F}_c = i_c[A_1 \cos (\theta_a + 120^\circ) + A_3 \cos 3(\theta_a + 120^\circ) + A_5 \cos 5(\theta_a + 120^\circ)] \\
= I_a \cos (\omega t + 120^\circ)[A_1 \cos (\theta_a + 120^\circ) + A_3 \cos 3\theta_a + A_5 \cos (5\theta_a - 120^\circ)]
\]

The total mmf will be
\[ F_{\text{tot}} = F_a + F_b + F_c \]
\[ = \frac{3}{2} I_a [A_1 \cos (\theta_a - \omega t) A_5 \cos (5\theta_a + \omega t)] \]
\[ = \frac{3}{2} I_a [A_1 \cos (\theta_a - \omega t) A_5 \cos 5\left(\theta_a + \frac{\omega t}{5}\right)] \]

We see that the combined mmf contains only a fundamental space-harmonic component that rotates in the forward direction at angular velocity \(\omega\) and a 5'th space-harmonic that rotates in the negative direction at angular velocity \(\omega/5\).

**Problem 4.15**
The turns must be modified by a factor of
\[
\left(\frac{18}{24}\right) \left(\frac{1200}{1400}\right) = \frac{9}{14} = 0.64
\]

**Problem 4.16**
\[
\Phi_p = \frac{30E_a}{N(\text{poles})n} = 6.25 \text{ mWb}
\]

**Problem 4.17**
part (a):
\[
\Phi_p = \left(\frac{2}{\text{poles}}\right) 2B_{\text{peak}} lr = \left(\frac{2}{4}\right) \times 2 \times 1.25 \times 0.21 \times (0.0952/2) = 12.5 \text{ mWb}
\]
\[
N_{\text{ph}} = \frac{V_{\text{rms}} \times \text{poles}}{\sqrt{2} \pi f_m e_k \Phi_p} = \frac{(230/\sqrt{3}) \times 4}{\sqrt{2} \times 60 \times 0.925 \times 0.0125} = 43 \text{ turns}
\]
part (b): From Eq. B.27
\[
L = \frac{16\mu_0 l \sigma}{\pi g} \left(\frac{k_w N_{\text{ph}}}{\text{poles}}\right)^2 = 21.2 \text{ mH}
\]

**Problem 4.18**
part (a):
\[
\Phi_p = \frac{V_{\text{rms}}}{\sqrt{2} \pi N_{\text{ph}}} = 10.8 \text{ mWb}
\]
\[
B_{\text{peak}} = \frac{\Phi_p}{2lr} = 0.523 \text{ T}
\]
part (b):

\[ I_f = \frac{\pi B_{\text{peak}} \theta}{2 \mu_0 k_r N_r} = 0.65 \text{ A} \]

part (c):

\[ L_{\text{af}} = \frac{L_{a,\text{peak}}}{I_f} = \frac{\sqrt{2} V_{\text{rms}}/\omega}{I_f} = 0.69 \text{ H} \]

**Problem 4.19**

No numerical solution required.

**Problem 4.20**

\[ \Phi_{\text{peak}} = \left( \frac{2 D l}{\text{poles}} \right) B_{\text{peak}} \]

\[ F_{r,\text{peak}} = \frac{4 k_r N_r I_{r,\text{max}}}{\pi \times \text{poles}} \]

\[ T_{\text{peak}} = \frac{\pi}{2} \left( \frac{\text{poles}}{2} \right)^2 \Phi_{\text{peak}} F_{r,\text{peak}} = 4.39 \times 10^6 \text{ N} \cdot \text{m} \]

\[ P_{\text{peak}} = T_{\text{peak}} \omega_m = 828 \text{ MW} \]

**Problem 4.21**

\[ \Phi_{\text{peak}} = \left( \frac{2 D l}{\text{poles}} \right) B_{\text{peak}} \]

\[ F_{r,\text{peak}} = \frac{4 k_r N_r I_{r,\text{max}}}{\pi \times \text{poles}} \]

\[ T_{\text{peak}} = \frac{\pi}{2} \left( \frac{\text{poles}}{2} \right)^2 \Phi_{\text{peak}} F_{r,\text{peak}} = 16.1 \text{ N} \cdot \text{m} \]

\[ P_{\text{peak}} = T_{\text{peak}} \omega_m = 6.06 \text{ kW} \]

**Problem 4.22**

part (a):

\[ T = i_a \frac{d M_{a f}}{d \theta} + i_b \frac{d M_{b f}}{d \theta} = M i_f (i_b \cos \theta_0 - i_a \sin \theta_0) \]
This expression applies under all operating conditions.

part (b): 

\[ T = 2M \dot{I}_0^2 (\cos \theta_0 - \sin \theta_0) = 2\sqrt{2} M \dot{I}_0^2 \sin (\theta_0 - \pi/4) \]

Provided there are any losses at all, the rotor will come to rest at \( \theta_0 = \pi/4 \) for which \( T = 0 \) and \( dt/d\theta_0 < 0 \).

part (c): 

\[ T = \sqrt{2} M \dot{I}_a I_f (\sin \omega t \cos \theta_0 - \cos \omega t \sin \theta_0) \]
\[ = \sqrt{2} M \dot{I}_a I_f (\omega t - \theta_0) = \sqrt{2} M \dot{I}_a I_f \sin \delta \]

part (d): 

\[ v_a = R_a i_a + \frac{d}{dt} (L_{aa} i_a + M_{af} i_f) \]
\[ = \sqrt{2} \dot{I}_a (R_a \cos \omega t - \omega L_{aa} \sin \omega t) - \omega M I_f \sin (\omega t - \delta) \]

\[ v_b = R_a i_b + \frac{d}{dt} (L_{aa} i_b + M_{bf} i_f) \]
\[ = \sqrt{2} \dot{I}_a (R_a \sin \omega t + \omega L_{aa} \cos \omega t) + \omega M I_f \cos (\omega t - \delta) \]

**Problem 4.23**

\[ T = MI_f (i_b \cos \theta_0 - i_a \sin \theta_0) \]
\[ = \sqrt{2} MI_f [(I_a + I'/2) \sin \delta + (I'/2) \sin (2\omega t + \delta)] \]

The time-averaged torque is thus

\[ <T> = \sqrt{2} MI_f (I_a + I'/2) \sin \delta \]

**Problem 4.24**

part (a): 

\[ T = \frac{i_a^2}{2} \frac{dL_{aa}}{d\theta_0} + \frac{i_b^2}{2} \frac{dL_{bb}}{d\theta_0} + i_a i_b \frac{dL_{ab}}{d\theta_0} + i_a i_i \frac{dM_{ai}}{d\theta_0} + i_b i_i \frac{dM_{bi}}{d\theta_0} \]
\[ = \sqrt{2} I_a I_f M \sin \delta + 2I_a^2 L_2 \sin 2\delta \]

part (b): Motor if \( T > 0, \delta > 0 \). Generator if \( T < 0, \delta < 0 \).

part (c): For \( I_f = 0 \), there will still be a reluctance torque \( T = 2I_a^2 L_2 \sin 2\delta \) and the machine can still operate.
Problem 4.25
part (a):

\[ v = \frac{f}{\lambda} = 25 \text{ m/sec} \]

part (b): The synchronous rotor velocity is 25 m/sec.
part (c): For a slip of 0.045, the rotor velocity will be \((1 - 0.045) \times 25 = 23.9 \text{ m/sec}\).

Problem 4.26

\[ I_{\text{rms}} = \frac{B_{\text{peak}}}{\sqrt{2}} \left( \frac{g}{\mu_0} \right) \left( \frac{2}{3} \right) \left( \frac{\pi}{4} \right) \left( \frac{2p}{k_w N_{\text{ph}}} \right) \]

\[ = \frac{1.45}{\sqrt{2}} \left( \frac{9.3 \times 10^{-3}}{\mu_0} \right) \left( \frac{2}{3} \right) \left( \frac{\pi}{4} \right) \left( \frac{2 \times 70.91 \times 280}{0.91 \times 280} \right) = 218 \text{ A} \]

Problem 4.27
part (a): Defining \( \beta = 2\pi/\text{wavelength} \)

\[ \Phi_p = w \int_0^{\pi/\beta} B_{\text{peak}} \cos \beta x dx = \frac{2wB_{\text{peak}}}{\beta} = 1.48 \text{ mWb} \]

part (b): Since the rotor is 5 wavelengths long, the armature winding will link 10 poles of flux with 10 turns per pole. Thus, \( \lambda_{\text{peak}} = 100\Phi_p = 0.148 \text{ Wb} \).
part (c): \( \omega = \beta v \) and thus

\[ V_{\text{rms}} = \frac{\omega \lambda_{\text{peak}}}{\sqrt{2}} = 34.6 \text{ V, rms} \]