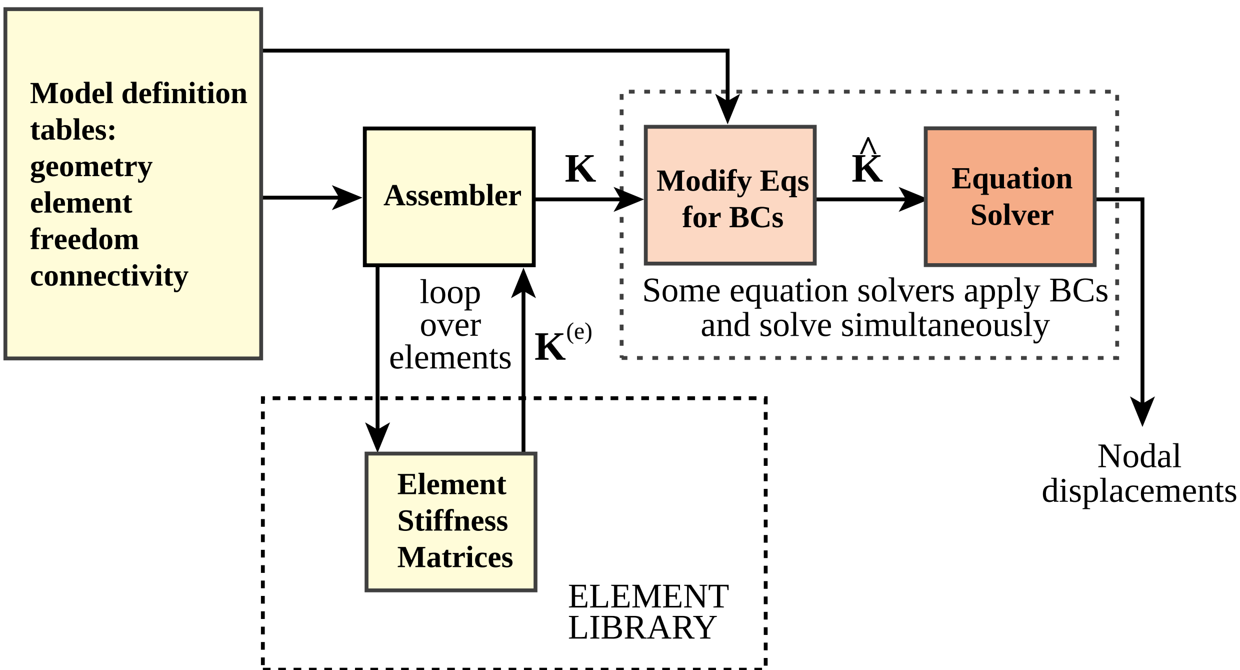
*Introduction to FEM*

# 26
# Solving FEM Equations

*Introduction to FEM*

# Role of the Solver in a FEM Code



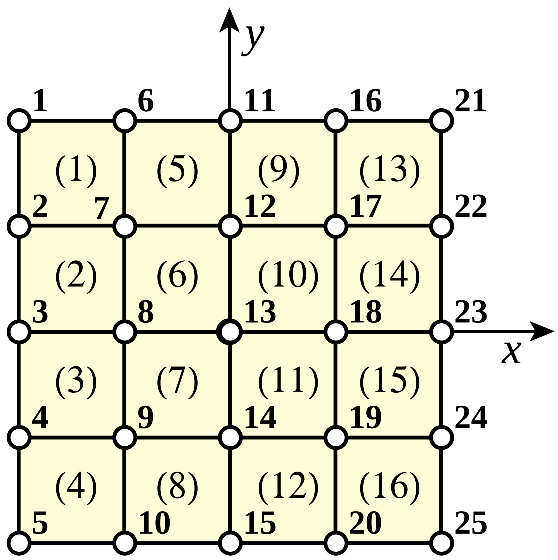
# Computer Resources Req'd by FEM Solver

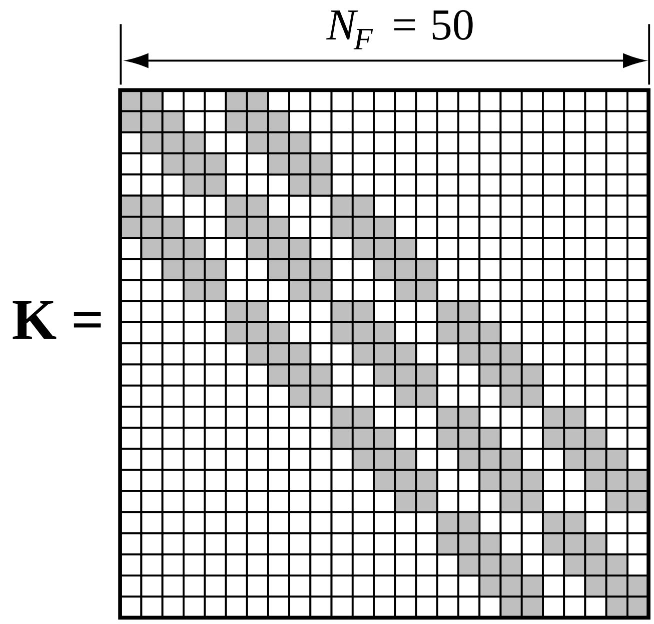
## Storage and Solution Times for a Fully Stored Stiffness Matrix

| Matrix order $N$ | Storage (double prec) | Factor op. units | Factor time workstation/PC | Factor time supercomputer |
|---|---|---|---|---|
| $10^4$ | 800 MB | $10^{12}/6$ | 3 hrs | 2 min |
| $10^5$ | 80 GB | $10^{15}/6$ | 4 mos | 30 hrs |
| $10^6$ | 8 TB | $10^{18}/6$ | 300 yrs | 3 yrs |

time numbers last adjusted in 1998
to get current times divide by 10-20

# Typical Stiffness Matrix Sparsity Pattern



# Computer Resources Req'd by FEM Solver

## Storage and Solution Times for a Skyline Stiffness Matrix Assuming $B = \sqrt{N}$

| Matrix order $N$ | Storage (double prec) | Factor op. units | Factor time workstation/PC | Factor time supercomputer |
|---|---|---|---|---|
| $10^4$ | 8 MB | $10^8/2$ | 5 sec | 0.05 sec |
| $10^5$ | 240 MB | $10^{10}/2$ | 8 min | 5 sec |
| $10^6$ | 8000 MB | $10^{12}/2$ | 15 hrs | 8 min |

time numbers last adjusted in 1998
to get current times divide by 10-20

# What We Will Cover Today

**How the Master Stiffness Equations are Stored in a commonly used "skyline" sparse format**

**How to Mark BC on the Master Stiffness Eqs (if you write your own solver)**

**The Basic Solution Steps**

**(Implementation Details will be Skipped since Built-in Mathematica Solver will be used for Demo Programs)**

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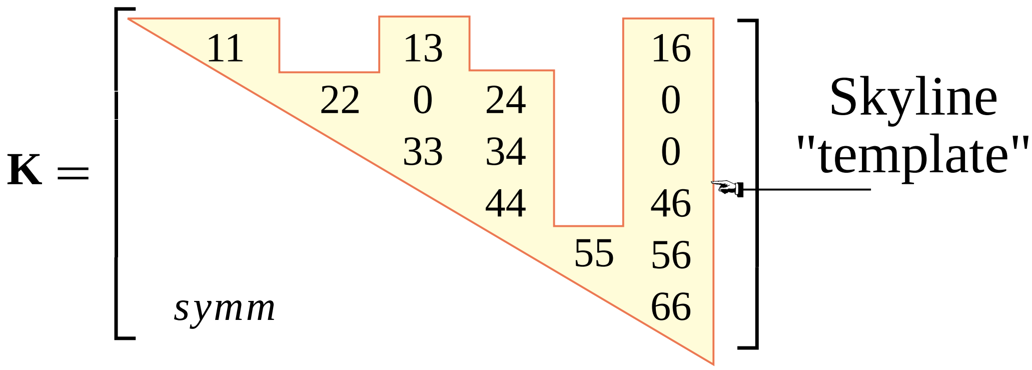
# Skyline Storage
# (aka Profile or Variable-Band Storage)




*Mathematica* **representation**

```
p= { 0,1,2,5,8,9,15};
s= { 11,22,13,0,33,24,34,44,55,16,0,0,46,56,66};
S= { p, s};
```

**or**

```
S={{0,1,2,5,8,9,15},{11,22,13,0,33,24,34,44,55,16,0,0,46,56,66}};
```

# Marking Displacement BCs

Equations for which the displacement component is known or prescribed are identified by a ***negative*** diagonal location value. For example, if $u_3$ and $u_5$ are prese4cribed displacement components in the sample system,

$$\texttt{p}: \quad [\,0,\ 1,\ 2,\ -5,\ 8,\ -9,\ 15\,]$$

# Solution Steps for K u = f

## Factorization

$$\mathbf{K} = \mathbf{LDU} = \mathbf{LDL}^T = \mathbf{U}^T\mathbf{DU}$$

## Solution

$$\textit{Forward reduction}: \quad \mathbf{L}\,\mathbf{z} = \mathbf{f}$$

$$\textit{Diagonal scaling}: \quad \mathbf{D}\,\mathbf{y} = \mathbf{z}$$

$$\textit{Back substitution}: \quad \mathbf{U}\,\mathbf{u} = \mathbf{y}$$

# SkySolver Implementation (sketched only, Notes give details)

```
S = { p, s }
```

$$\mathbf{K} = \begin{bmatrix} 11 & & 13 & & & 16 \\ & 22 & 0 & 24 & & 0 \\ & & 33 & 34 & & 0 \\ & & & 44 & & 46 \\ & & & & 55 & 56 \\ symm & & & & & 66 \end{bmatrix}$$

# SkySolver Implementation (cont'd)

*Mathematica* representation of skyline array

```
p= { 0,1,2,5,8,9,15};
s= { 11,22,13,0,33,24,34,44,55,16,0,0,46,56,66};
S= { p, s};
```

or directly

```
S={{0,1,2,5,8,9,15},
     {11,22,13,0,33,24,34,44,55,16,0,0,46,56,66}};
```

Further implementation details are given in Chapter 26 of Notes in sections marked *, but these are intended for a more advanced course