Problem 1: (2.5 points) Calculate the equivalent transfer function \( G_N(s) = \frac{Y(s)}{N(s)} \) of the control system shown in the figure 1.

Solution:
- Signal flow graph (0.5 điểm) (other signal flow graphs are also accepted)

- Forward path from N(s) to Y(s): (0.5 point) 
  \[ P_1 = -G_2G_3 \]
  \[ P_2 = G_6 \]

- Loop: (0.5 point)
  \[ L_1 = -G_2G_4G_1 \]
  \[ L_2 = -G_3G_5 \]
  \[ L_3 = -G_2G_3G_1 \]

- Determinant and cofactors: (0.5 point)
  \[ \Delta = 1 - (L_1 + L_2 + L_3) + L_1L_2 \]
  \[ \Delta_1 = 1 \]
  \[ \Delta_2 = 1 - L_1 \]

- The equivalent transfer function from N(s) to Y(s): (0.5 point)
  \[
  G_N(s) = \left. \frac{Y(s)}{N(s)} \right|_{R(s)=0} = \frac{1}{\Delta} \left( P_1\Delta_1 + P_2\Delta_2 \right)
  \]
  \[
  = \frac{1}{1 - (-G_2G_4G_1 - G_3G_5 - G_2G_3G_1) + G_1G_2G_3G_4G_5} (-G_2G_3 + G_6(1 + G_2G_4G_1))
  \]
**Problem 2:** (1.5 points - 2.0 points) Write the state equations of the control system with the pre-specified state variables in the Figure 2:

![Figure 2](image)

**Solution:**
From the block diagram, we have the following relationship:

\[
X_1(s) = \frac{1}{s+2}(X_2(s) - X_3(s)) \quad \Rightarrow \quad \dot{x}_1(t) = -2x_1(t) + x_2(t) - x_3(t) \quad (0.5 \text{ point})
\]

\[
X_2(s) = \frac{2}{s+5}(R(s) - X_1(s)) \quad \Rightarrow \quad \dot{x}_2(t) = -2x_1(t) - 5x_2(t) + 2r(t) \quad (0.5 \text{ point})
\]

\[
X_3(s) = \frac{3}{s+1}X_1(s) \quad \Rightarrow \quad \dot{x}_3(t) = 3x_1(t) - x_3(t) \quad (0.5 \text{ point})
\]

The state space equation in matrix form:

\[
\dot{x}(t) = \begin{bmatrix} -2 & 1 & -1 \\ -2 & -5 & 0 \\ 3 & 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} r(t)
\]

\[
y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x(t)
\]

**Problem 3:** (3.0 points) Given a unity negative feedback control system with the open loop TF:

\[
G(s) = \frac{K(s+1)}{s(s+10)}
\]

Draw the root locus of the system when \(0 \leq K < +\infty\). Give your comment about the stability of the closed loop control system based on the root locus.

**Solution:**
+ Poles: \(p_1 = p_2 = 0; p_3 = -10\) (0.5 point)
+ Zeros: \(z_i = -1\);

+ Break-in/break-away points:

\[
K = -\frac{s^3 + 10s^2}{s+1}
\]

\[
\frac{dK}{ds} = 0 \Leftrightarrow (3s^2 + 20s)(s+1) - s^3 - 10s^2 = 0
\]

\[
\Leftrightarrow 2s^3 + 13s^2 + 20s = 0
\]

\[
\Leftrightarrow \begin{cases} s = 0 \quad (\text{accepted}) \\ s = -2.5 \quad (\text{accepted}) \\ s = -4 \quad (\text{accepted}) \end{cases}
\]


Asymptotes:
+ Angles: \( \frac{180}{3-2} = \pm 90^\circ \);
+ Intersection with the real axis: \( OA = \frac{0 + 0 - 10 + 1}{3 - 1} = -4.5 \)  
\( \{ \) (0.5 point)

Root locus: (1 point)

\[ \begin{align*}
\text{Problem 4: (3.0 points)} & \quad \text{Given the control system in the Figure 3.} \\
& \quad \\
& \quad \text{4.1 With } K_C = 1, \text{ plot the Bode diagram (magnitude and phase) of } G(s) \text{ in the frequency range } [0.01 \rightarrow 20\text{rad/s}]. \text{ Determine the gain margin and phase margin of the open loop system. Is the closed-loop system stable? Why?} \\
& \quad \text{4.2 Find the range of } K_C \text{ so that the closed loop system is stable.} \\
& \quad \\
& \quad \text{Solution:} \\
& \quad \text{4.1 With } K_C = 1, \text{ the open loop transfer function is:} \\
& \quad G(s) = \frac{100(s + 0.5)e^{-0.1s}}{s^2(s + 10)(s + 5)} \quad \Leftrightarrow \quad G(s) = \frac{(1s + 1)e^{-0.1s}}{s^2(0.1s + 1)(0.2s + 1)} \\
& \quad \end{align*} \]
Corner frequencies: $\omega_1 = 0.5 \text{ (rad/s)}$, $\omega_2 = 5 \text{ (rad/s)}$, $\omega_3 = 10 \text{ (rad/s)}$  

Point A:  

$$A = \begin{cases} \omega_0 = 0.01 \text{ (rad/s)} \\ L(\omega_0) = 20 \log(1) - 2 \log(0.01) = 80 \text{ dB} \end{cases}$$  

Phase formula:  

$$\varphi(\omega) = -180 + \arctan(2\omega) - \arctan(0.1\omega) - \arctan(0.2\omega) - 0.1 \omega$$  

Phase table:  

<table>
<thead>
<tr>
<th>$\omega$ (rad/s)</th>
<th>0.1</th>
<th>0.5</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi(\omega)$ (deg)</td>
<td>-171</td>
<td>-146</td>
<td>-139</td>
<td>-196</td>
<td>-259</td>
<td>-345</td>
</tr>
</tbody>
</table>

Bode diagram (1.0 point)

Gain crossover frequency: $\omega_c = 2 \text{ (rad/s)}$,  
Phase crossover frequency: $\omega_{\pi} = 2 \text{ (rad/s)}$  
Gain margin: $GM = 5 \text{ (dB)}$, phase margin: $\Phi M = 30^0$  
Using Bode criterion, we can conclude that the closed-loop system is stable. (0.5 point)

4.2 (0.5 point)  

Critical gain: $K_C = 10^{\frac{GM}{20}} = 10^{\frac{5}{20}} = 1.8$  
$\Rightarrow 0 < K_C < 1.8$  

(End)