Introduction to FEM

28

Stress Recovery

Stress Recovery

Processing **phase has solved for node displacements from the (modified) master stiffness equations**

$$\mathbf{K}\,\mathbf{u} = \mathbf{f}$$

Postprocessing **phase now starts to get derived quantities. Among them are internal forces and stresses.**

The process of computing stresses from node displacements is called *stress recovery*.

General Comments

Stresses recovered from low order elements (e.g. 3-node triangles and 4-node quads) often display large ***interelement jumps***.

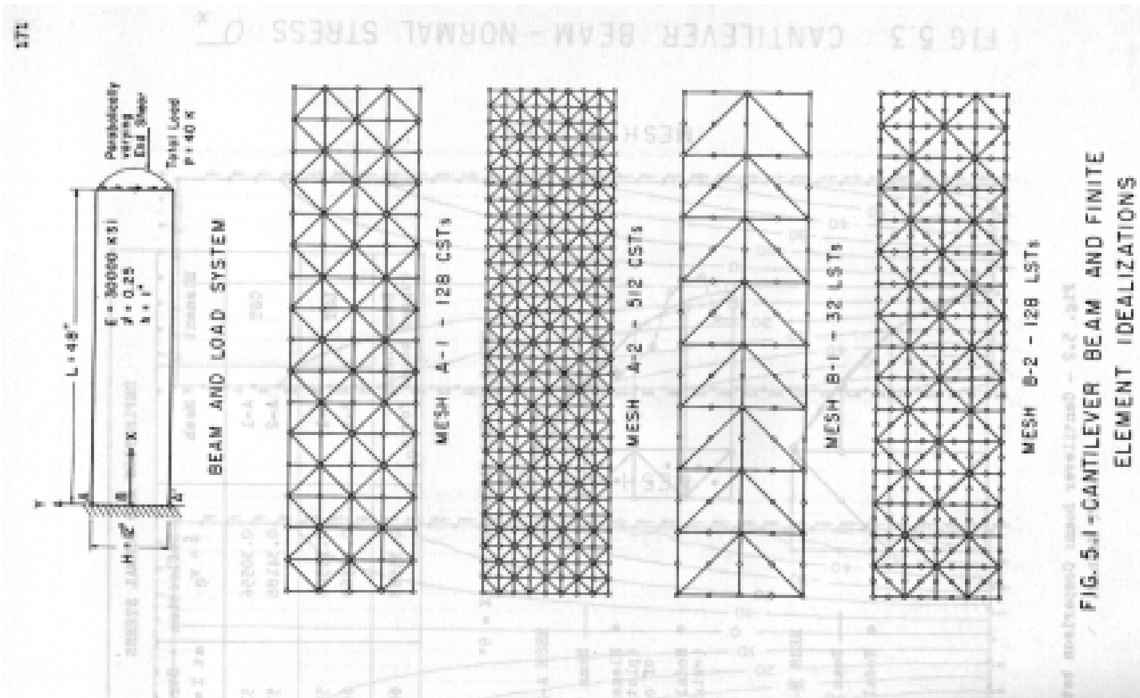
In-plane bending situations are particularly troublesome

Jumps can be eliminated by ***interelement averaging at nodes*** This usually improves the stress quality at interior nodes, but may not be effective at boundary nodes.

Stress recovery over quadrilateral elements can be improved by ***extrapolation from Gauss sample points***

Introduction to FEM

The Berkeley Cantilever



Nodal Stress Averaging

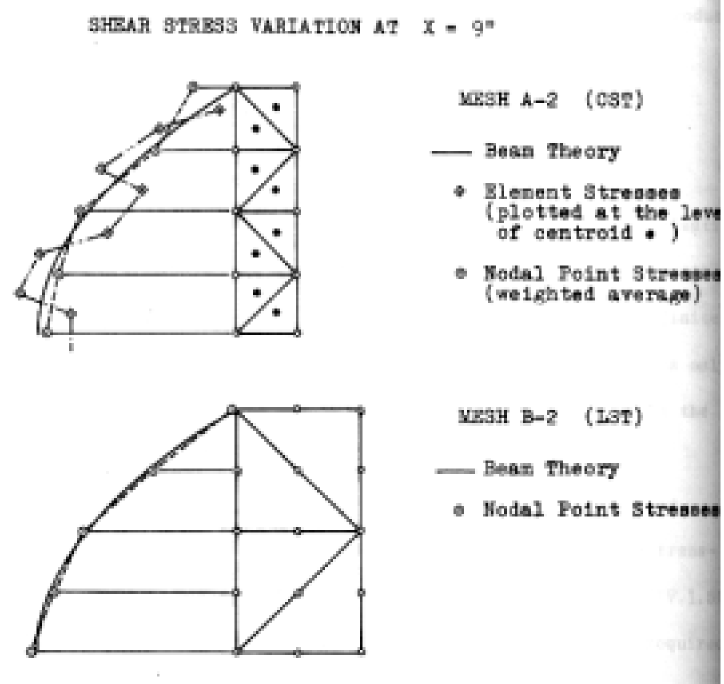


Fig. 5.2 - Cantilever Beam: Comparison between LST and CST Meshes.

Gauss Elements

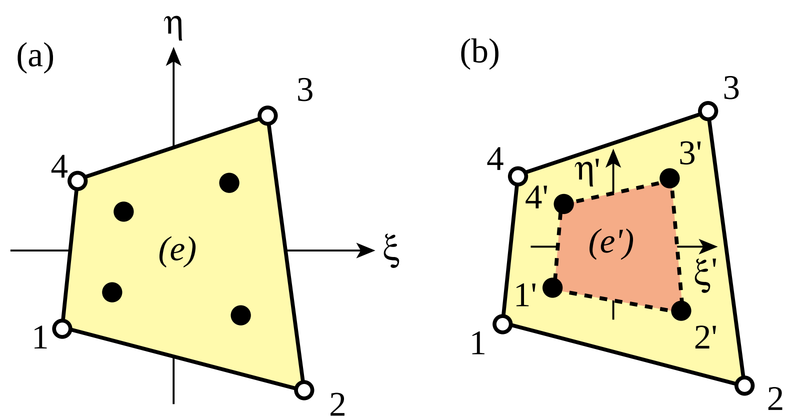


Table 29.1 Natural Coordinates of Bilinear Quadrilateral Nodes

Corner node	ξ	η	ξ'	η'	Gauss node	ξ	η	ξ'	η'
1	-1	-1	$-\sqrt{3}$	$-\sqrt{3}$	1’	$-1/\sqrt{3}$	$-1/\sqrt{3}$	-1	-1
2	$+1$	-1	$+\sqrt{3}$	$-\sqrt{3}$	2’	$+1/\sqrt{3}$	$-1/\sqrt{3}$	$+1$	-1
3	$+1$	$+1$	$+\sqrt{3}$	$+\sqrt{3}$	3’	$+1/\sqrt{3}$	$+1/\sqrt{3}$	$+1$	$+1$
4	-1	$+1$	$-\sqrt{3}$	$+\sqrt{3}$	4’	$-1/\sqrt{3}$	$+1/\sqrt{3}$	-1	$+1$

Gauss nodes, and coordinates ξ' and η' are defined in §29.4 and Fig. 29.1

Extrapolation to the Corner Points

Shape functions of "Gauss element"

$$\begin{aligned}
N_1^{(e')} &= \tfrac{1}{4}(1-\xi')(1-\eta'),\\
N_2^{(e')} &= \tfrac{1}{4}(1+\xi')(1-\eta'),\\
N_3^{(e')} &= \tfrac{1}{4}(1+\xi')(1+\eta'),\\
N_4^{(e')} &= \tfrac{1}{4}(1-\xi')(1+\eta').
\end{aligned}$$

To extrapolate, replace the ξ' and η' corner coordinates of the actual element:

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} 1+\frac{1}{2}\sqrt{3} & -\frac{1}{2} & 1-\frac{1}{2}\sqrt{3} & -\frac{1}{2} \\ -\frac{1}{2} & 1+\frac{1}{2}\sqrt{3} & -\frac{1}{2} & 1-\frac{1}{2}\sqrt{3} \\ 1-\frac{1}{2}\sqrt{3} & -\frac{1}{2} & 1+\frac{1}{2}\sqrt{3} & -\frac{1}{2} \\ -\frac{1}{2} & 1-\frac{1}{2}\sqrt{3} & -\frac{1}{2} & 1+\frac{1}{2}\sqrt{3} \end{bmatrix} \begin{bmatrix} w_1' \\ w_2' \\ w_3' \\ w_4' \end{bmatrix}$$

Introduction to FEM

Other "Gauss Element" Configurations