The Causal Bond Graph: 
an efficient tool for energetic system design

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A. Methodological issues : the Causal Bond Graph

1) Introduction: general concepts for energy system modelling
2) Basics of Bond-Graphs
3) Basic components of Bond-Graphs
4) Construction of Bond Graphs in electricity, mechanics and hydraulics
5) Multidisciplinary examples : Electro-Hydraulic Actuator, Photovoltaic generator,…
6) Causal properties in the Bond Graphs : physical meaning, mathematical issues
7) From the causal Bond Graph till system analysis : state equation, transfer function derivation from causal paths

B. Applications : the Bond Graph in electrical engineering

1) Modelling of switching cells & static converters in power electronics
2) Modelling of electromechanical (electrical machines) & electrochemical devices
3) Examples of systems in electrical engineering : hybrid systems for renewable energy
Introduction: general concepts for energy system modelling

**Context of the system design**
- Living in a more and more complex world (even for energy systems)
- Increasing demand of economic competitiveness (world global market)
  » reduction of development costs: virtual prototyping
  » more and more innovation, more and more quickly
- Associating multidisciplinary principles: Electricity, Electronics, Mechanics, Thermics, Hydraulics, Chimics Automation, Computer sciences

**Example of automotive**
» Harsh evolution in terms of organisation: creating multi competence technical framework for given functions (Braking, power network,...)

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**Design cycle**

- **1990**
  - 5 ans
  - 5-10%

- **2000**
  - 3 ans
  - 20-30%

**rate of electronic devices in automotive**

→ **Mechatronics, integrated design, system approach**

✓ **need of system oriented design method**
A typical example of integrated design in a multi-disciplinary context: « reverse osmosis desalination unit powered by PV-Wind hybrid system »

Multi-disciplinary approach: thinking by energetic « analogies »
High level couplings
« top down » & global approach
Integrated design: Architecture - Sizing - Management
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2) Basics on Bond Graphs (BGs)

- **Bond-Graphs origin**
  - Invented at MIT (Boston) by H. Paynter (61), formalised by D. Karnopp et R. Rosenberg
  - Arrived in Europe end of 70 (Pays Bas, Twente), France (Alstom)

- **Used in industry by**:
  - Automotive (PSA, Renault, Ford, Toyota, EDF, Thomson, CEA, Airbus, GM, Hélion, CNES,...)

- **Main Characteristics and potentialities**
  - Modeling of Power/Energy transfers => universal language
  - Unified formalism for any physical domain (based on energetic analogies)
  - BGs describe system architecture : based on localized parameter modeling
  - BGs Facilitate functional & structural decomposition of complex systems : word BG
  - “Causal property inside” : a prime importance for energy systems
    - Derivation of Mathematical Structure (transfer functions, state equations)
    - Warning on conflicts of association
    - Structural Analysis (observability, controllability, stability), modal analysis (model reduction)
2) Basics on Bond Graphs (BGs)

- **BGs approach**: for any physical system energy is continuous & power is conservative

\[
P = e \cdot f
\]

Positive way:
\[
P = V \cdot I
\]
\[
P = F \cdot V
\]

- Paynter’s tetrahedron

\[
P = v \cdot i
\]
\[
P = F \cdot V
\]
\[
P = e \cdot f = v \cdot i
\]
\[
E = p \cdot q = \phi \cdot q = \int_0^t e(\tau) \cdot f(\tau) \, d\tau
\]
## 2) Basics on Bond Graphs (BGs)

The Bond Graph: an homogeneous & multi domain modeling approach

<table>
<thead>
<tr>
<th>Domains</th>
<th>Effort (e)</th>
<th>Flux (f)</th>
<th>Moment (p)</th>
<th>Déplacement (q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity</td>
<td>voltage (V)</td>
<td>Current (A)</td>
<td>magnetic flux (Wb)</td>
<td>Charge (C)</td>
</tr>
<tr>
<td>Mech Translation</td>
<td>Force (N)</td>
<td>speed (m/s)</td>
<td>Impulsion (Ns)</td>
<td>Displacement (m)</td>
</tr>
<tr>
<td>Mech Rotation</td>
<td>Torque (N.m)</td>
<td>speed (Rd/s)</td>
<td>Impulsion (Nms)</td>
<td>Angle (Rd)</td>
</tr>
<tr>
<td>Hydraulics</td>
<td>Pressure (N/m2)</td>
<td>Vol flow (m³/s)</td>
<td>impulsion of P</td>
<td>Volume (m³)</td>
</tr>
<tr>
<td>Magnetic</td>
<td>M.M.F (A)</td>
<td>flux derivative (V)</td>
<td>xxxx</td>
<td>Flux (Wb)</td>
</tr>
<tr>
<td>Chemical</td>
<td>Chemical Potential</td>
<td>Molar Flux</td>
<td>xxxx</td>
<td>Molar mass</td>
</tr>
<tr>
<td>Thermodynamics</td>
<td>Temperature</td>
<td>entropic Flux</td>
<td>xxxx</td>
<td>entropy</td>
</tr>
<tr>
<td>Acoustics</td>
<td>Pressure (N/m2)</td>
<td>Vol flow (m³/s)</td>
<td>Impulsion</td>
<td>Volume (m³)</td>
</tr>
</tbody>
</table>
2) Basics on Bond Graphs (BGs)

- Word Bond Graph: functional & Structural decomposition
  - allows fixing interfaces

- Power Bonds
- Information signals

Wind turbine: $C = \Delta V I$

Generator: $V = I$

DC Bus: $0 = V I$

Motor: $C = \Delta V I$

Pump: $P = Q$

PV cell array: $V = I$

Accumulator: $T = \dot{S}$

Hydraulics
- Pressure flow

Chimics
- Enthalpy flux

Electricity
- Voltage
- Current

Mechanics
- Torque, Force
- Speed

5 domains, 8 inter-domain crossings
3) Basic elements of BGs (monoport case)

Monoports Basic elements

<table>
<thead>
<tr>
<th>Effort Source</th>
<th>Se : E(t)</th>
<th>Flux Source</th>
<th>Sf : F(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>e(t)</td>
<td>f</td>
<td>e(t)</td>
<td>f</td>
</tr>
</tbody>
</table>

R element: generic relation between effort \% flux: \( R(e,f) = 0 \)

- Linear case: \( e = R_1 f \)
- Electricity: resistance: \( v = R i \)
- Mechanics: friction, damping effect: \( T = F \Omega \) (rotation)
- Hydraulics: restriction, load loss: \( P = R_1 |Q| \)
3) Basic elements of BGs (monoport case)

Monoports Basic elements

C element (potential storage) : relation effort % flux integral \( C(e,q) = 0 \)
- Linear case : \( q = C1.e \)
- Electricity: \( q = C1.v \); \( V = (1/C) \int i.dt \)
- Mechanics: \( F = k.(x1-x2) = k \int (v1-v2).dt \)
- Hydraulics: tank storage \( Q = C \cdot (dP/dt) \)

I element (kinetic storage) : relation moment % flux: \( I(p,f) = 0 \)
- Linear case: \( p = I1.f = \int e.dt \)
- Electricity: \( \phi = L.i ; v = L.d\phi/dt \) \((I1 = L)\)
- Mechanics \( T = J.d\phi/dt \); \( F = m.dv/dt \)

Electrical equivalence
3) Basic elements of BGs (mono-port case)

Junctions (power conservative)

**O Junction: Common effort**

\[
\begin{align*}
e_1 & = e_2 = e_3 \\
\sum (input \ flux) & = \sum (output \ flux)
\end{align*}
\]

Electricity: circuit nodes
Hydraulics: pressure nodes

**1 Junction: Common flux**

\[
\begin{align*}
e_1 & = e_2 + e_3 \\
\sum (input \ efforts) & = \sum (output \ efforts)
\end{align*}
\]

Electricity: voltage drop, circuit mesh
Mechanics: speed nodes
Hydraulics: pressure drop
3) Basic elements of BGs (mono_port case)

Junctions (power conservative)

- Transformer junction : TF
  
  ![Transformer Diagram]

  
  \[ e_1 \rightarrow m \rightarrow e_2 \]

  \[ f_1 \rightarrow TF \rightarrow f_2 \]

  \[ e_1 = m.e_2 \]

  \[ f_2 = m.f_1 \]

  electricity : transformer, « switching cell »

  mechanics : pulley, gearing \((R1.\Omega_1 = R2.\Omega_2)\)

  hydraulics : jack \((P=F/S, \ Q=S.V)\)

- Gyrator junction : GY
  
  ![Gyrator Diagram]

  \[ e_1 \rightarrow r \rightarrow e_2 \]

  \[ f_1 \rightarrow GY \rightarrow f_2 \]

  \[ E = \phi. \Omega \]

  \[ C_{em} = \phi.I \]

  \[ e_1 = r.f_2 \]

  \[ e_2 = r.f_1 \]

  electricity : hall effect sensor

  electromechanical transformation
3) Basic elements of BGs (mono-port case)

- Detectors: link between energy / signal parts (links with control unit)
  No puissance involved                        information link →
  \[ e \rightarrow D_e \quad e = 0 \rightarrow D_f \]
  \[ f = 0 \quad f \]

- Multiports: representation of coupled circuits, coupled energy (thermodynamics),
  Vector model (ex: 3D mechanics)

  Example of a I multiport: coupled inductive circuit
  \[
  E (\phi) = \frac{1}{2} \phi^T \begin{bmatrix}
  L1 & M12 & M13 \\
  M21 & L2 & M23 \\
  M31 & M32 & L3
  \end{bmatrix} \begin{bmatrix}
  i1 \\
  i2 \\
  i3
  \end{bmatrix}
  \]

  \[ I: [L] \quad I: [L] \quad I: [L] \]
4) Rules for establishing BGs: electricity domain
"the academic way"

1. Define a positive convention for currents in each element of the graph
2. Choose a reference potential
3. Choose a number for each circuit node and represent it on the BG by a 0 junction
4. Represent voltage drops across each element by introducing a 1 junction with 3 bonds, two being linked to the 0 junctions in the neighborhood of each considered element
5. Place elements (R,C,I) on the free bonds of the 1 junctions
6. Simplification of the BG: eliminate all 0 junctions linked with reference potentials. Then, suppress all free bonds linked with those reference nodes. Finally, eliminate all 0 and 1 junctions with only 2 bonds without power inversion:

![Diagram of a circuit with nodes and elements]
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[Diagram showing a circuit with elements R, C, and I, and nodes N0, N1, and N2, with annotations for simplification steps.]
4) Rules for establishing BGs: electricity domain
"the branch & mesh way"

1. Consider the 2 branches in // (C,L) as a macro component $Z_{L//C}$.

2. Represent the mesh (series connection) $E$, $Ra$, $Z_{L//C}$ by a 1 junction 1 (iso-current) on which one connects those 3 elements.

3. Represent by a 0 junction (iso-voltage) the 3 parallel branches $(E,Ra)$ // $C$ // $L$.

![Diagram](image-url)
4) Rules for establishing BGs: mechanical domain (adapt for mechanical rotation)

1. Orientate the translation axis (position & speed). The orientation of power bonds with follow this orientation.

2. For each connection between elements, affect an absolute speed vector and associate a 1 junction. Place I elements on each 1 junction, then affect force sources (don’t forget gravity force for vertical axis moving) and speed sources.

3. Between two successive 1 junctions related to different speeds, insert a 0 junction with three bonds oriented to make appear the correct relative speed on the free bond. Place R & C elements on this free bond. If several elements have the same relative speed, insert a 1 junction on the free bond of the 0 junction then insert these elements.

4. Simplification of the BG: eliminate all 1 junctions with null speed (mechanical reference). Then simplify the BG by eliminating all junctions with only 2 bonds with the same orientation.

\[
\dot{V}_M(t) - M \frac{dV_M}{dt} - F_K - F_\delta = 0 \quad (\sum (e_{sortant} - e_{rentrant}) = 0)
\]

\[
J \ 0 \ (C : 1 / K) : V_M - (V_M - 0) - 0 = 0 ; \quad F_K = K \int (V_M - 0) \ dt
\]
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\[ J_1(V_M) : F(t) - M \frac{dV_M}{dt} - F_K - F_\delta = 0 \quad \left( \sum (e_{sortant} - e_{rentran}) = 0 \right) \]

\[ J \ 0 \ (C : 1 / K) : V_M - (V_M - 0) - 0 = 0 \quad ; \quad F_K = K \int (V_M - 0) \, dt \]
4) Rules for establishing BGs: electromechanical domain

4.3) Electromechanical conversion

\[ U = R_m I_m + L_m \frac{dI_m}{dt} + \Phi \Omega_m \]

\[ T_m = \Phi I_m = F_m \Omega_m + J_m \frac{d\Omega_m}{dt} \]

\[ E_m = \Phi \Omega_m \]

\[ T_m = \Phi I_m \]

\[ (\Omega=0) \]

\[ R: R_m \]

\[ I: L_m \]

\[ I: J_m \]

\[ T_{load} \]
4) Rules for establishing BGs: electricity vs mechanical

- Energetic analogies: Electricity
  \[ \frac{1}{2} C V^2 \]
  \[ \frac{1}{2} L I^2 \]
- Mechanics
  \[ \frac{1}{2} K x^2 = \frac{1}{2} \left( \frac{1}{K} \right) F^2 \]

- Kirchoff analogies: Electricity
  \[ \Sigma I (\text{elec node}) = 0 \]
- Mechanics
  \[ \Sigma F (\text{mec node}) = 0 \]
4. One electrical example: « do it yourself »
4. Photovoltaic cell example: « do it yourself »

\[ I_D = I_s \exp\left( \frac{V_D}{K.T} \right) - 1 \]
4) Rules for establishing BGs: hydraulic domain

Rules for BG setting (very close to electrical domain)
- Set a way of flow define as positive way of power;
- Seek all nodes with different pressures and place a 0 junction;
- Place a 1 junction between two 0 junctions to set the pressure drop and place elements with the corresponding pressure difference;
- Choose a reference pressure (generally $P_{\text{atmosphere}}$) and delete the associated 0 junctions with all linked bonds. Then, simplify the BG:

Example: system “pump-jack” of an EHA

- **Volumetric pump**
  \[
  \frac{T_p}{\Omega} \cdot \frac{D}{TF} \frac{\Delta P_p}{Q_p} \cdot Q_p(m^3/s) = D (m^3). \Omega (Rd/s) \]
  \[
  T_p = D. \Delta P_p
  \]

- **Hydraulic jack**
  \[
  \frac{\Delta P_v}{V_v} \cdot \frac{S^{-1}}{TF} \frac{\Delta F}{V_v} \cdot \Delta P_v (N/m^2) = \Delta F(N)/S(m^2)
  \]
  \[
  Q(m^3/s) = S(m^2). V_v(m/s)
  \]
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Example: system “pump-jack” of an EHA

• Simplified 2 lines BG
4) *Rules for establishing BGs: hydraulic domain*

Rules for BG setting (very close to electrical domain)
- Set a way of flow define as positive way of power;
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Example: system “pump-jack” of an EHA

![Diagram of Volumetric Pump and Hydraulic Jack]

- Simplified 1 line BG

$$\begin{align*}
S_f: \Omega(t) & \frac{\Delta T_p}{\Omega} \frac{D}{TF} \frac{\Delta P_p}{Q} & I & \frac{\Delta P_v}{Q} \frac{S}{TF} \frac{\Delta F}{V_v} & \text{load}
\end{align*}$$
4) Physical BG of an EHA
4) Physical BG of an EHA

Hydraulic part

Validated in static and dynamic modes
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5. Causality in Bond-Graphs

- Characteristics of causality in energetic systems (one of major interest of BGs!)
  - Cause - effect relationships shown off;
  - Analysis of interactions (causal path and loops);
  - Display warnings when non physical (energetically) associations
  - Possibilities of systemic analysis: mode (simplification of models) &
  structural analysis (controllability, observability,…);
  - structure of mathematical equations:
    - integral (physical) causality: Ordinary Differential state Equations;
    - existence of derivative causality: Algebro Differential state Equation;
    - systematic equation derivation (state equations, transfer function/matrix)
5. Causality in Bond-Graphs

- Convention: causal stroke
A imposes an effort (e) to B, whose effect sets the flow f towards A

\[ A \xrightarrow{e} B \Rightarrow A \xleftarrow{f} B \]

Causality stroke: “close to the element for which the effort is a given data”

B imposes an effort (e) to A, whose effect sets the flow f towards B

\[ B \xrightarrow{e} A \Rightarrow B \xleftarrow{f} A \]

Do not confuse causality way With power transfer orientation!

Accumulator

\[ V \text{ is imposed} \]

Effort, V

\[ \Sigma \xrightarrow{Flux, I} \]

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5. Causality in Bond-Graphs

- SCAP procedure (Sequential Causality Assignment Procedure)

- Mandatory causality for sources

\[
\text{Se: } U \xrightarrow{e} I \quad (U > 0 \text{ ou } U < 0) \\
U \xrightarrow{i} \text{Se: } U \xrightarrow{e} I \\
U \xrightarrow{i} \text{Se: } -U \xrightarrow{e} I
\]

- Integral Causality for I & C elements (if possible to fulfill energy rules)

\[
\text{I: } C \quad (f: \text{ given data}) \quad e = \frac{1}{C} \int f \, dt \quad ; \quad e = \frac{1}{C_s} f \\
I \xleftarrow{f} \quad (e: \text{ given data}) \quad f = \frac{1}{L} \int e \, dt \quad ; \quad f = \frac{1}{I_s} e
\]

- Contextual causality of R elements (arbitrary for linear relations)

\[
\text{R: } f \quad (f \text{ given data}) \quad e = R \cdot f \\
R \xleftarrow{e} \quad (e \text{ given data}) \quad f = \frac{e}{R}
\]

Remark : for non linear systems, the causality depends on the non linearity calculation
5. Causality in Bond-Graphs

- Causality for junctions
  - 0 Junction (common effort)
    which effort imposes its value to the others? => Only 1 causal stroke close to the 0 junction
  
  - 1 Junction (common flux)
    which flux imposes its value to the others? => Only 1 causal stroke far from the 1 junction
  
  - TF Junction: only 1 causal stroke close to the TF junction
    \[
    e_1 := m \cdot e_2 \\
    f_2 := m \cdot f_1
    \]
    Causal writing
    \[
    e_2 := e_1 \\
    e_3 := e_1 \\
    f_1 := f_2 + f_3
    \]
  
  - GY Junction: 0 ou 2 causal stroke close to the junction
    \[
    e_1 := r \cdot f_2 \\
    e_2 := r \cdot f_1
    \]
    \[
    f_1 := e_2 / r \\
    f_2 := e_1 / r
    \]
5. Causality in Bond-Graphs

- **Sequential Causality Application Procedure (SCAP)**
  1. Affect mandatory causality for sources and for non linear elements R
  2. Affect integral causality for C & I elements
  3. Propagate the causality for junctions
  4. Propagate causality for R elements

- **Example for Sequential Causality Application (SCAP)**

![Diagram of a bond-graph with elements labeled N0, N1, N2, E, Ra, C, L]
5. Causality conflicts in Bond-Graphs:

- DC current machine with rigid speed reducer

\[ V = R_m I + L_m \frac{dI}{dt} + k\Omega \]
\[ C = k I = J_m \frac{d\Omega}{dt} + C_{ch} \]

Derivative causality due to rigid link between the 2 state variables:
- coupled dynamics: only 1 independent dynamic
- Need of introducing a flexible link inside the reducer
5. Double causality in Bond-Graphs:

- Case of a double causality

- 2 valuable causalities (due to the interaction between both resistances)
- equivalent solutions if linear but different if non linear
- possibility of generating an Algebraic (ADE) model state => adapted solver
5. Double causality in Bond-Graphs:

- Example of PV array

\[ I_D = I_s \cdot \exp \left( \frac{V_D}{K_T} \right) - 1 \]
6. Properties issued from Causal BG: towards system analysis

- Definition of causal properties allows writing inter elements relationships with systematic and organised application rules. One can deduce:
  - State equations;
  - Transfer functions or block diagrams;
  - Controllability & observability properties;
  - Mode evaluation & coupling seek;
  - Model reduction;
  - Synthesis by Graph inversion for sizing: bicausality concept

- Those issues derivates from “causal paths” & “causal loops” definition.
6. Properties issued from Causal BG : towards system analysis

**Causal Path (CP):** bond series (chain) with the same causal orientation: for one given node, 2 bonds of the CP have an opposed causal orientation:

Two ports have a causal influence if the output variable from P1 (here e) influence the input variable for P2 (here e).

"Indirect CP": CP crossing through a passive element (R,C,I)
6. Properties issued from Causal BG : towards system analysis

- **Causal Loop (CL)**: closed CP between two R,C, I elements with no bonds crossed by following the same (effort or flux variable) more than one time (=> no indirect CL).

\[
\begin{align*}
C & \quad e \quad | \quad 0 \quad e \\
\downarrow & \quad f \\
R & \quad e \quad | \quad 1 \quad e
\end{align*}
\]

- **Gain of R,C, I elements**

\[
\begin{align*}
R & \quad \Rightarrow \quad f := e / R \\
\quad & \quad e \quad | \quad R \\
C & \quad \Rightarrow \quad e := f / (C.s) \quad \text{(integral causality)} \\
\quad & \quad e \quad | \quad C \\
I & \quad \Rightarrow \quad f := e / (I.s) \quad \text{(integral causality)} \\
\quad & \quad e \quad | \quad I
\end{align*}
\]
6. Properties issued from Causal BG : towards system analysis

• Gain of Causal Path (CP) and causal loop (CL)

➢ Gain of a CP

where: \( n_0 = \) number of power inversion at \( J_0 \) when following the flux variable;
\( n_1 = \) number of power inversion at \( J_1 \) when following the effort variable;
\( m_j, r_k \) are the gains of TF and GY taking into account the causal orientation

Note : for an indirect CL crossing a R,C,I, element, multiply the CP gain by the element gain.

\[
T_i = (-1)^{n_0+n_1} \cdot \prod_{j,k} \left( m_j \text{ ou } \frac{1}{m_j} \right) \cdot \left( r_k \text{ ou } \frac{1}{r_k} \right)
\]

➢ Gain of a CL :

\[
T_i = (-1)^{n_0+n_1} \cdot \prod_{g} \cdot \prod_{j,k} \left( m_j \text{ ou } \frac{1}{m_j} \right)^2 \cdot \left( r_k \text{ ou } \frac{1}{r_k} \right)^2
\]

where \( \prod \) are the gains of R,C,I elements crossed inside the CL (also for start and arrival elements)
6. Properties issued from Causal BG: towards system analysis

- Example of CP (following the effort variable):

\[ T_{P1, P2} = (-1)^0 + 1 \cdot \frac{1}{m} \]

- Example of CL:

\[ T_{R, L} = (-1)^{1+2} \cdot \left(\frac{1}{m}\right)^2 \cdot \left(\frac{R}{L \cdot S}\right) \]
6. Properties issued from Causal BG : towards system analysis

Example of causal loops

\[ (Ra \leftrightarrow L) \rightarrow G_{(Ra,L)} = -\frac{Ra}{L.s} \]

- 3 causal loops:
  \[ (Rb \leftrightarrow C) \rightarrow G_{(Rb,C)} = -\frac{1}{Rb.C.s} \]
  \[ (L \leftrightarrow C) \rightarrow G_{(L,C)} = -\frac{1}{L.C.s^2} \]
6. Properties issued from Causal BG: towards system analysis

- Example of causal loops (1/4 vehicle transmission line): « do it yourself »
6. Properties issued from Causal BG : towards state equations

- **State Variables**
  - Power state variables : \( X^T = [e_c, f_I] \);
  - Energy state variables : \( X^T = [p_I, q_C] \); \( \frac{dX^T}{dt} = [e_I, f_C] \);
  - Both choices are equivalent in linear cases.
  - For non linear model, better choose energy state variables (models often simpler mathematically).

- **Example of a machine with magnetic saturation** :
  - model with magnetic fluxes :
    \[
    \begin{cases}
    \frac{d\phi(i,\theta)}{dt} = V - RL^{-1}\phi(i,\theta) \\
    \phi(i,\theta) = L(i,\theta)i
    \end{cases}
    \]
  - model with currents :
    \[
    \begin{cases}
    \frac{d\phi(i,\theta)}{dt} = L(i,\theta)\frac{di}{dt} + \frac{dL(i,\theta)}{dt}i \\
    \frac{dL(i,\theta)}{dt} = \frac{dL(i,\theta)}{d\theta}\frac{d\theta}{dt} = L_{dyn}(i,\theta).\omega \\
    \frac{di}{dt} = L^{-1}(i,\theta)[V - (R + L_{dyn}(i,\theta).\omega)i]
    \end{cases}
    \]
6. Properties issued from Causal BG : towards state equations

**Calculation from structural and internal relationships**

- **number the bonds**
- **write structural relationships for all junctions**
- **write internal relations for all R,C,I elements**

**Example of equation derivation**

*Structural equations*  
\[ J_1 : e_2 = E - e_3 - e_4 \]
\[ f_1 = f_2; f_3 = f_2; f_4 = f_2 \]
\[ J_0 : f_6 = f_4 - f_5 \]
\[ e_4 = e_6; e_5 = e_6 \]

*Internal equations*  
\[ L: \quad \dot{f}_2 = \frac{e_2}{L}; \quad f_2 = \frac{p_2}{L} \]
\[ Ra: \quad e_3 = Ra \cdot f_3 \]
\[ Rb: \quad f_5 = \frac{e_5}{Rb} \]
\[ \dot{e}_6 = \frac{f_6}{C}; \quad e_6 = \frac{q_6}{C} \]

*Power variables*  
\[ \dot{f}_2 = \frac{e_2}{L} = \frac{E - e_3 - e_4}{L} = \frac{E - Ra \cdot f_3 - e_6}{L} = \frac{E - Ra \cdot f_2 - e_6}{L} \]
\[ \dot{e}_6 = \frac{f_6 - f_5}{C} = \frac{f_2 - e_5/Rb}{C} = \frac{f_2 - e_6/Rb}{C} \]

*Energy variables*  
\[ \dot{p}_2 = e_2 = E - e_3 - e_4 = E - Ra \cdot f_3 - e_6 = E - Ra \cdot f_2 - q_6/C = E - Ra \cdot \frac{p_2}{L} - q_6/C \]
\[ \dot{q}_6 = f_6 = f_4 - f_5 = f_2 - e_5/Rb = \frac{p_2}{L} - e_6/Rb = \frac{p_2}{L} - q_6/(Rb \cdot C) \]

\[
\begin{bmatrix}
\dot{p}_2 \\
\dot{q}_6
\end{bmatrix} = \begin{bmatrix}
-Ra/L & -1/L \\
1/C & -1/Rb.C
\end{bmatrix} \begin{bmatrix}
p_2 \\
q_6
\end{bmatrix} + \begin{bmatrix}
E/L \\
0
\end{bmatrix}
\]
6. Properties issued from Causal BG : towards state equations

Find the state matrix from CL & CP

éléments de la diagonale : éléments $a_{ii} = \text{somme des gains de toutes les BC couplant les R aux éléments dynamiques (I,C) auquel correspond chaque variable d'état de la matrice } A$..

éléments $a_{ij}$ : donnés par les CC ( directs ou indirects passant par des éléments R) couplant les éléments dynamiques (C,I) entre eux. En particulier, $a_{ij}$ est donné par le gain du CC partant de l'élément correspondant à la j$_éme$ variable d'état et allant jusqu'à l'élément correspondant à la i$_éme$ variable d'état en multipliant ce gain par le gain de l'élément de départ du CC.

éléments $b_i$ : $CC$ allant des entrées du système (sources) aux éléments dynamiques correspondants à chaque variable d'état (chaînes d'action).

Remarque : on retrouve le gain des BC entre 2 éléments dynamiques I,C lorsqu'on fait le produit $a_{ij}a_j$
6. Properties issued from Causal BG : towards transfer function

Find transfer functions from Causal Loops and Paths : Mason Rule

If \( y(t) \) and \( u(t) \) are respectively the system output and input, one can define the transfer function by:

\[
\frac{Y(s)}{U(s)} = \frac{\sum_k T_k(s) \cdot D_k(s)}{D(s)}
\]

with:

- \( T_k(s) \) - gain of the direct or indirect path \( N^k \) linking the input to the output
- \( D(s) \) - is the bond-graph determinant which can be calculated as:

\[
D(s) = 1 - \sum_j B_j(s) + \sum_{j,k} B_j(s) \cdot B_k(s) - \sum_{j,k,l} B_j(s) \cdot B_k(s) \cdot B_l(s) + \ldots
\]

where: the first term deals with the gain of CL (\( \Sigma B_j \)),

the second is the multiplication of 2 by 2 disjoint CL (\( \Sigma B_j, B_k \),...)

\( D_k(s) \) - the calculation is the same as for \( D(s) \), but only with the CL \( B_{j,k,l}(s) \) which don’t touch the action chain (causal path linking input and output)
6. Properties issued from Causal BG : towards transfer function

Find transfer functions from Causal Loops and Paths : Mason Rule

Problem : calculate $G(s) = V_C / E$

- 3 causal loops

- 1 direct chain $E \rightarrow V_C$ : no disjoint CL $D_1 = 1$

Denominator : 2 disjoint CL

$$D(s) = 1 - \left( -\frac{1}{R_bC_s} \right) \left( -\frac{R_a}{Ls} \right) \left( -\frac{1}{L.C.s^2} \right) + \left( -\frac{1}{R_bC_s} \right) \left( -\frac{R_a}{Ls} \right)$$

$$G(s) = \frac{T_{\downarrow}D}{D(s)} = \frac{1}{L.C.s^2 + \left( R_aC + \frac{L}{R_b} \right)s + \frac{R_a}{R_b}}$$
A. Methodological issues : the Causal Bond Graph

1) Introduction: general concepts for energy system modelling
2) Basics of Bond-Graphs
3) Basic components of Bond-Graphs
4) Construction of Bond Graphs in electricity, mechanics and hydraulics
5) Multidisciplinary examples : electro-hydraulic actuator, Photovoltaic generator,…
6) Causal properties in the Bond Graphs : physical meaning, mathematical issues
7) From the causal Bond Graph till system analysis : state equation, transfer function derivation from causal paths

B. Applications : the Bond Graph in electrical engineering

1) Modelling of switching cells & static converters in power electronics
2) Modelling of electromechanical (electrical machines) & electrochemical devices
3) Examples of systems in electrical engineering : hybrid systems for renewable energy
1. Model of a switching cell: functional model

* Instantaneous functional model as an electrical transformer (no loss)

If $\eta$: state of the cell (high = 1; low = 0)

$$\begin{align*}
V_s &= \eta \cdot V_e \\
I_e &= \eta \cdot I_s
\end{align*}$$

* Average functional model as an electrical transformer (no loss)

$$\begin{align*}
\langle V_s \rangle &= \frac{T_{on}}{T_c} \cdot \langle V_e \rangle \\
\langle I_s \rangle &= \alpha^{-1} \cdot \langle I_e \rangle
\end{align*}$$
1. Model of a switching cell: functional model

- Logic of control
  
  \[ V_s = \eta \cdot V_e \]
  \[ I_e = \eta \cdot I_s \]

- Logic synthesis:

  \[ \eta = c_h + \text{Sgn}(I_s) \cdot c_b \]

  - Time Delay (dead times) taken into account
  - Discontinuous Conduction (\( I_s = 0 \)) cannot be simulated
  - Losses (switching & conduction) possible to estimate

- **Is** > 0 \( \Rightarrow \) \( \text{Sign}(I_s) = 1 \); Is < 0 \( \Rightarrow \) \( \text{Sign}(I_s) = 0 \)
- Ti On \( \Rightarrow \) \( c_i = 1 \); Ti Off \( \Rightarrow \) \( c_i = 0 \)

<table>
<thead>
<tr>
<th>( c_h )</th>
<th>( c_h )</th>
<th>Sign(I_s)</th>
<th>Conduction</th>
<th>Vs</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>( T_H )</td>
<td>( V_e )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>( D_b )</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>( D_b )</td>
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<td>1</td>
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<td>( D_h )</td>
<td>( V_e )</td>
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<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>( T_B )</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( D_h )</td>
<td>( V_e )</td>
</tr>
</tbody>
</table>
1. Model of a switching cell: functional model

* Functional model “as an electrical transformer” with losses

Switching loss estimation in IGBT (builder data)
- Transistor & diode conduction
- Transistor switching

Losses simulated as a voltage drop
Whole losses simulated as a resistance

Note: average losses (for 1 switching period) can be included in the average model
1. Model of a switching cell: “component model”

\[ \text{J1 : } e_3 := V_e - e_2 \quad \text{J0 : } f_3 := f_4 + I_s \]

\[ \begin{align*}
\text{f1 := } & f_3; \quad \text{f2 := } f_3 \\
\text{e3 := } & e_4; \quad \text{e4 := } e_3
\end{align*} \]

\[ \Rightarrow \text{Calculation of } V_s : \]

\[ V_s := e_3 := V_e - e_2 := V_e - R_h \cdot f_3 := V_e - R_h \cdot (f_4 + I_s) := V_e - R_h \cdot (e_4/R_B + I_s) \]

\[ e_3 = e_4 : \text{non causal writing} \]

\[ e_3 = V_s \quad \Rightarrow \]

\[ V_s = V_e - R_h \cdot (V_s/R_B + I_s) \]

(1 BG, 2 causality cases)

* Internal equations

\[ R_h: \quad e_2 = R_h \cdot f_2 \]

\[ R_B: \quad f_4 = e_4/R_B \]

* Structural equations

\[ \begin{align*}
\text{J1 : } & e_3 := V_e - e_2 \\
\text{f1 := } & f_3; \quad \text{f2 := } f_3 \\
\text{f3 := } & f_4 + I_s \\
\text{e3 := } & e_4; \quad \text{e4 := } e_3
\end{align*} \]
1. Model of a switching cell: “component model”

- which issue?
  - solving the double causality issue (algebraic loop)
  - defining generic model of switching cells with fixed external causality (« fractal modelling »)

- One solution: considering the physical behavior of power semiconductor: capacitor behavior during switching

Consider the cable inductance in each leg to break the capacitive mesh and cancel the causality conflict.
1. Model of a switching cell: “component model”

- 3rd order model \((L_{câb}, C_H, C_B)\): modular, fractal, causal

Switching Model (time, waveforms)
- Losses (switching, conduction)
- Computation cost: order 3, thin calculation step,
1. Model of a switching cell: “component model”

- Component model transistor / diode

- Macro Component with capacitance

- 3rd order model of the switching cell

- Switching Model (time, waveforms)
- Losses (switching, conduction)
- Physical phenomena (stored charges, saturation,...)
- computation cost: order 3, thin calculation step,

MC: Switching Cell
1. Model of a switching cell: example of a boost converter

Model with instantaneous values
\[ V' = (1-C_B) \cdot V_C \]
\[ I_s = (1-C_B) \cdot I_L \]

Average model: replace \( C_B \) by \( \alpha \):
\[ V' = (1-\alpha_B) \cdot V_C \]
\[ I_s = (1-\alpha_B) \cdot I_L \]
1. Model of a switching cell: 3 phase case

- Model of a voltage source 3 phase inverter

\[
\begin{align*}
V_e & \quad I_e \\
0 & \quad T_{iH} \\
1 & \quad I_{ei} \\
2 & \quad T_{iB} \\
3 & \quad I_{si} \\
\text{(i=1,2,3)} & \quad V_{si} \\
\end{align*}
\]

\[\text{Se:Ve} \quad \begin{array}{c}
V_e \\
I_e \\
\eta_i^{-1} \\
TF \\
V_{i1} \\
I_{i1} \\
S_f: I_s1 \\
V_{i2} \\
I_{i2} \\
\eta_i^{-1} \\
TF \\
V_{i2} \\
I_{i2} \\
S_f: I_s2 \\
V_{i3} \\
I_{i3} \\
\eta_i^{-1} \\
TF \\
V_{i3} \\
I_{i3} \\
S_f: I_s3
\end{array}\]

* Feeding voltages \(V_{i0}\) \(\Rightarrow\) load voltage \(V_{iN}\)

Hyp: balanced load impedances:

\[
\begin{align*}
V_{1N} + V_{2N} + V_{3N} &= Z_{ch} \left( I_{s1} + I_{s2} + I_{s3} \right) = 0 \\
(V_{1O} + V_{2O} + V_{3O}) + (3 \cdot V_{ON}) &= 0
\end{align*}
\]

\[
\begin{align*}
V_{1N} &= \frac{1}{3} (2V_{10} - V_{20} - V_{30}) = \frac{V_e}{3} (2\eta_1 - \eta_2 - \eta_3) \\
V_{2N} &= \frac{1}{3} (2V_{20} - V_{10} - V_{30}) = \frac{V_e}{3} (2\eta_2 - \eta_1 - \eta_3) \\
V_{3N} &= \frac{1}{3} (2V_{30} - V_{20} - V_{10}) = \frac{V_e}{3} (2\eta_3 - \eta_2 - \eta_1)
\end{align*}
\]
1. Model of a switching cell: 3 phase case

- Model of a voltage source 3 phase inverter

* Input current calculation:

\[
\begin{align*}
I_e &= I_{e1} + I_{e2} + I_{e3} = \eta_1 I_{s1} + \eta_2 I_{s2} + \eta_3 I_{s3} \\
I_{s1} + I_{s2} + I_{s3} &= 0 \\
I_e &= \left(\frac{2\eta_1 - \eta_2 - \eta_3}{3}\right)I_{s1} + \left(\frac{2\eta_2 - \eta_1 - \eta_3}{3}\right)I_{s2} + \left(\frac{2\eta_3 - \eta_2 - \eta_1}{3}\right)I_{s3}
\end{align*}
\]

- 3 phase « functional model » (DC voltage / load voltages)

\[
\begin{align*}
I_e &= \eta_{13} I_{s1} + \eta_{23} I_{s2} + \eta_{33} I_{s3} \\
V_{1N} &= \eta_{13} V_e \\
V_{2N} &= \eta_{23} V_e \\
V_{3N} &= \eta_{33} V_e
\end{align*}
\]
2. Model of a chopper fed DC machine

- DC Machine model

- DC Machine Model fed by a bridge structure of chopper
  Functional model
2. Model of a chopper fed DC machine

- DC Machine model

- DC Machine Model fed by a bridge structure of chopper
  > Component model

- simplified BG : B = reference

Integral causality
2. Model of a 3φ inverter fed PM synchronous machine

- 3 phase model

- Inverter fed PM synchronous machine (functional model)

\[ \sum_{i=1,2,3} E_i I_i = T_{em,1} \Omega + T_{em,2} \Omega + T_{em,3} \Omega = T_{em} \Omega \]

\[ \Phi_i (\Theta) = \Phi_{magnet} \cdot \cos (\Theta_{elec} - \varphi_i) \]

\[ E_i (\Theta) = \frac{d\Phi_i (\Theta)}{dt} = -Np \Omega \Phi_{magnet} \sin (\Theta - \varphi_i) \]
2. Constant Power Park’s & Concordia transformations

\[
\begin{bmatrix}
X_\alpha \\
X_\beta \\
X_0
\end{bmatrix} = \sqrt{\frac{2}{3}}
\begin{bmatrix}
1 & 1 & 1 \\
2 & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix}
\begin{bmatrix}
X_a \\
X_b \\
X_c
\end{bmatrix}
\]

\[
\begin{bmatrix}
X_d \\
X_q
\end{bmatrix} =
\begin{bmatrix}
\cos(\psi) & \sin(\psi) \\
-\sin(\psi) & \cos(\psi)
\end{bmatrix}
\begin{bmatrix}
X_\alpha \\
X_\beta
\end{bmatrix}
\]
2. Squirrel cage induction machine model

- Model with losses merged to the stator side

\[ T_{em} = p \left( \Phi_{r\beta} I_{r\alpha} - \Phi_{r\alpha} I_{r\beta} \right) \]

\[ R_r' = \left( \frac{L_m}{L_r} \right)^2 R_r \]

\[ F_{em}' = \frac{L_m}{L_r} F_{em} \]

\[ I_r' = \frac{L_r}{L_m} I_r \]

\[ \Phi_r' = \frac{L_m}{L_r} \Phi_r \]
2. Electrochemical transformation

• Generalised Variables for chemical systems:
  • effort = chemical potential (or molar energy : $\Delta H$)
  • flux = molar flux (\(\xi\) en mol / s)

• Energy that can be transformed in electricity: Gibbs free energy ($\Delta G$):

The oxydo-reduction reaction with given pressure and temperature involves a total energy corresponding with the constant pressure heat $Q_p$ associated with the enthalpy variation $\Delta H$. Regarding the 2\textsuperscript{nd} thermodynamic principle, one part of this total energy can be converted in work.

Thus, for thermodynamic equilibrium, by neglecting the influence of other potential fields (electromagnetic & gravitation), this maximal transferred energy (exergy) can be associated to the free enthalpy variation (Gibbs free energy), derived versus enthalpy and entropy variations:

\[ \Delta G = \Delta H - T \cdot \Delta S \]

• Example of a fuel cell: \( H_2 + \frac{1}{2}O_2 \rightarrow H_2O \) \( \Rightarrow \) \( \Delta G = G_{H_2O} - G_{H_2} - \frac{1}{2}G_{O_2} \)
2. Electrochemical transformation

If the oxydo reduction transformation gives $Z$ moles of electrons by product moles $M$ (with $N_M$ the quantity of the reagent M in mol):

- Electrical charge of $z$ Mol of e-:
  
  \[ q = - z.F \]
  
  $F$, Faraday constant (i.e.: charge of one electron mole).

- Charge of $z.N_M$ mol of e-:
  
  \[ Q = N_M.q = -N_M.z.F \]

- Then for a reaction flux given by:
  
  \[ \xi_M = dN_M/dt \]

  \[ i = dQ/dt = \xi_M z F \]

- Chemical converted power:
  
  \[ P_{chimie} = - \xi_M \Delta G \]

- Energy conservation relationship:
  
  \[ E.i = - \xi_M \Delta G \]

\[ E = - \Delta G / (z.F) \]

- $\Delta G$ (J.mol$^{-1}$)
  
  \[ z F \]

  $T$ (F)

  $E$ (V)

  $\xi_M$ (mol.s$^{-1}$)

  $i$ (A)
2. Electrochemical transformation
3. Example of multidisciplinary electrical engineering system

Wind turbine

Generator

PV cell array

DC Bus

Motor

Pump

inverse osmosis

Accumulator

5 domains, 8 inter-domain crossings

Hydraulics

Chimics

Electricity

Mechanics

Thermics

Pressure flow

Δ–enthalpy

Molar flux

Voltage

Current

Torque, Force

Speed

Voltage

Current

Mechanics

Torque, Force

Speed

5 domains, 8 inter-domain crossings
3. Example of multidisciplinary electrical engineering system

BG model of a wind turbine driving a DC generator

- The causality conflict due to the coupling of both inertia I elements can be solved by:
  - gathering the 2 I elements (generator inertia neglected or merged with the turbine inertia);
  - by inserting flexible elements in the mechanical transmission.
3. Example of multidisciplinary electrical engineering system

Energy management and control of the wind turbine system:
Due to the high inertia, better to control the generator torque

- MPPT with torque control:

\[ C_p(\lambda) = C_p^{\text{opt}} \; ; \; P_{\text{opt}} = K_{\text{opt}} \Omega_{\text{opt}}^3 \]

\[ K_{\text{opt}} = \frac{1}{2} \frac{C_p^{\text{opt}} \cdot \rho \cdot S \cdot R}{\lambda^{3 \text{opt}}} \]

\[ T_{\text{em}}^{\text{ref}}(\Omega) = K_{\text{opt}} \cdot \Omega^2 \]
3. Example of multidisciplinary electrical engineering system

Characteristic of a PV array

BG of a PV cell (R_{shunt} neglected)
3. Example of multidisciplinary electrical engineering system

Simplified Model of a lead acid accumulator

The high value capacitance ($Cb$) conducts a current with the same amplitude and phase as the actual cell with in the same excitation conditions
3. Example of multidisciplinary electrical engineering system

Inverse osmosis process

Spiral module for an inverse osmotic process
3. Example of multidisciplinary electrical engineering system

\[ P = (\rho \cdot r^2 \cdot \Omega - \frac{\rho \cdot \cot \beta}{2 \cdot \pi \cdot k} Q) \cdot \Omega \]

\[ Tm = (\rho \cdot r^2 \cdot \Omega - \frac{\rho \cdot \cot \beta}{2 \cdot \pi \cdot k} Q) \cdot Q \]
3. Example of multidisciplinary electrical engineering system

![Diagram of inverse osmosis system]

- **Débit d’alimentation**: Input flow
- **Membrane**: Inverse osmosis membrane
- **Pump output**: Pump output
- **Rejet**: Reject flow
- **Eau pure**: Pure water flow

**Calcul de la conductivité**

- **P**: Pump
- **C**: Concentrate
- **MR**: Retentate
- **C**: Permeate
- **R**: Resistance

**BG of inverse osmotic process**

- **Passage de l’eau à travers la membrane**: Passage of water through the membrane

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3. Example of multidisciplinary electrical engineering system

**System oriented energy management strategy & sizing**

- **The storage sub system** is controlled as a V source: DC bus voltage control by balancing (PV + wind) power with load powers (inverse osmosis + auxiliaries).
- The energy of producers (PV + wind) is maximised: MPPT strategy

**System Sizing** : all system couplings have to be considered:
- climatic statistics (Weibull for wind, insulation statistic for solar power)
- statistic need for loads: pure water demand for a given period;
- energy efficiency (losses) of each sub system
- energy efficiency of producers (quality of the MPPT)
3. Example of multidisciplinary electrical engineering system

1) Accumulator directly connected to the DC Bus

**Advantage**: simple and low cost structure

**Drawbacks**: the bus voltage is floating following the SOC. Need of a series connection with a high number of cells (voltage balance potentially problematic)
3. Example of multidisciplinary electrical engineering system

2) Accumulator is connected to the DC Bus through a DC DC chopper

Advantages: Controlled and stabilised bus voltage, lower number of cells in series (easier balance voltage).

Drawbacks: cost, complexity

\[ I_{PV+E} = I_{cons} + I_{stock} + I_{bus} \]
\[ I_{bus} = \frac{C_{bus}}{p} V_{bus} \]
\[ I_{accu} = \frac{1}{(R_b + 1/C_b p + L_f p)} (V_H - E_b) \]
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