Index properties

9.1 Introduction

Determining reliable values for soil parameters, either for use in an elastic–plastic soil model such as Cam clay or to obtain values of strength and stiffness for some less elaborate analysis, usually requires that laboratory tests such as triaxial or oedometer tests be performed on undisturbed samples of soil. To obtain good quality undisturbed samples is usually expensive and frequently difficult. Performance of good-quality triaxial and oedometer tests requires time and skill and is also expensive. It is possible to characterise or classify soils with quicker, less-sophisticated tests which do not require undisturbed samples of the soils. This characterisation for cohesive, clayey soils is achieved by using index tests which determine the natural water content and the so-called liquid limit and plastic limit of the soil.*

Although the procedures which have been adopted for performing these index tests may appear quaint, it is possible, using the ideas of critical state strengths and models of soil behaviour such as Cam clay, to relate values of index properties to other properties of engineering importance. Empirical correlations between index properties and strengths and compressibilities have been used for many decades. Critical state soil mechanics points the way to a rational basis for many of these correlations.

It would be extreme to suggest that the availability of these correlations makes it unnecessary to perform any tests more sophisticated than the index tests. Their value lies in two areas. Where it is difficult to obtain good soil samples or where results of other tests are not yet available,

*The term plastic is traditionally used to describe soils for which it is possible to determine liquid and plastic limits. This is a confusing term since it is a theme of this book that elastic–plastic soil models are useful for describing the behaviour of soils such as sands, which according to this definition would be classified as non-plastic.
these correlations can be used to give preliminary values of soil properties for use in feasibility studies. At a later stage, when more data are available from a wide range of tests, such correlations can be used to check the internal consistency of the data that have been obtained and to draw attention to apparent anomalies.

The fall-cone test is described in Section 9.2. An analysis of this test in terms of critical state strengths shows it to be an extremely powerful index test. The range of correlations which emerge is presented in Section 9.3, together with examples of their applications. Subsequent sections then provide the theoretical background to and comment on these correlations. Readers who are concerned only to know the correlations themselves need proceed no further than Section 9.3.

9.2 Fall-cone test as index test

Static indentation tests have been used since 1900 to provide quick estimates of the hardness of metals (Tabor, 1951). Creation of an
indentation is an almost entirely plastic process, and the ‘hardness’ of a metal, determined from the indentation force and the dimensions of the indentation (spherical, conical, or pyramidal), is directly related to its yield strength. Indentation tests have been used in Scandinavia since 1915 to provide a measure of the ‘consistency’ of clays in routine geotechnical investigations (Bjerrum and Flodin, 1960). Olsson (1921) describes the fall-cone apparatus which he developed for this purpose (Fig. 9.1); a cone is allowed to fall freely under its own weight from a position at rest with the point of the cone just touching the surface of the clay. This is no longer a static indentation test; the cone accelerates initially and then decelerates to rest and penetrates a distance which is greater than the penetration at which the soil resistance is equal to the weight of the cone (where the acceleration of the fall-cone is zero).

A detailed study of the relationship between cone penetration and soil strength is reported by Hansbo (1957). The variables governing the problem are (Fig. 9.2) the mass \( m \) and tip angle \( \alpha \) of the cone, the penetration \( d \), and the undrained shear strength of the soil \( c_u \). Dimensional analysis (Wood and Wroth, 1978) then shows that

\[
\frac{c_u d^2}{mg} = f(\alpha, \chi) \quad (9.1)
\]

where the parameter \( \chi \) allows for surface effects between the soil and cone (for example, friction or adhesion), and \( g \) is the acceleration due to gravity (9.81 m/s\(^2\)). For a given material of cone and for related soils (perhaps soils of similar activity, see Section 9.4.3), the cone angle is the dominant factor in \( f(\alpha, \chi) \), so (9.1) can be rewritten as

\[
\frac{c_u d^2}{mg} = k_\alpha \quad (9.2)
\]

where \( k_\alpha \), the cone factor, certainly depends on the angle of the cone.

In Chapter 6, critical state lines for clay were proposed, supported by

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![Fig. 9.2 Fall-cone test.](image)
9.2 Fall-cone test as index test

Experimental evidence, of the form

\[ v_{es} = \Gamma - \lambda \ln p'_{es} \]
\[ q_{es} = M p'_{es} \]

Since undrained strength \( c_u \) is just half the deviator stress at failure,

\[ c_u = \frac{q_{es}}{2} \]

and since in undrained deformation the specific volume does not change, expressions (9.3)–(9.5) can be combined to give a general relationship between undrained strength \( c_u \) and specific volume \( v \):

\[ v = \left[ \Gamma + \lambda \ln \left( \frac{M}{2} \right) \right] - \lambda \ln c_u \]

or

\[ c_u = \frac{M}{2} \exp \left( \frac{\Gamma - v}{\lambda} \right) \]

For saturated soils, a direct relationship exists between water content \( w \) and specific volume \( v \):

\[ v = 1 + G_s w \]

where \( G_s \) is the specific gravity of the soil particles. Combination of equations (9.2)–(9.8) then shows that if a series of fall-cone tests is performed using the same cone (with \( m \) and \( \alpha \) chosen) on samples of the same clay prepared at various water contents, then the measured penetrations \( d \) are related to water content \( w \) by

\[ w = \frac{2\lambda}{G_s} \ln d + \frac{\Gamma - 1}{G_s} + \frac{\lambda}{G_s} \ln \left[ \frac{M}{2k_m mg} \right] \]

A plot of water content \( w \) against logarithm of cone penetration \( d \) should give a straight line with slope \( 2\lambda/G_s \) (Fig. 9.3), and the compressibility \( \lambda \) of the soil can be determined from these cone tests.

If cone tests are performed with two geometrically similar cones of different masses \( m_1 \) and \( m_2 \), then plots of water content against cone penetration should produce two parallel lines with a water content

*The relationship between strengths measured with various devices is discussed in Section 10.6; in this chapter it is assumed that this strength represents the maximum shear stress which the soil can support, independent of the mode of shearing. The justification for this is that we are trying to establish approximate correlations rather than exact analyses in this chapter. However, this assumption is in essence the Tresca criterion discussed in Section 3.2 and is basic to many of the plasticity analyses of undrained loading of soils (Atkinson, 1981; Houlsby, 1982).*
separation $\Delta w$ (Fig. 9.3) where, from (9.9),

$$\Delta w = \frac{\lambda}{G_s} \ln \frac{m_2}{m_1}$$  \hspace{1cm} (9.10)

and this gives a second route by which $\lambda$ can be deduced from the results of fall-cone tests. The slight advantage of (9.10) over (9.9) is that, because the fall-cone test is a dynamic rather than a static test, there may be effects associated with the rate at which the clay is being deformed which should cancel out if the geometry of the penetration process is kept constant by comparing results obtained at identical penetrations. Satisfactory results have been obtained for Cambridge gault clay (Fig. 9.4) using a ratio of cone masses $m_2/m_1 = 3$. Working from the gradients of the parallel lines (9.9) and taking $G_s = 2.75$ gives a value of $\lambda = 0.30$. Working from the spacing between the lines (9.10) gives an independent value of $\lambda = 0.31$.

These equations lead to two ways in which fall-cone tests can be used.

Fig. 9.3 Relationship between water content and cone penetration.

Fig. 9.4 Fall-cone tests on Cambridge gault clay.
9.2 Fall-cone test as index test

to provide strength indices for soils. Olsson (1921) devised a strength number (hålfasthetstala) proportional to the mass of a cone of tip angle \( \alpha = 60^\circ \) which would produce a penetration \( d = 10 \text{ mm} \) in the soil. Of course, only a certain number of combinations of cone mass and cone angle were available, and charts and tables were produced from which the penetration of any given cone could be converted to a strength number. A strength number of 10 was assigned to a cone mass of 60 g. From (9.2), the strength number is directly proportional to undrained strength \( c_u \).

Olsson went further and defined an index property, fineness number (finlekstala), as the water content of soil having a strength number 10, in other words, the fineness number is the water content at which a 60° cone of mass 60 g would penetrate 10 mm when allowed to fall under its own weight. This fineness number is now accepted in Sweden and other Scandinavian countries as equivalent to the liquid limit (Karlsson, 1977). Other cone-penetration tests have been used elsewhere to estimate liquid limits of soils (Sherwood and Ryley, 1970), and the test which is now the preferred British Standard test 2(A) for determining the liquid limit (BS 1377, 1975) is a fall-cone test using a 30° cone of mass 80 g. The liquid limit \( w_L \) has thus become the water content at which soil has a standard strength. (The background to the measurements of liquid limit is discussed in Section 9.4.1.)

Karlsson (1977) states that a 10-mm penetration of a 60° cone of mass 60 g corresponds to a soil strength of 1.7 kPa. Fall-cone and miniature vane tests performed by Wood (1985a) produce average values of cone factors,

\[
k_a = 0.85 \quad \text{for } \alpha = 30^\circ
\]

and

\[
k_a = 0.29 \quad \text{for } \alpha = 60^\circ
\]

which lead from (9.2) to estimates of strength at the liquid limit of 1.71 kPa according to the Swedish definition and 1.67 kPa according to the British definition. For the purposes of the approximate correlations which are set out in Section 9.3, however, we assume that the standard undrained shear strength of soils at their liquid limit \( c_u \) is approximately 2 kPa.

Another use of fall-cone tests proceeds from (9.10) to obtain an index of the way in which strength changes with water content. Specifically, if the same penetration \( d \) is obtained when cones of two different masses penetrate two different samples of the same soil, then, from (9.2), the ratio of the strengths of the soil samples is equal to the ratio of the masses of the cones. The change in water content \( \Delta w \) in (9.10) is that necessary to change the strength of the soil by the factor \( m_2/m_1 \). A useful index might
then be the water content change $\Delta w_{100}$ required to produce a 100-fold change in strength. Direct determination of this would require cone tests to be performed with a ratio of cone masses $m_2/m_1 = 100$. The standard cones used in Scandinavia and in the United Kingdom have masses $m_1 = 60$ g and 80 g, and so the parallel series of tests would have to be performed with $m_2 = 6$ kg or 8 kg. Such cones would be cumbersome to use, and it is consequently more appropriate to extrapolate to $\Delta w_{100}$ using a smaller ratio of cone masses. Then from (9.10),

$$\Delta w_{100} = \Delta w \frac{\ln 100}{\ln (m_2/m_1)} \quad (9.11)$$

The results for Cambridge gault clay (Fig. 9.4) were obtained using a ratio of cone masses $m_2/m_1 = 3$. Since $\ln 3 \approx (\ln 100)/4$, this implies an approximate 4-fold extrapolation, $\Delta w_{100} \approx 4\Delta w$.

The determination of this strength change index by extrapolation relies on the value of $\lambda$ in (9.6) being a constant. The value of $\lambda$ that is being determined with (9.9) or (9.10) is valid for water contents close to the liquid limit. It typically drops with increasing pressure and decreasing water content (the water content after all cannot fall below zero, so the line $w = 0$ must provide an asymptote to the volume change characteristics), and so the extrapolation implied in (9.11) could be expected to yield values of $\Delta w_{100}$ which are too high.

The fall-cone test is being used as an index test to give direct information about strengths and about change in strength with water content, which from critical state lines can be interpreted as information about compressibility such as the value of the parameter $\lambda$.

### 9.3 Properties of insensitive soils

Many of the values of liquid limit quoted in published work have been obtained using the Casagrande apparatus described in Section 9.4.1. It is assumed that these values also correspond to a strength of 2 kPa, so that as far as the relations between soil properties to be described here are concerned, liquid limit values obtained with the various devices are assumed to be interchangeable.

The other index test that is widely used for classifying clayey soils is the so-called plastic limit test, described in Section 9.4.2. This plastic limit test is probably a less consistent strength test than the fall-cone liquid-limit test, but study of the data of undrained shear strength and water content for remoulded soils collected by Mitchell (1976) (Fig. 9.5) suggests that at the plastic limit (with water content $w_p$), also, the range of strengths is reasonably small, having an approximate average value of around 200 kPa,
9.3 Properties of insensitive soils

that is about 100 times the strength at the liquid limit. The difference in water content between the liquid limit and plastic limit is known as the plasticity index \( I_p \); it appears consequently that this is approximately equivalent to the quantity \( \Delta w_{100} \) introduced in the previous section.

The data of water contents in Fig. 9.5 have been plotted in terms of liquidity index \( I_L \) which scales water contents of soils in a standard way between their liquid and plastic limits:

\[
I_L = \frac{w - w_p}{w_L - w_p} = \frac{w - w_p}{I_p}
\]  

(9.12)

Liquidity index thus takes values of 1 and 0 when the water content of a soil is at the liquid or plastic limit, respectively.

Two simple relationships between index properties and other properties of insensitive or remoulded soils can now be deduced. Setting \( I_p = \Delta w_{100} \), we find that from (9.10) and (9.11) compressibility \( \lambda \) can be linked with

Fig. 9.5 Variation of remoulded undrained strength \( c_u \) with liquidity index \( I_L \) for (1) Horten clay \( (w_L = 0.30, w_p = 0.16) \); (2) London clay \( (w_L = 0.73, w_p = 0.25) \); (3) Shellhaven clay \( (w_L = 0.97, w_p = 0.32) \); (4) Gosport clay \( (w_L = 0.80, w_p = 0.30) \) (after Skempton and Northev, 1953); range of strength data collected by Mitchell (1976) shown shaded.
plasticity $I_p$:

$$\lambda = \frac{I_p G_s}{\ln 100}$$  \hspace{1cm} (9.13)$$

$$\approx 0.6 I_p.$$  \hspace{1cm} (9.14)

using a typical value for $G_s \approx 2.7$.

The parameter $\lambda$ is the compressibility of the soil in the expression

$$v = v_1 - \lambda \ln p'$$  \hspace{1cm} (9.15)(4.2bis)

for a normal compression line. The compressibility $C'_c$, which is the slope of the normal compression line when written as

$$v = v_1 - C'_c \log_{10} p'$$  \hspace{1cm} (9.16)

is perhaps more often quoted. The two compressibilities are simply related,

$$C'_c = \lambda \ln 10 = 2.303 \lambda$$  \hspace{1cm} (9.17a)

Fig. 9.6 Values of compressibility ($C'_c$ and $\lambda$) and plasticity index ($I_p$) for soils from the Mississippi delta and Gulf of Mexico ( + ) (data from McClelland, 1967); Egyptian soils (o) (data from Yousef, el Ramli, and el Demery, 1965); scabed soils off Tel Aviv (a) (data from Almagor, 1967); soils from Britain and elsewhere (x) (data from Skempton, 1944).
or
\[ \lambda = 0.4343C'_e \]  
(9.17b)
and (9.14) becomes
\[ C'_e = \frac{I_p G_s}{2} \]  
(9.18)
\[ \approx 1.35I_p \]  
(9.19)

Data of compressibility \( \lambda \) or \( C'_e \) and plasticity \( I_p \) in Fig. 9.6 have been culled from a number of published sources: soils from the Mississippi delta and Gulf of Mexico (McClelland, 1967), Egyptian soils (Youssef, el Ramli, and el Demery, 1965), seabed soils of Tel Aviv (Almagor, 1967), and various soils from Britain and elsewhere (Skempton, 1944). Relationship (9.14) is included and provides a reasonable fit to the data though there is, of course, a lot of scatter.

It is important to note that the data of undrained strengths in Fig. 9.5 were obtained from remoulded soils, and the mention of a critical state line at the beginning of this section also implies that remoulded strengths are being considered. The compressibility which is calculated using (9.14) or (9.19) is a compressibility for a remoulded, structureless soil. Undisturbed soils usually have a higher value of compression index than remoulded soils because of the extra compression associated with destruction of their structure. Schmertmann (1955), for example, shows

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Fig. 9.7 Effect of disturbance on slope of one-dimensional normal compression line for marine organic silty clay (after Schmertmann, 1955).
that disturbance of soil samples has an effect on compression index equivalent to partial remoulding (Fig. 9.7); a steeper normal compression line implies a higher value of λ or \( C'_{e} \) and data of measured compressibilities are expected to congregate in the upper part of Fig. 9.6.

Having assigned strengths of 2 kPa and 200 kPa to soils at their liquid and plastic limits, if the spread and curvature of the liquidity-logarithm-of-strength relationship can be neglected, we can estimate the remoulded strength of a soil with knowledge only of its liquidity index, in other words, with knowledge only of its liquid and plastic limits and natural water content, by using the expression

\[
c_{u} = 2 \times 100^{(1 - L)} \text{ kPa}
\]  

(9.20)

An example of the application of (9.20) is shown in Fig. 9.8 for a site on the Thistle field in the North Sea. Some undrained strength data from unconfined triaxial compression tests (\( \sigma_{r} = 0 \)) are included for comparison. The agreement between these measured strengths and estimates made on the basis of values of liquidity index is reasonable, but even more encouraging is the match of the pattern of observed and estimated variations of strength with depth. The measured strength profile (which is typical of some North Sea sites) shows an initial rapid rise in strength to a depth of about 5 m, followed by a drop to a minimum at a depth of about 15 m, and followed by a rapid increase in strength at a depth of about 20 m; all these variations are reflected in the profile of liquidity

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**Fig. 9.8** Measured (●) and calculated (+) profiles of strength with depth for location in Thistle field, North Sea.
index with depth. It is precisely at such offshore locations that it may be difficult to retrieve good soil samples for direct determination of in situ strength. An independent estimate of strength using (9.20) thus provides an invaluable confirmation of those measured trends that are observed.

Expression (9.20) is sensitive to errors in liquidity index. Water contents and index properties are typically quoted to 0.01 (1%), so for soils of low plasticity, estimates of liquidity index, and thence of strength can very easily be in error. Expression (9.20) is plotted as line \( A \) in Fig. 9.9.

![Fig. 9.9 Approximate unique relationship expected between liquidity index \( I_L \) and remoulded undrained shear strength \( c_u \) (line \( A \)).](image)

![Fig. 9.10 Profile of water content \( w \), index properties \( w_L \) and \( w_p \), and undrained shear strength \( c_u \) with depth for boring at Drammen, Norway (after Bjerrum, 1954).](image)
Fig. 9.11  (a) Sample of undisturbed Norwegian quick clay supporting load of 4 kgf (39.2 N); (b) same sample of Norwegian quick clay after remoulding at natural water content (photographs courtesy of Norwegian Geotechnical Institute, Oslo).
9.3 Properties of insensitive soils

Terzaghi (1936), introducing the parameter liquidity index (9.12), notes that the value of liquidity index 'indicates the consistency of the clay after remoulding it without changing the water content'. Critical state strength too is a remoulded soil strength. Actual field or laboratory determinations of strength may, in a strength:liquidity diagram (Fig. 9.9), appear inconsistent with the simple expression (line A) if the strength that is determined is not properly a remoulded strength. Two possibilities have been indicated on Fig. 9.9.

A typical profile of water content and strength from a boring in soft clay at Drammen, Norway is shown in Fig 9.10 (from Bjerrum, 1954). Here the water content is near the liquid limit \( (w \sim w_L, I_L \sim 1) \) near the surface, dropping below the liquid limit with depth. However, the strengths measured with in situ vane tests would plot well to the right of line A in Fig. 9.9. These are undisturbed strengths, and this is a sensitive clay, that is, the natural clay has some structure which is destroyed by remoulding so the ratio of undisturbed to remoulded strength is significantly greater than unity (for this clay, the sensitivity is typically around 10 when the liquidity is close to 1). A pedagogic example of the character of a sensitive clay is shown in Fig. 9.11: an undisturbed sample can support a sizeable load, but once the structure is disturbed and the soil remoulded, the sample flows like a liquid with extremely low strength. We should regard the

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**Fig. 9.12** Profile of water content \( w \), index properties \( w_c \) and \( w_p \), and undrained shear strength \( c_u \) with depth for boring at Paddington, London (after Skempton and Henkel, 1957).
undisturbed strength data plotting to the right of line A in Fig. 9.9 with some caution. The strength that is appropriate to the in situ liquidity of the soil is the remoulded strength; it may be dangerous to rely on the peak, undisturbed strength for design purposes. Some properties of sensitive soils are discussed in Section 9.5.

A typical water content and strength profile from a boring in heavily overconsolidated London clay is shown in Fig. 9.12 (from Skempton and Henkel, 1957). The water contents below the top few metres are around the plastic limit \( w \sim w_p \), \( I_L \sim 0 \) and the strengths, measured in undrained triaxial tests, around and above 200 kPa. These strengths and liquidities plot close to line A in Fig. 9.9. However, it was noted in Section 7.7 that when old failure surfaces are present in heavily overconsolidated clays, the clay structure adjacent to the failure surfaces may have been modified by the sliding so that particles are oriented parallel to the failure surface. The residual strength which could be mobilised on such surfaces can then be lower than the strength of the remoulded soil, in which the clay structure is completely random. The operational strength of such soils which can be relied upon for design purposes plots to the left of line A in Fig. 9.9. Such heavily overconsolidated soils are likely to have water contents close to their plastic limits, \( I_L \sim 0 \), so these strengths plot in the lower shaded region in Fig. 9.9.

So far, liquidity index has been linked with undrained strength, that is, strength defined in terms of total stresses. If a soil is experiencing a combination of total stresses such that a particular undrained strength is being mobilised, it must also be experiencing a combination of effective stresses such that a criterion of effective stress failure is being satisfied.

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**Fig. 9.13** (a) Mohr's circles of total stress \( (T) \) and effective stress \( (E) \) for soil failing with mobilised undrained strength \( c_L \); (b) effective stress state on critical state line (csl).
other words, a pore pressure (or pore suction) must exist in order that total and effective stress failure conditions can occur simultaneously.

Section 9.2 showed that soils with a water content equal to their liquid limit \( w = w_L \) had an undrained shear strength \( c_L = 2 \text{kPa} \). If a soil with a water content equal to its liquid limit is being sheared in undrained triaxial compression, then the undrained strength \( c_L \) defines the radii of the Mohr circles of total and effective stress (Fig. 9.13a) and specifies the deviator stress \( q_L = 2c_L \) mobilised on the critical state line for the soil (Fig. 9.13b). Calculation of the effective mean stress

\[
P'_L = \frac{q_L}{M} = \frac{2c_L}{M}
\]

(9.21)

requires a value for the slope of the critical state line \( M \) which is related to the effective angle of friction \( \phi' \):

\[
M = \frac{6 \sin \phi'}{3 - \sin \phi'}
\]

(9.22)(7.9bis)

Data of angle of friction (plotted as \( \sin \phi' \)) and plasticity index \( I_p \) have been assembled by Mitchell (1976) and are shown in Fig. 9.14; a corresponding scale of values of \( M \) for triaxial compression, calculated from (9.22), is included. Mitchell observes that 'the peak value of \( \phi' \) decreases with increasing plasticity index and activity'. The spread is large but can be approximately described by the equation

\[
\sin \phi' = 0.35 - 0.1 \ln I_p
\]

(9.23)

However, for present purposes an average value \( M \approx 1.0 \) (corresponding to \( \phi' = 25.4^\circ \)) can be taken. Then with \( c_L = 2 \text{kPa} \), the mean effective stress

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**Fig. 9.14** Relationship between \( M \) or \( \sin \phi' \) and plasticity index \( I_p \) for normally compressed soils [after Mitchell, 1976, with additional data (o) from Brooker and Ireland, 1965].
on the critical state line at a water content equal to the liquid limit is, from (9.21), \( p'_L \sim 4 \text{kPa} \).

This statement about a mean effective stress at the critical state for soil having a particular liquidity can be extended to a proposition about liquidity index and effective stress in normal compression if a model of soil behaviour such as one of the elastic-plastic models discussed in Chapters 4 and 5 is available. The relative positions in the compression plane of the critical state line and a normal compression line are controlled by the geometries of the yield loci and plastic potential curves in the \( p' : q \) plane. As an example, details of the calculation which can be made using the Cam clay model of Chapter 5 are presented in Section 9.4.5. It emerges there that an approximate estimate of the vertical effective stress in one-dimensional normal compression at a given specific volume, water content, or liquidity index can be obtained by doubling the mean effective stress on the critical state line at the same value of the volumetric variable.

If the chosen value of water content is the liquid limit \( w_L \), then the corresponding mean effective stress on the critical state line is \( p'_L \sim 4 \text{kPa} \), and the deduced corresponding vertical effective stress in one-dimensional normal compression is \( \sigma'_v \sim 8 \text{kPa} \). All these calculations are independent of the chosen value of water content. At the plastic limit, the remoulded undrained strength is assumed to be 100 times higher than at the liquid limit, and the mean effective stress on the critical state line and the vertical effective stress on the one-dimensional normal compression line are similarly 100 times higher. Assuming a constant compressibility between the liquid limit and plastic limit implies a relationship similar to (9.20) linking liquidity and vertical effective stress:

\[
\sigma'_v = 8 \times 100^{(1 - I_L)} \text{kPa}
\]  
(9.24)

'Sedimentation compression curves' (i.e. data of vertical effective stress and specific volume) for one-dimensionally normally compressed 'argillaceous deposits' at 21 sites gathered by Skempton (1970a) are shown in Fig. 9.15a. The slope of each group of data points is an indication of the compressibility of that particular deposit, and the wide range of compressibility apparent in Fig. 9.15a is an indication of a similarly wide range of values of plasticity index. The data in Fig. 9.15a are brought together into a narrower band when the volumetric parameter is converted to liquidity index for each sample (Fig. 9.15b). These data confirm the proposition of an approximately unique relationship between vertical effective stress and liquidity index. Expression (9.24), plotted in Fig. 9.15b, provides a reasonable match to the assembled data.

Most soil deposits are not normally compressed, but are overconsolidated
because of erosion of overburden or other effects. A typical soil element such as $A$ in Fig. 9.16 may have experienced the history shown in the compression plane: normal compression to $A_1$ followed by swelling to $A_2$ with removal of overburden. An average one-dimensional normal compression line relating liquidity index and vertical effective stress for

Fig. 9.15 (a) Data of specific volume $v$ or water content $w$ and vertical effective stress $\sigma'$, for normally compressed argillaceous sediments; (b) spread of data from (a) with water contents normalised to liquidity index $I_L$ (after Skempton, 1970a).
insensitive soils has been proposed. A soil which is overconsolidated has a combination of liquidity index and vertical effective stress which does not plot on this line. The compression and swelling processes shown in the $\sigma'_v:V$ compression plane in Fig. 9.16 can just as well be plotted in a $\sigma'_v:1_L$ compression plane (Fig. 9.17), and the position of point $A_2$ relative to the normal compression line $LA_1P$ can be used to estimate the in situ overconsolidation ratio $n$ of the soil.

It is convenient to assume that volume changes occur on unloading according to an expression of the form

$$v = v_s - \kappa^* \ln \sigma'_v$$

(9.25)

In other words, the line linking $A_2$ and $A_1$ in Figs. 9.16 and 9.17 is straight. (The details of this one-dimensional unloading process and the relationship of $\kappa^*$ to other soil properties are considered in Section 10.3.2.) If the ratio of slopes of unloading and normal compression lines $\kappa^*/\lambda$ is known, the point $A_1$ can be re-established by projection at the appropriate slope from

Fig. 9.16 (a) Normal compression of soil at $A$; (b) overconsolidation of soil at $A$ caused by erosion of overburden.

Fig. 9.17 Estimation of past maximum vertical effective stress from in situ combination of liquidity index and vertical effective stress.
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$A_2$ to intersect the normal compression line as shown in Fig. 9.17 to define past maximum stress $\sigma'_{vm}$ and $n = \sigma'_{vm}/\sigma'_{vi}$ (a procedure suggested by Wroth, 1979). This procedure can be described analytically as

$$\frac{\ln n}{\ln 100} = \frac{(1 - I_{Ld}) - \ln(\sigma'_{vi}/\sigma'_{vl})/\ln 100}{\Lambda^*}$$  \hspace{1cm} (9.26)

where

$$\Lambda^* = 1 - \frac{\kappa^*}{\lambda}$$  \hspace{1cm} (9.27)

and $\sigma'_{vl} = 8$ kPa. Expression (9.26) can be conveniently rewritten

$$\log_{10} n = \frac{2(1 - I_{Ld}) - \log_{10}(\sigma'_{vi}/8)}{\Lambda}$$  \hspace{1cm} (9.28)

with $\sigma'_{vi}$ measured in kilopascals. This expression, with $\Lambda^* = 0.8$ (implying $\kappa^*/\lambda = 0.2$), has been used to estimate the variation of overconsolidation ratio with depth for the site on the Thistle field in the North Sea, for which strength estimates were shown in Fig. 9.8. Only very limited oedometer data are available to provide corroborative evidence for these estimates of overconsolidation ratio; but a plausible trend is indicated (Fig. 9.18), with a general fall of overconsolidation ratio towards 1 in the top 18 m and below this depth a higher value, where the strength data in Fig. 9.8 showed also a sharp increase.

More generally, charts of strength and liquidity and of overburden pressure (vertical effective stress) and liquidity can be used to make qualitative statements about the consolidation history of soil deposits (Wood, 1985b). Line X is expression (9.20) in Fig. 9.19a, a chart of liquidity

Fig. 9.18 Values of overconsolidation ratio for location in Thistle field, North Sea (\(\cdot\), estimated values, \(\times\), values from oedometer tests).
Index and strength (cf. Fig. 9.9); in Fig. 9.19b, a chart of liquidity index and vertical effective stress, line Y is expression (9.24) (cf. Fig. 9.15). Four possibilities are distinguished in Figs. 9.19a, b.

Normally consolidated insensitive soils have strengths dependent on their liquidity which plot around line X (A in Fig. 9.19a). Such soils also have vertical effective stresses which plot around line Y (A in Fig. 9.19b). Since strength is a function primarily of water content, or liquidity index, overconsolidated soils also plot around line X (B in Fig. 9.19a). However, overconsolidated soils have lower liquidities than normally compressed soils and consequently have vertical effective stresses which plot below line Y (B in Fig. 9.19b).

Lines X and Y in Fig. 9.19 relate to soil which has been remoulded and disturbed and has lost its natural structure. Natural sensitive soils can have peak strengths considerably higher than their remoulded strengths and plot to the right of line X (C in Fig. 9.19a). When their structure is disturbed, the reliable strength for their in situ liquidity is considerably lower than this peak strength. Such soils contain too much water to be good for them (i.e. their structure is much more open than their present stress level implies); they therefore plot above line Y (C in Fig. 9.19b). When their structure is disturbed, they may reach eventual equilibrium at the low liquidity corresponding roughly to the same vertical effective stress but with the expulsion of a large volume of water from their collapsing pores.

Another consolidation possibility is so-called underconsolidation. Typically, in a saturated soil deposit with the water table at the surface,

Fig. 9.19  Plotting of field data of undrained strength $c_u$, vertical effective stress $\sigma'_v$, and liquidity index $I_L$ in relation to proposed unique relationships.
the effective vertical stress at depth $z$ is
\[ \sigma' = \gamma z \]  
(9.29)
However, if the rate of deposition of a soil is high, then part of the total overburden pressure at any depth
\[ \sigma_v = \gamma z \]  
(9.30)
is supported by excess pore pressures instead of by effective stresses. The pore pressure exceeds the hydrostatic pore pressure,
\[ u > \gamma_w z \]  
(9.31)
and the effective stresses are lower than expected from (9.29),
\[ \sigma' < (\gamma - \gamma_w)z = \gamma' z \]  
(9.32)
If values of vertical effective stress are calculated on the assumption that only hydrostatic pore pressures
\[ u = \gamma_w z \]  
(9.33)
are present, then the liquidity:vertical effective stress combination will plot to the right of line $Y$ ($D$ in Fig. 9.19b).
Underconsolidated soils are in fact normally compressed soils (i.e. the current vertical effective stress is the largest the soil has known and is the same as the preconsolidation pressure $\sigma'_{vc} = \sigma'_v$), but the vertical effective stresses are lower than expected. If the actual pore pressures $u$ were known, then the effective stresses for these soils, calculated from
\[ \sigma'_v = \sigma_v - u = \gamma z - u \]  
(9.34)should plot around line $Y$. The strengths of underconsolidated soils should still correspond with their actual liquidities since the in situ liquidities reflect the effective stresses of which the soil is actually aware. Thus, $D$ in Fig. 9.19a lies around line $X$, and in terms of strength and liquidity, the soil looks just like any other young normally compressed deposit.
An indication that a particular soil is probably underconsolidated is of importance because large settlements are to be expected as the soil comes into equilibrium, with pore pressures falling to hydrostatic values, even before settlements occur under imposed surface loads.

9.4 Background to correlations

9.4.1 Liquid limit
Albert Atterberg (1911), a Swedish agricultural engineer, described a suite of tests that might be performed to determine the water contents at which the character of the mechanical behaviour of a clay went through various transitions. Of these, two have been absorbed into the soil mechanics canon: the liquid and plastic limit tests which were reckoned
by Atterberg to define the range of water content for which a clay could be described as a plastic material.

Atterberg's test to determine the liquid limit (flytgränsen: literally, flowing limit) of soil involved mixing a pat of clay 'in a little round-bottomed porcelain bowl of 10–12 cm diameter'. A groove was cut through the pat of clay with a spatula, and the bowl was then struck many times against the palm of one hand. If after many blows the groove was only closed to an insignificant height at the base of the pat, then the clay was

Fig. 9.20 (a)–(c) Casagrande liquid limit apparatus (after British Standard BS1377:1975); (d) variation with water content of number of blows to close groove in Casagrande liquid limit test (after Casagrande, 1932); (e) section through groove in Casagrande liquid limit test.
defined to be at its liquid limit: its water content was just not high enough for it to flow like a liquid given the opportunity to do so.

To make the measurement of the liquid limit more scientific, the test was standardised using the apparatus proposed by Casagrande (1932). This is a device (Figs. 9.20a, b) for performing mechanically the procedure described by Atterberg. The standard metal bowl containing the pat of soil is given repeated standard blows by being dropped through 10 mm onto a hard rubber base. The procedure is further tightened by requiring that, if a soil is at its liquid limit, the groove made in the pat of soil with a standard grooving tool (Fig. 9.20c) should close at its base over a distance of 13 mm in exactly 25 blows. In practice, liquid limit is usually obtained by interpolation from 'flow curves' (Casagrande, 1932) relating water content with number of blows (Fig. 9.20d).

In Section 9.2, the liquid limit of soils was associated with a particular undrained strength. The Casagrande liquid limit test requires the failure of miniature soil slopes (cross section, Fig. 9.20e): each slope is formed with a height of 8 mm and slope angle of 60.6°. Failure is induced by means of a series of shock decelerations as the bowl of the apparatus strikes the hard rubber base. The undrained stability of these slopes, of height $H$ and formed in soil of strength $c_u$ and total unit weight $\gamma$, can be studied in terms of the dimensionless group $c_u/\gamma H$. Since the unit weight $\gamma$ depends on the value of the liquid limit, the strength $c_u$ of a soil at its Casagrande liquid limit also depends on the value of that liquid limit.

Data of strengths of soils at water contents around their Casagrande liquid limits have been reported by Youssef, el Ramli, and el Demery.

![Variation of undrained strength with water content at water contents close to liquid limit (after Youssef, el Ramli, and el Demery, 1965).](image-url)
(1965) (Fig. 9.21). The strengths were measured by means of a laboratory miniature vane. There is a clear trend for the strength at the liquid limit $c_L$ to fall as the liquid limit (determined with the Casagrande apparatus) increases, following the average line which has been drawn in the figure. The extreme values on this line are, for soil $A$: $w_L = 0.3$, $c_L = 2.4$ kPa; and for soil $B$: $w_L = 2.0$, $c_L = 1.3$ kPa. If we assume that the specific gravity of soil particles is the same for all the soils, $G_s = 2.7$, then water contents can be converted into unit weights using

$$
\gamma = \frac{G_s(1 + w)\gamma_w}{1 + G_s w}
$$

(9.35)

For soil $A$, $\gamma = 1.94 \gamma_w$ and $c_u/\gamma H = 15.8$; for soil $B$, $\gamma = 1.27 \gamma_w$ and $c_u/\gamma H = 13.0$. The closeness of the values of the dimensionless group $c_u/\gamma H$ shows the way in which the differences in strength and in unit weight for the soils balance each other.

The fall-cone test is now preferred to the Casagrande apparatus for determining the liquid limit, not only because it is more repeatable and consistent, and less operator sensitive but also because it provides a direct strength index; it is clear that the Casagrande apparatus cannot do this. Even though the combinations of cone parameters adopted in Scandinavia and Britain do appear to give liquid limits which correspond reasonably with Casagrande liquid limits, the very different procedures of the fall-cone test and the Casagrande device must lead to different values of liquid limits for extreme soil types.

9.4.2 Plastic limit

Atterberg’s (1911) test to determine the plastic limit (utrullgränser, literally, rolling-out limit) of soil involved rolling the clay on a piece of paper with the fingers until it formed ‘fine threads’. The fine threads were put together and rolled again, and the process was repeated until the clay could no longer be rolled but broke into pieces. The clay was then defined to be at its plastic limit: its water content was just not high enough for it to be capable of plastic deformation and moulding.

The procedure for determining the plastic limit $w_p$ has also been tightened. Casagrande (1932) notes that the water content at which a thread of soil crumbles depends on the diameter of the thread, and a diameter of 3 mm is now standard. British Standard test 3 specifies other details necessary for the correct performance of this test.

In Section 9.3, a strength of $\sim 200$ kPa was proposed for soils at their plastic limit. Schofield and Wroth (1968) have suggested that the plastic-limit test ‘implies a tensile failure, rather like the split-cylinder or Brazil
9.4 Background to correlations

test of concrete cylinders. In the split-cylinder test (Kong and Evans, 1975), a cylinder of concrete is loaded across a diameter (Fig. 9.22a); and when the load is increased, the cylinder eventually splits on this diameter. For an elastic cylinder of diameter \( D \), loaded with point loads \( P \) per unit length, Timoshenko (1934) shows that there is a uniform tensile stress

\[
\sigma_t = \frac{-2P}{\pi D}
\]  

(9.36)

acting across this diameter \( AA \).* The normal compressive stress acting on the transverse diameter \( BB \) has the distribution shown in

Fig. 9.22  (a) Cylindrical specimen loaded diametrically in compression; (b) stresses on transverse diameter \( BB \); (c) Mohr’s circles of total (T) and effective (E) stress for element at centre of specimen (shear stress \( \tau \), total normal stress \( \sigma \), effective normal stress \( \sigma' \)).

*For loads applied, practically, over a small part of the circumference, Wright (1955) shows that most of the diameter is still subjected to a tensile stress given by (9.36).
9 Index properties

Fig. 9.22b, with a maximum at the centre,

\[ \sigma_c = \frac{6P}{\pi D} \]  

(9.37)

The vertical compressive stress has this magnitude at all points on the vertical diameter AA. The Mohr circle of total stress at all points on this diameter is therefore as shown in Fig. 9.22c. For soil which fails according to a frictional Mohr–Coulomb failure criterion, as discussed in Chapter 7, such a total stress state can be sustained only if there are negative pore pressures in the soil so that the Mohr circle of effective stress lies to the right of the total stress circle (Fig. 9.22c). The magnitude of this negative pore pressure depends, of course, on the effective angle of friction of the soil and on the size of the total stress circle, the undrained strength that is mobilised, which depends on the normal load \( P \) (Fig. 9.22a).*

If the strength at the plastic limit \( c_p \sim 200 \text{kPa} \), then \( \sigma_c \sim 300 \text{kPa} \) and \( \sigma_t \sim 100 \text{kPa} \). The total stress along the axis of the thread of clay being rolled is roughly zero, so the total mean stress is \( p_r \sim 67 \text{kPa} \). It was suggested in Section 9.3 that the mean effective stress at the critical state would be about twice the undrained strength at the same water content. That implies a mean effective stress \( p'_e \sim 400 \text{kPa} \) and a suction \( u \sim -333 \text{kPa} \). Suctions are able to exist in soils because of the action of surface tension at the air–water interface, where pore water menisci span between soil particles. Quite high suctions can exist without the soil becoming unsaturated, that is, without the menisci being drawn back from the surface of the soil. Suctions between 200 and 300 kPa are shown by Brady (1988) for London clay, with no external loading, at water contents near the plastic limit. The plastic limit test is probably a less consistent strength test than the Casagrande liquid limit test, but large suctions are certainly present in soils near their plastic limits, which can account for the mobilisation of significant undrained shear strengths.

9.4.3 Plasticity and compressibility, liquidity and strength

The simple correlations proposed in Section 9.3 between plasticity and compressibility, and between liquidity and strength, worked because of the assumption that there was a ratio of 100 between strengths of soils at their plastic and liquid limits. This figure was chosen as a convenient round number based on scattered data shown in Fig. 9.5. In the absence of any other information, this provides a good starting point for estimating

*The British Standard (BS 1377: 1975) says only that ‘it is important to maintain uniform rolling pressure throughout the test’.
9.4 Background to correlations

strengths and compressibilities from index properties. If other information is available, then it may be possible to improve these estimates.

It is proposed here that correlations are likely to work most satisfactorily for groups of related soils. Many different clay minerals contribute to the claylike properties of soils, such as plasticity. The plasticity of a soil depends on the type of clay mineral present as well as on the amount of clay present.* Soil samples taken over a limited geographical area are likely to have a common geological history, and though there may be variations in the proportion of clay present in individual samples, the nature of the clay mineral is likely to show much less variability.

Data of plasticity \( I_p \) and clay content \( C \) from a number of sites are shown in Fig. 9.23, adapted from Skempton (1953). Each set of points lies around a line through the origin, and Skempton defines the activity \( A \) of a soil as the slope of this line:

\[
A = \frac{I_p}{C}
\]  

(9.38)

Fig. 9.23 Relationship between plasticity index \( I_p \) and clay fraction \( C \) for natural clays and clay minerals, with values of activity \( A = I_p/C \) indicated (after Skempton, 1953).

* A soil which consists only of sand particles does not exhibit any plasticity characteristics which can be observed in these standard tests for the liquid and plastic limits, and such soils are traditionally recorded as non-plastic.
Some activity values for soils containing pure clay minerals are also shown. Evidently, the activity of the soil is the plasticity of the pure clay fraction.

Variations of strength with liquidity index for two sets of artificial soils are shown in Fig. 9.24. The data are taken from tests on mixtures of kaolinite and sand, and mixtures of montmorillonite and sand, reported by Dumbleton and West (1970); the proportions of clay varied from 25 to 100 per cent. The authors report liquid limits determined using the Casagrande apparatus (Section 9.4.1). However, from their data of strengths (measured with a miniature laboratory vane), we could extract for each mixture the water contents corresponding to a strength of 1.7 kPa, to make the data consistent with the fall-cone index test of Section 9.2. Values of liquidity index could be calculated using these pseudo-fall-cone liquid limits.

The data for the montmorillonite and kaolinite mixtures diverge towards their plastic limits (as $I_L \to 0$), and though the spread within each band is reasonably small, the ratio of strengths at plastic and liquid limits is of the order of 100 for the montmorillonite mixtures but is much nearer to

Fig. 9.24 Variation of remoulded undrained strength $c_u$ with liquidity index $I_L$ for mixtures of (K) kaolinite and (M) montmorillonite with natural quartz sand (values of clay content $C$ marked for each curve) (data from Dumbleton and West, 1970).
30 for the kaolinite mixtures. So the ratio 100 introduced in Section 9.3 can be replaced by a more general ratio \( R = c_p / c_L \) which probably depends on the activity of the clay mineral present.

A more general form of (9.13) is then

\[
\lambda = \frac{I_p G_s}{\ln R} \tag{9.39}
\]

and the value of \( R \) can be estimated for a group of soils by inserting into (9.39) the value of compressibility \( \lambda \) measured in an oedometer test over a reasonable range of effective stress.

A corresponding more general form of (9.20) is then

\[
c_u = c_L R^{(1 - f_L)} \tag{9.40}
\]

where \( c_L \) is the strength of soils at their liquid limit, which was shown to be 1.7 kPa in Section 9.2 and was approximated to 2 kPa in Section 9.3.

### 9.4.4 Liquidity and critical states

The simple experimentally observed form of critical state lines

\[
q = M \rho' \tag{9.41}(\text{cf. 9.21})
\]

\[
V = \Gamma - \lambda \ln \rho' \tag{9.42}(\text{cf. 9.3})
\]

requires three soil parameters for its definition: its slope in the compression plane \( \lambda \) (or \( C_c \)), which has been shown in Section 9.3 to be related to plasticity index (9.13); its slope in the effective stress plane \( M \) (or \( \phi' \)), which has been shown to be weakly related to plasticity index (Fig. 9.14); and its location in the compression plane, the parameter \( \Gamma \), which is the value of specific volume at a mean effective stress \( \rho' = 1 \) unit of stress. Conventionally, the unit of stress is 1 kPa, but any units can be used without changing the position of the critical state line in the compression plane.

A link between \( \Gamma \) and \( w_L \) can now be deduced. Equation (9.42) can be written in terms of a reference pressure \( p'_L \), the mean effective stress at the critical state for a water content equal to the liquid limit (9.21). It then becomes

\[
v = v_L - \lambda \ln \frac{p'}{p'_L} \tag{9.43}
\]

where

\[
v_L = 1 + G_s w_L \tag{9.44}
\]

This is the specific volume for the soil at its liquid limit and has replaced \( \Gamma \). If \( \Gamma \) is to be retained, then from expressions (9.42)–(9.44) (Figs. 9.25a, b),

\[
\Gamma = 1 + G_s w_L + \lambda \ln p'_L \tag{9.45}
\]
or with (9.39),
\[ \Gamma = 1 + G_s \left( w_L + I_p \frac{\ln p_L'}{\ln R} \right) \]  
(9.46)

For \( p_L' = 4 \text{kPa} \) and \( R = 100 \), (9.46) becomes
\[ \Gamma = 1 + G_s (w_L + 0.3 I_p) \]  
(9.47)

Although this may seem a little devious, it is an indication that the parameter \( \Gamma \) is essentially equivalent to liquid limit; a soil with a high liquid limit has a high value of \( \Gamma \).

For the Weald clay (for which data of critical states were discussed in Section 6.3), Roscoe, Schofield, and Wroth (1958) quote liquid limit \( w_L = 0.43 \), plastic limit \( w_p = 0.18 \), and clay content \( C = 0.4 \). Then (9.47), using \( G_s = 2.75 \), leads to \( \Gamma = 2.389 \). In Section 6.3, a value of \( \Gamma \) of 2.072 was deduced from the experimental data, which were obtained over a range of mean effective stress of about 70–700 kPa. The value of \( \Gamma \) quoted there was itself an extrapolation from about 70 kPa to 1 kPa. Since it has

---

Fig. 9.25 (a,b) Relationship between \( \Gamma \) and liquid limit \( w_L \) \((I_L = 1)\) for reference volumetric parameter on critical state line (csl); (c) linear approximations to curved critical state line.
already been noted that the slope of critical state lines tends to fall as the mean stress increases, it is not surprising that this value of $\Gamma$ is lower than that calculated from (9.47) (Fig. 9.25c). Of course, in practice, if a simple relationship such as (9.42) is to be used as an approximation for calculation purposes, then the values of $\lambda$ and $\Gamma$ should be chosen to give the best possible fit in the region of effective stresses of engineering interest in any particular problem.

Soils of different plasticities have different values of compressibility $\lambda$ and produce different critical state lines when these are plotted in terms of water content or specific volume and mean effective stress (Fig. 9.26). The critical state lines for any two soils intersect at a point. Schofield and Wroth (1968) suggested that the critical state lines for all soils might pass through a single point in the $\ln p' - v$ plane, which they called the $\Omega$-point. This is probably too bold a generalisation. Here the proposal that critical state lines for related soils (soils perhaps of similar activity) pass through an $\Omega$-point (Fig. 9.26) is explored.

Such an $\Omega$-point constitutes a relationship between liquid limit and plasticity for soils. If $p'_{\Omega}$, $v_{\Omega}$, and $w_{\Omega}$ are values of mean effective stress, specific volume, and water content at the $\Omega$-point, then for any one of the related soils,

$$\lambda = \frac{(w_L - w_{\Omega})G_s}{\ln(p'_L/p'_L)} \quad (9.48)$$

and with (9.39),

$$I_p = \frac{(w_L - w_{\Omega}) \ln R}{\ln(p'_L/p'_L)} \quad (9.49)$$

The first expression, (9.48), is a relationship between compressibility $\lambda$ and

Fig. 9.26 Critical state lines for related soils $A$, $B$, and $C$ passing through single $\Omega$-point in compression space.
liquid limit $w_L$ similar to the approximate relationship proposed by Terzaghi and Peck (1948) for remoulded clays, on the basis mainly of data collected by Skempton (1944) for a wide range of soils:

$$\lambda = 0.3(w_L - 0.1)$$  \hspace{1cm} (9.50)

The second expression, (9.49), is a relationship of the same type as Casagrande's (1947) $A$-line (Fig. 9.27a), which is used for classification of soil types:

$$I_p = 0.73(w_L - 0.2)$$  \hspace{1cm} (9.51)

This line is produced by Casagrande as an 'empirical boundary between typical inorganic clays which are generally above the $A$-line, and plastic soils containing organic colloids which are below it. Also located below the $A$-line are typical inorganic silts and silty clays...'. If the results of index tests on a number of samples of related soils are plotted on this plasticity chart, the points tend to lie on a straight line which is often approximately parallel to the $A$-line.

Given a group of related soils which plot on a line

$$I_p = A(w_L - B)$$  \hspace{1cm} (9.52)

comparison with (9.49) shows that this is equivalent to defining an $\Omega$-point given by

$$\ln \frac{p'_\Omega}{p'_L} = \frac{1}{A} \ln R$$  \hspace{1cm} (9.53)

$$w_\Omega = B$$  \hspace{1cm} (9.54)

or

$$v_\Omega = 1 + G_s B$$  \hspace{1cm} (9.55)

Then if $p'_L$ is fixed, for example at 4 kPa, and $R$ is taken as 100, the position of each of the family of critical state lines in Fig. 9.26 immediately follows. Thus, $A = 0.73$ implies $p'_\Omega = 2197$ kPa, and $B = 0.2$ implies $w_\Omega = 0.2$ and $v_\Omega = 1.54$, for $G_s = 2.7$.

Plasticity data of Norwegian marine clays, from Bjerrum (1954), are plotted in Fig. 9.27b. These lie on a line nearly parallel to Casagrande's $A$-line

$$I_p = 0.79(w_L - 0.17)$$  \hspace{1cm} (9.56)

which, with $G_s = 2.7$, $R = 100$, and $p'_L = 4$ kPa, implies $p'_\Omega = 1383$ kPa and $v_\Omega = 1.464$. These clays were deposited in a saline environment, but as a result of sea-level changes fresh water now flows through them. This fresh water leaches the salt from the clays and leaves them in a sensitive state (see Section 9.5). The effect of leaching the salt is to reduce the plasticity and liquid limit of the clay. Bjerrum (1954) quotes the results of a
laboratory experiment in which two clay samples were sedimented in salt water; one was leached and the other left unleached, and index properties for the two samples were measured. The path of the leaching process is shown in Fig. 9.27b. The clay both before and after leaching is a Norwegian marine clay; the effect of leaching is merely to move the clay down the line (9.56) in the plasticity chart. In terms of critical state lines, this implies

Fig. 9.27 Relation between liquid limit \( w_L \) and plasticity index \( I_p \). (a) Typical soils: 1. gumbo clays (Mississippi, Arkansas, Texas); 2. glacial clays (Boston, Detroit, Chicago, Canada); 3. clay (Venezuela); 4. organic silt and clay (Flushing Meadows, Long Island); 5. organic clay (New London, Connecticut); 6. kaolin (Mica, Washington); 7. organic silt and clay (Panama); 8. micaceous sandy silt (Cartersville, Georgia); 9. kaolin-type clays (Vera, Washington, and South Carolina) (after Casagrande, 1947). (b) Norwegian marine clays (data from Bjerrum, 1954, 1967).
a reduction in compressibility without changing the \( \Omega \)-point: leaching changes the critical state line from \( A \) to \( B \) to \( C \) in Fig. 9.26.

Thus, given some knowledge about a group of soils, the liquid limit \( w_L \) is sufficient to place a soil within this group and to obtain estimates for the location and slope of the critical state line for that soil.

### 9.4.5 Liquidity and normal compression

Elastic-plastic models constructed within the general framework discussed in Chapter 4 can be used to calculate the relative positions in the \( \ln p':v \) compression plane of the critical state line and a normal compression line, for example, the one-dimensional normal compression line. This relative position is most conveniently described in terms of the ratio of stresses on the two lines, which in Chapter 4 have been assumed to be parallel, at any particular water content or specific volume.

In general, plastic potentials and yield loci in the \( p':q \) effective stress plane are not identical. It was noted in Section 6.1 that the critical state

---

![Diagram](image)

---

**Fig. 9.28** Relation between values of mean effective stress \( p' \) on normal compression line (ncl) and critical state line (csl) at same specific volume (yl, yield locus; pp, plastic potential): (a) \( p':q \) effective stress plane; (b) \( v:p' \) compression plane; (c) \( v:p' \) compression plane with \( p' \) plotted on logarithmic scale.
9.4 Background to correlations

The value of \( \Lambda \) indicates the relative magnitudes of the slopes of the normal compression and unloading-reloading lines (Fig. 9.28c). From (9.59), \( \kappa/\lambda = 1 \) implies \( \Lambda = 0 \); \( \kappa/\lambda = 0.5 \) implies \( \Lambda = 0.5 \); \( \kappa/\lambda = 0 \) implies \( \Lambda = 1 \). Although the compressibility \( \lambda \) is well linked with plasticity of soils (Section 9.3), it is harder to make definitive statements about the elastic volumetric parameter \( \kappa \). This is partly because \( \kappa \), which represents volumetric changes occurring during isotropic unloading and reloading, is hardly ever measured and partly because a single unloading-reloading line is at best an idealisation of a real hysteretic response (Fig. 3.13a). Given such a response, the value of \( \kappa \), determined perhaps as the average slope of the hysteresis loop, depends on the size of that loop and the extent to which the maximum mean stress is removed.

Much more often, a swelling index \( \kappa^* \) is measured which represents volume changes occurring during one-dimensional unloading (Fig. 9.30a) according to an expression of the form

\[
u = v_s - \kappa^* \ln \sigma'_v
\]

(9.25bis)

which involves only the vertical effective stress \( \sigma'_v \) applied in an oedometer test. It will be seen in Section 10.3.2 that changes in stress ratio \( \eta \) occur during one-dimensional unloading so that changes in mean effective stress and vertical effective stress are not in constant proportion, as implied by

Fig. 9.29 Ratio \( r = p'_w/p'_d \) of mean effective stresses on one-dimensional normal compression line and critical state line for same yield locus related to angle of shearing resistance \( \phi' \) according to Cam clay model.
(9.63) during one-dimensional normal compression, and the connection between $\kappa$ and $\kappa^*$ is not straightforward.

Hysteresis loops seen in one-dimensional unloading and reloading tend to be much larger than those seen in unloading and reloading at constant stress ratio. A standard procedure for determining $\kappa^*$ might involve

Fig. 9.30 (a) One-dimensional compression and unloading of clay from Gulf of Mexico (depth 0.82 m below seabed, water depth 3.7 km) (after Bryant, Cernock, and Morelock, 1967); (b) possible standard procedure for determining average slope of unloading-reloading cycle in oedometer by reducing vertical effective stress by factor of 4.

Fig. 9.31 Data of parameter $\Lambda$ and plasticity index $I_p$ (data collected by Mayne, 1980).
unloading the vertical effective stress by a definite factor of, for example, 2 or 4 (Fig. 9.30b). In the absence of such a standard procedure, the interpretation of published values of swelling index \( r^* \) is difficult.

The parameter \( \Lambda \) was previously introduced in Section 7.2 as an exponent in an expression, (7.27), linking undrained strength and overconsolidation ratio. Mayne (1980) and Mayne and Swanson (1981) have fitted an expression of this form to published strength and overconsolidation data for many soils in order to deduce the value of \( \Lambda \) of which each soil seems to be aware. Their 105 values have been plotted against plasticity index \( I_p \) in Fig. 9.31. There is great scatter and no obvious trend, and some of the values seem implausibly low or high. There is possibly a fall of \( \Lambda \) with increasing plasticity. (The mean value of \( \Lambda \) is 0.63 with a standard deviation of 0.18.)

Leaving the choice of \( \Lambda \) open, we can evaluate the quantity \( 6r^3/[M(1 + 2K_{one})] \) from (9.65) for various values of angle of friction \( \phi' \) (which controls \( M, K_{one} \), and, through the Cam clay model, \( r \)) and different values of \( \Lambda \). Curves are plotted in Fig. 9.32. For average soils with \( \phi' \) in the range of 20° to 25° and \( \Lambda \) in the range of 0.6 to 0.8, a convenient value of this quantity is about 4. Thus, at a given specific volume or liquidity index, the vertical effective stress in one-dimensional normal compression is about four times the undrained strength and about twice the mean effective stress at the critical state. This is the value that was adopted in Section 9.3.

Fig. 9.32 Ratio of vertical effective stress in one-dimensional normal compression, and undrained strength at same specific volume as function of \( \Lambda \) and angle of shearing resistance \( \phi' \) according to Cam clay model.
9.5 Sensitive soils

An example of a sensitive soil was shown in Fig. 9.11. Such soils have undisturbed strengths which may be considerably higher than their disturbed strengths. The ratio of undisturbed to remoulded strength is defined as the sensitivity $S_i$:

$$S_i = \frac{c_u}{c_{ur}}$$

(9.70)

Data of sensitivity and liquidity index for certain sensitive soils from various parts of the world are presented in Fig. 9.33. Related soils show a clear trend of increasing sensitivity with increasing liquidity index. For example, for the data of Norwegian marine clays in Fig. 9.33 (from Bjerrum, 1954), a relationship of the form

$$S_i = \exp(kI_L)$$

(9.71)

with $k \approx 2$ provides a reasonable fit. This implies a sensitivity $S_i \approx 7.4$ for a clay at its liquid limit ($w = w_L$, $I_L = 1$). A value $k = 0$ implies insensitive soil. Insensitive behaviour is usually observed for clays at or below their plastic limit, so (9.71) should be used only for $I_L > 0$.

Equations (9.70) and (9.71) can be combined with (9.20) to produce an expression with which undisturbed strengths can be estimated:

$$c_u = c_L R \exp[(k - \ln R)I_L] \quad \text{for} \quad I_L > 0$$

(9.72)

This equation is best used for related soils with known values of $k$ and $R$.

Fig. 9.33 Interrelationship between sensitivity and liquidity index for natural clays (△, Skempton and Northey, 1953; ◦, Bjerrum, 1954; ○, Bjerrum and Simons, 1960; +, Bjerrum, 1967).
Bjerrum (1954) shows that the sensitivity of Norwegian clays is closely linked to the salt concentration in their pore water. It was noted in Section 9.4.4 that as the salt concentration changes, the index properties of the clay also change. As Fig. 9.34 shows, the plastic limit falls very slightly as the salt is leached out but the liquid limit falls markedly, and hence the plasticity drops considerably. The water content remains essentially constant because the structure of the clay does not change, so there is a marked rise in liquidity index. Data from adjacent borings in leached and unleached clay in Drammen are presented by Bjerrum (1967). The water content is more or less the same in both, but the liquidity index changes from about 1.1 to 2. An increase of sensitivity from 8 to 200–300 is quoted by Bjerrum (this leaching path is plotted on Fig. 9.33), whereas (9.71) with \( k = 2 \) suggests an increase from 9 to 55. High values of sensitivity are not likely to be accurate, and the precise value is of somewhat academic interest since it indicates primarily that it may be extremely dangerous to rely on the undisturbed strength for engineering purposes. Nevertheless, the link

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**Fig. 9.34** Changes in properties of a normally compressed Norwegian marine clay when salt concentration is reduced by leaching with fresh water: (a) sensitivity \( S_t \); (b) undisturbed and remoulded strengths; (c) water content \( w \), liquid limit \( w_L \), and plastic limit \( w_p \) (after Bjerrum, 1954).
between sensitivity and liquidity at constant water content is clear. The effect of leaching is to move the state of the soil up line \( B \) (for \( k = 2 \)) in Fig. 9.35: as the liquidity index increases, the separation of line \( B \) and line \( A \) (\( k = 0 \) for remoulded soil) increases, and hence the sensitivity increases.

A similar discussion can be applied to expression (9.24) relating vertical effective stress and liquidity index for normally compressed soils. It is evident from Fig. 9.34 that sensitive soils are being left with water contents that are too high to be good for them. The high liquidity and low in situ effective stress leaves a surplus of liquidity over that predicted by (9.24): this is an indication of the large amount of water that will be released.

**Fig. 9.35** Expected approximate relationships between liquidity index \( I_L \) and undrained shear strength \( c_u \) for insensitive or remoulded soil (line \( A \)) and for undisturbed, sensitive soil (line \( B \)).

**Fig. 9.36** Idealised relationship between liquidity index and effective overburden pressure, with contours of sensitivity (——) (after U.S. Dept. of the Navy, 1971) and expression (9.73) (—–).
when the structure of the soil is disturbed and the soil tries to reach remoulded equilibrium under the in situ vertical effective stress. Slope failures in such soils can have devastating consequences.

The expression equivalent to (9.72) is

$$\sigma_v' = \sigma_{vL}' R \exp[(k - \ln R)I_L] \quad \text{for } I_L > 0$$

(9.73)

Plotting this with $R = 100$, $\sigma_{vL}' = 8 \text{kPa}$, and various values of $k$ gives a set of lines equivalent to the curves in Fig. 9.36, based on a chart from the design manual Navfac DM-7 (U.S. Department of the Navy, 1971).

Some field data of effective overburden pressure and liquidity index of undisturbed clays collected by Skempton and Northey (1953) are shown in Fig. 9.37. Expression (9.73) is plotted in this figure with values of $k$ between 0 and 3. A value of $k = 3$ provides a reasonable upper limit to the data of sensitivity and liquidity of clays in Fig. 9.33 and also a reasonable outer limit to the data in Fig. 9.37.

Results of one-dimensional normal compression of slurried clays from Gosport and Horten are also shown in Fig. 9.37 for comparison with the

Fig. 9.37 Data of liquidity index and effective overburden pressure for European clays (after Skempton and Northey, 1953) (• and – field data; --- oedometer data for slurried clays).
undisturbed compression curves. For both these clays, the slurried data lie close to the line for insensitive soil with \( k = 0 \). However, whereas the Gosport clay shows negligible in situ sensitivity, with the slurried and undisturbed data lying almost on top of each other, the in situ Horten clay plots close to the line for \( k = 3 \), indicating a high in situ sensitivity.

Expression (9.73) can be rewritten as

\[
I_L = \left[ \frac{\ln R}{\ln R - k} \right] \left[ 1 - \frac{\ln(\sigma'_u/\sigma'_L)}{\ln R} \right]
\]

The ratio of the compressibilities of the undisturbed and remoulded soils is

\[
\frac{k_u}{k_r} = \frac{\ln R}{\ln R - k}
\]

Fig. 9.38 (a) Construction of relationship between specific volume \( v \) and vertical effective stress \( \sigma'_v \) expected for sensitive soil; (b) one-dimensional compression of (1) undisturbed and (2) remoulded Leda clay (after Quigley and Thompson, 1966).
Terzaghi and Peck (1948) suggest that this ratio should be about 1.3 'for an ordinary clay of medium or low sensitivity'. With $R = 100$, $\lambda_u/\lambda_e = 1.3$ in (9.75) implies $k \sim 1$, which in Fig. 9.33 does indeed correspond to clay of medium or low sensitivity.

Yudhbir (1973) suggests a procedure by which field, in situ normal compression curves might be reconstructed or laboratory compression curves corrected for disturbance that occurred during sampling (as in Fig. 9.7). It is implicit in (9.74) that undisturbed and disturbed compression curves intersect at a water content equal to the plastic limit ($I_L = 0$); at this liquidity, sensitivity seems in general to be close to unity. With knowledge of index properties, (9.74) can be converted to a relationship between specific volume and vertical effective stress for completely disturbed soil ($k = 0$, line $A$ in Fig. 9.38a), which can be expected to apply anyway for water contents below the plastic limit, beyond point $P$, with $v_P = 1 + G_s w_p$. If actual oedometer data on compression of disturbed samples were available, they could be used instead. From strength measurements (e.g. with a field vane or a fall-cone), some estimate of the sensitivity of the soil at its in situ water content can be obtained and used to estimate a value for $k$ in (9.71). The normal compression line for in situ soil can then be generated from (9.74), using this value of $k$ (line $B$ in Fig. 9.38a). Recompression of sensitive soils at their in situ specific volume is often a very stiff process, particularly compared with subsequent normal compression (Figs. 9.30a and 9.38b) and so a third line can be drawn in Fig. 9.38a, a horizontal line $C$ at the in situ specific volume. The resulting trilinear relationship, $C-B-A$, suitably smoothed to accommodate nature's abhorrence of sharp corners, is then a more realistic field compression curve (compare Fig. 9.38b).

### 9.6 Strength and overburden pressure

In Section 9.4.5, a convenient average value of 4 was chosen for the ratio of vertical effective stress during one-dimensional normal compression to the undrained strength of a soil at the same water content. This link between strength and overburden pressure should be examined further.

Many soil deposits are not normally compressed, so a general approximate relationship links strength with the equivalent consolidation pressure $\sigma'_{ve}$ – the vertical effective stress which, in one-dimensional normal compression, would produce the current specific volume or water content (see Sections 6.2 and 7.4.1). The general relationship is

$$\frac{c_u}{\sigma'_{ve}} \approx 0.25$$  \hspace{1cm} (9.76)
The strength of a soil is linked with the size of the current, in situ yield locus. The Cam clay model was used to help justify (9.76), but Fig. 9.28 illustrated how a more general elastic-plastic model might be used. The size of an in situ yield locus can be characterised by the preconsolidation pressure $\sigma'_{pc}$, the yield point observed in an oedometer test (Section 3.3). For lightly overconsolidated clays, the difference between the preconsolidation pressure (point $A_1$ in Fig. 9.16b) and the equivalent consolidation pressure (point $C$ in Fig. 9.16b) is small since the slope of the unloading line in this region is usually considerably flatter than the slope of the

![Graph](image-url)

Fig. 9.39  Data of ratio of undrained strength to vertical effective stress ($c_u/\sigma_v'$), and plasticity index $I_p$: (a) after Skempton (1957); (b) after Leroueil, Magnan, and Tavenas (1985), with relationships proposed by Skempton (1957) (line X) and Bjerrum (1973) (line Y) (*, organic content < 3%; o, organic content > 3%).
normal compression line. The consequences of assuming the relationship

\[
\frac{c_u}{\sigma'_{ve}} = 0.25
\]  \hspace{1cm} (9.77)

to estimate strengths of clays are explored here.

An expression that is widely used in practice to assess strength data is that due to Skempton (1954b, 1957). Skempton showed data of \(c_u/\sigma'_v\) for a number of soils that were supposedly normally compressed (Fig. 9.39a) and deduced the relationship

\[
\frac{c_u}{\sigma'_{ve}} = 0.11 + 0.37I_p
\]  \hspace{1cm} (9.78)

which links the ratio \(c_u/\sigma'_{ve}\) with plasticity \(I_p\). The undrained strengths were peak values obtained from field vane tests. For normally compressed soils, the in situ vertical effective stress and the preconsolidation pressure are identical, \(\sigma'_v = \sigma'_{ve}\), and in plotting the data for Fig. 9.39a, Skempton did not distinguish between them. With the gathering of more data, the scatter becomes greater and the nature of the dependence of \(c_u/\sigma'_{ve}\) on plasticity becomes less clear. Data collected by Leroueil, Magnan, and Tavenas (1985) are shown in Fig. 9.39b: here the undrained strengths are indeed divided by preconsolidation pressures. Expression (9.78) (line X) appears to provide a reasonable fit to the data.

Bjerrum (1972, 1973) analysed a large number of field records and proposed relationships between the strength ratio \(c_u/\sigma'_v\) and effective overconsolidation ratio \(\sigma'_{ve}/\sigma'_v\) which reduce to the curve shown in Fig. 9.39b (line Y) relating the strength ratio \(c_u/\sigma'_{ve}\) and plasticity index \(I_p\). However, when many case histories of failures of embankments on soft clays are back-analysed, we find that the strengths estimated from Bjerrum’s curve in Fig. 9.39b give a false estimate of stability: the factor of safety calculated at failure differs from unity by an amount dependent, roughly, on plasticity (Fig. 9.40a). Bjerrum (1972) drew an average straight line through his data of estimated factor-of-safety at failure (Fig. 9.40a) and then inverted this relationship to give a factor \(\mu\), which varied with plasticity (Fig. 9.40b), by which field vane strength data should be modified to give a better estimate of the strength which could be mobilised in the field. Bjerrum (1973) suggests that this \(\mu\) factor may arise partly from effects of anisotropy of natural soils and partly from the dependence of strength on rate of shearing so the strength mobilised in a matter of seconds in vane tests is higher than the strength mobilised over a period of several weeks or months in embankment loading. Whatever the origin of this factor, when it is combined with Bjerrum’s curve for variation of \(c_u/\sigma'_{ve}\) with plasticity (Fig. 9.39b), the resulting field strength ratio is almost
Fig. 9.40 (a) Variation of expected factor of safety at failure with plasticity index $I_p$ with relationship proposed by Bjerrum (1972) (line Z) (after Leroueil, Magnan, and Tavenas, 1985); (b) factor $\mu$, for modifying measured vane shear strength, dependent on plasticity index $I_p$ (after Bjerrum, 1972).

Fig. 9.41 Ratio of undrained strength to preconsolidation pressure ($c_u/\sigma_{vc}'$) obtained from combination of relationships proposed by Bjerrum (1973) (after Mesri, 1975).
9.6 Strength and overburden pressure

Independent of plasticity (Fig. 9.41) with a roughly constant value of

\[ \frac{c_u}{\sigma'_{ve}} \approx 0.22 \]  \hspace{1cm} (9.79)

as shown by Mesri (1975). Larsson's (1980) back analyses of field failures confirm that (9.79) provides a reasonable estimate of field strengths for inorganic clays. Practically, then, an expression similar to (9.77) is found to provide a better estimate of the strength which can be relied upon in the field than an expression such as (9.78), in which the strength ratio is allowed to increase with plasticity.

An obvious reason for the unjustified optimism of analyses based on peak vane strengths is precisely that these are peak strengths. The development of a failure beneath an embankment or foundation requires strains and deformations which take many elements of the soil well past their peak condition (Fig. 9.42), and an integrated peak strength along a failure surface is bound to give an unsafe result. A critical state strength measured at large deformation is more conservative. Limited data collected by Trak, LaRochelle, Tavenas, Leroueil, and Roy (1980) do indeed suggest (Fig. 9.43) that a strength ratio \( c_u/\sigma'_{ve} \) calculated using a critical state strength which they term the 'undrained strength at large strains' (USALS) has a value \( \sim 0.22 \) which is almost independent of plasticity.

The number 0.25 in (9.76) or (9.77) was an average number emerging from a calculation made using the Cam clay model. In fact, according to the Cam clay model, this number depends on the angle of friction of the soil. Figure 9.32 shows that as the angle of friction increases and the soil becomes stronger, the ratio \( \sigma'_{ve}/c_u \) decreases and hence the inverse ratio

Fig. 9.42 Variation of shear strain \( \gamma \) and shear stress \( \tau \) mobilised along failure surface.
Fig. 9.43 Data of ratio of undrained strength at large strains to preconsolidation pressure, and plasticity index (after Trak, LaRochelle, Tavenas, Leroueil, and Roy, 1980).

\[
\frac{c_u}{\sigma'_{vc}} = 0.22
\]

Fig. 9.44 Consequences of constant ratio of strength to preconsolidation pressure, on path followed during undrained shearing from one-dimensional normal compression line (1-D ncl) to critical state line (csl): (a) \(\phi' = 20^\circ\), (b) \(\phi' = 25^\circ\), (c) \(\phi' = 30^\circ\), and (d) \(\phi' = 35^\circ\).
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$c_u/\sigma'_e$ increases. Angles of friction tend to fall with increasing plasticity (Fig. 9.14), so the Cam clay model on its own would produce a trend for variation of $c_u/\sigma'_e$ with plasticity $\lambda_p$ directly opposite to that described by expression (9.78) and to the general trend of the data in Fig. 9.39. The same applies to any other strength calculation based on friction (e.g. see Wroth, 1984).

If, in fact, field evidence supports a constant value of the ratio $c_u/\sigma'_e$ (calculated using critical state strengths) independent of plasticity and hence of angle of friction, then this has implications for the relative shapes of yield loci and plastic potentials to be used (Fig. 9.28) to describe the elastic–plastic behaviour of the soil. This can be demonstrated by considering stress states in a non-dimensional effective stress plane $p'/\sigma'_e:q/\sigma'_e$ (Fig. 9.44).

In one-dimensional normal compression, from (9.68) and (9.62),

$$\eta_{Ksc} = \frac{3 \sin \phi'}{3 - 2 \sin \phi'} \quad (9.80)$$

Since the vertical effective stress is equal to the preconsolidation pressure $\sigma'_v = \sigma'_e$, the initial in situ effective stress state must lie on the line

$$\frac{p'}{\sigma'_e} + \frac{2}{3} \frac{q}{\sigma'_e} = 1 \quad (9.81)$$

The initial effective stress state is fixed at the intersection of (9.80) and (9.81): points A in Fig. 9.44, plotted for $\phi' = 20^\circ, 25^\circ, 30^\circ$, and $35^\circ$.

Assume that failure is reached at large strains at a critical state,

$$\eta = M = \frac{6 \sin \phi'}{3 - \sin \phi'} \quad (9.22bis)$$

and also that from (9.77)

$$\frac{q}{\sigma'_e} = \frac{2c_u}{\sigma'_e} = 0.5 \quad (9.82)$$

This failure stress state is fixed at the intersection of (9.82) and (9.22): points B in Fig. 9.44. The path linking A and B cannot have the same shape for each of the four angles of friction, so it is an error to apply the Cam clay model without regard for soil type or plasticity and without regard for the marked difference between the shape of experimentally observed yield loci (in Chapter 3) and the assumed simple shape of the Cam clay yield loci (in Chapter 5).

In summary, the simple relationship

$$\frac{c_u}{\sigma'_e} \simeq 0.25 \quad (9.77bis)$$
provides a reasonable indication of the way in which the undrained strength which can be usefully mobilised in the field varies with pre-consolidation pressure. Any attempt to refine this expression must, however, be based on a realistic numerical model of soil behaviour.

9.7 Conclusion
In this chapter a number of simple relationships between soil properties have been established against a background of critical state lines and the importance of volumetric parameters in controlling soil response. We have not intended to suggest that all sophisticated testing can be dispensed with but rather to show that simple predictive charts and formulae can be produced against which one can test the results of the apparently crude index tests and other data. Some of the assumptions that have been made and, in particular, some of the numerical values that have been chosen can be regarded with suspicion, but the patterns that have emerged are not really dependent on precise numerical values. If it is found that certain data in a collection of data from related soils appear to conflict with a pattern, then those data should either be rejected or be studied in more detail to explain the discrepancies.

Exercises
E9.1. A site investigation has been carried out for a proposed road bridge. The results of a conventional one-dimensional compression test performed on a sample taken from a depth of 3.6 m are given in Fig. 9.E1. For this sample the natural water content \( w_i = 0.46 \), the

Fig. 9.E1 Oedometer test on sample of silty clay.
specific gravity of soil particles \( G_s = 2.63 \), the liquid limit \( w_L = 0.46 \), and plastic limit \( w_P = 0.25 \). The water table is at the ground surface. Estimate the in situ vertical effective stress at the depth of 3.6 m, and estimate the preconsolidation pressure and overconsolidation ratio from the results of the oedometer test.

The behaviour of the silty clay stratum from which the sample has been taken is to be represented by Cam clay. Adopting a value of \( \phi' = 30^\circ \), estimate suitable values of \( \lambda, \kappa, N \), and \( \Gamma \). Compare the value of \( \lambda \) with a value estimated from the given plasticity index.

Estimate the value of the undrained shear strength of the clay in triaxial compression. Compare this value with the measured strength of 9 kPa and with values estimated from knowledge of the plasticity of the soil and of the vertical effective stresses that it has experienced. Estimate the sensitivity of the clay and show that the liquidity index, vertical effective stress, and undrained strength data are broadly consistent.

E9.2. A bed of clay is being prepared for testing on a geotechnical centrifuge. The clay is prepared as a slurry at a water content twice the liquid limit and then compressed one-dimensionally under a uniform vertical effective stress which is increased in stages to 70 kPa. The block of clay is then 200 mm deep and is placed on the centrifuge and accelerated to 100 gravities with no surface loading but with the water table being maintained at the surface of the clay. The clay is allowed to reach equilibrium with zero excess pore pressure.

Estimate the variation of liquidity index, overconsolidation ratio, and undrained strength with depth through the clay.

E9.3. Obtain site investigation data from published papers or from files relating to a project with which you have been involved. Assess these data using the relationships between plasticity and compressibility, liquidity and strength, liquidity and vertical effective stress, and overconsolidation ratio that have been proposed in this chapter.