

Introduction to FEM

11

Superelements and Global-Local Analysis

Introduction to FEM

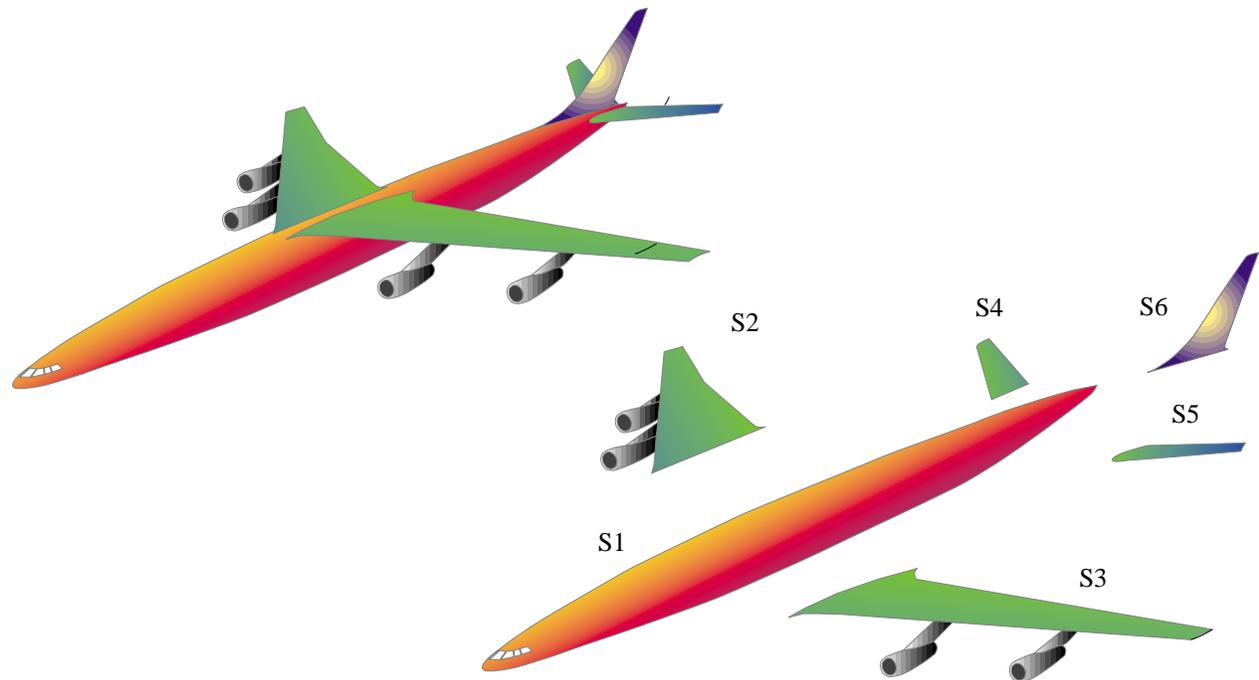
Superelements

Two extremes

Macroelements "bottom up"

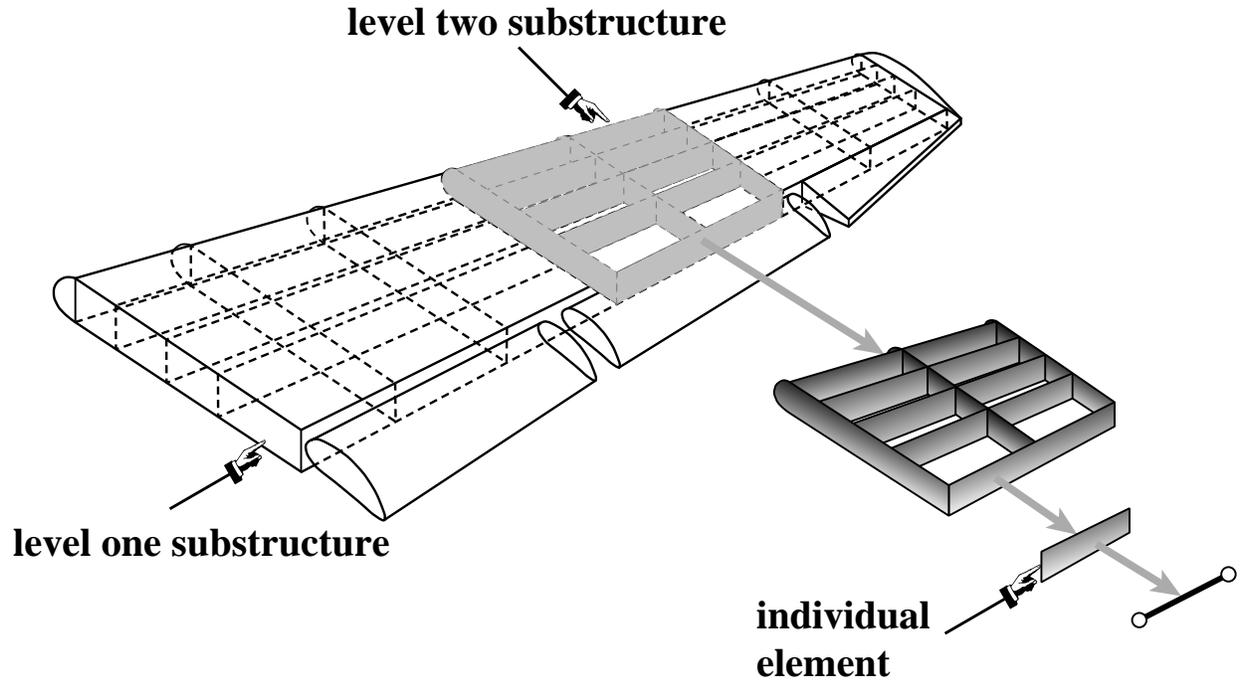
Substructures "top down"

Substructuring was Invented in the Aerospace Industry (early 1960s)



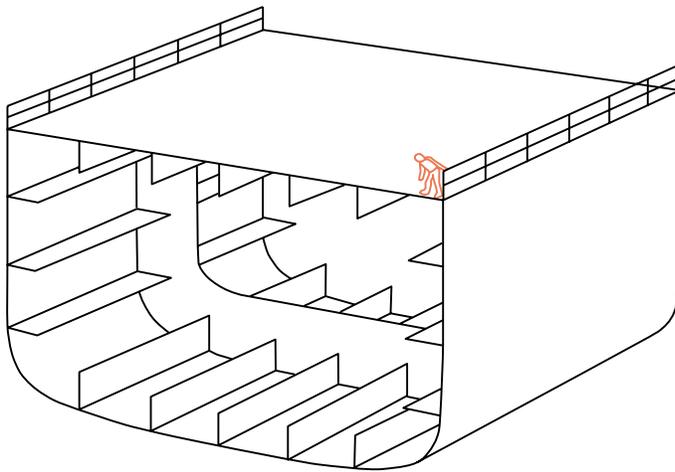
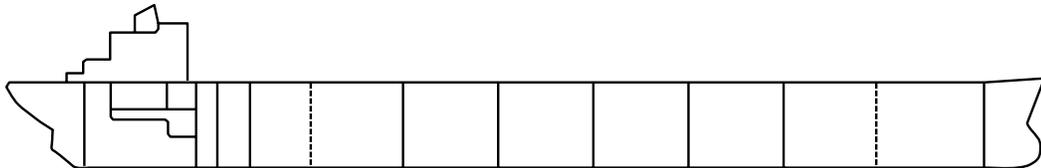
First level substructuring

Substructures (cont'd)



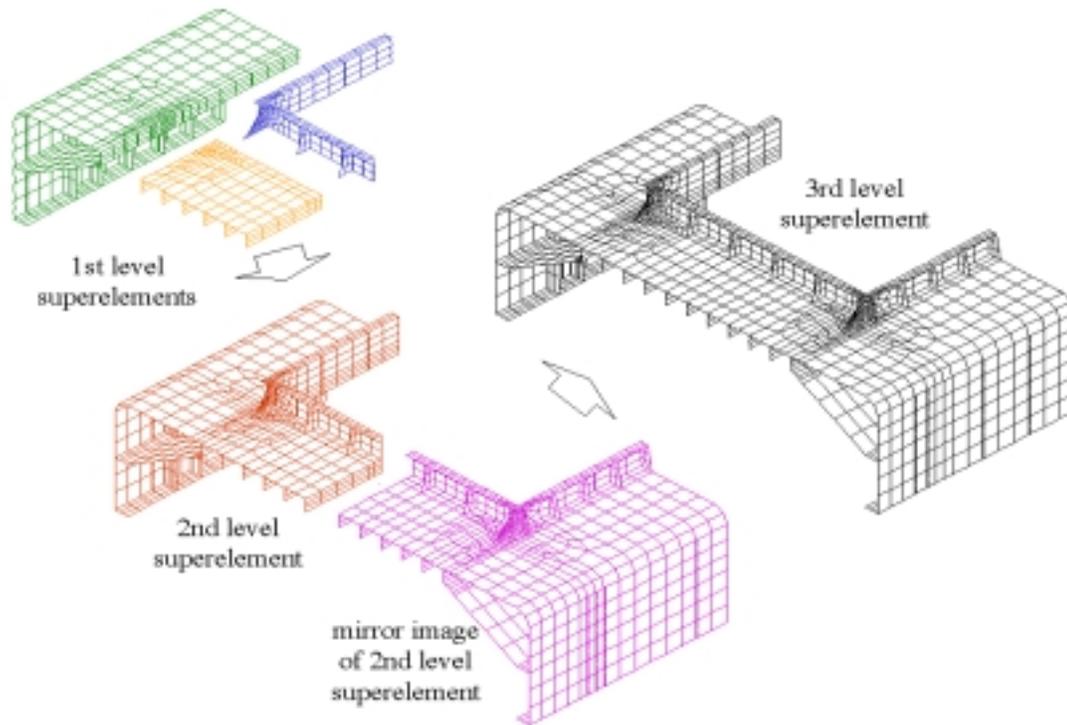
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Multilevel FEM Substructuring was Invented in the Norwegian Offshore Industry in the mid/late 60s



Among Other Things, to Take Advantage of Repetition

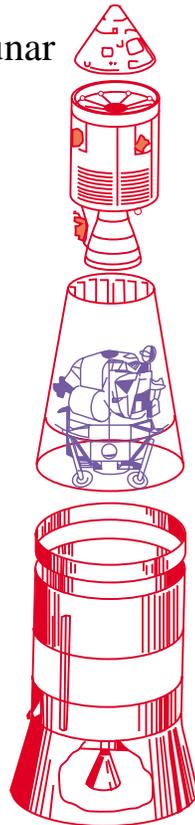
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From DNV (Det Norske Veritas) web-posted brochure.
 Permission requested for inclusion in book proper.

Multistage Rockets Naturally Decompose into Substructures

Short stack
Apollo/Saturn lunar
rocket



COMMAND MODULE

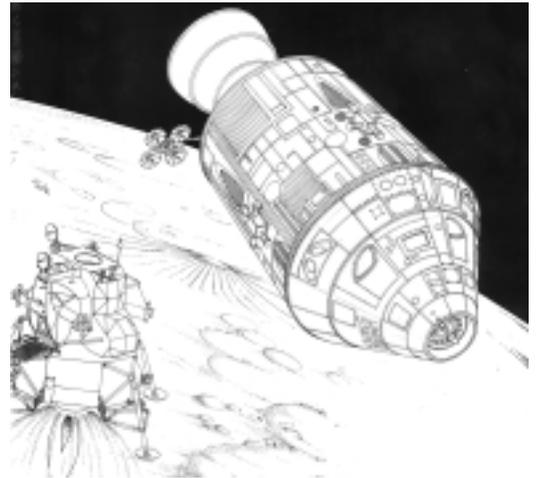
SERVICE MODULE

ADAPTER

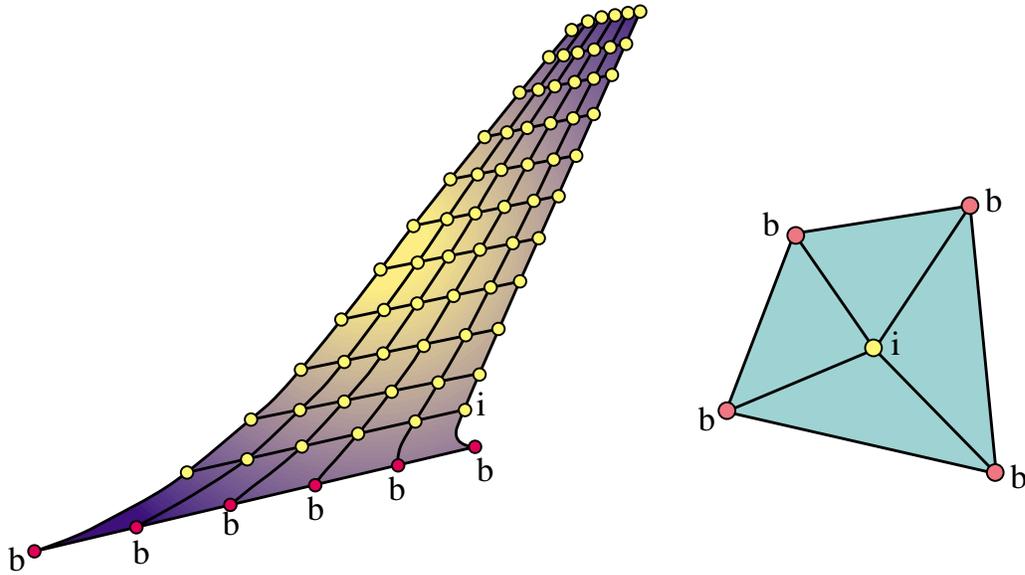
LUNAR MODULE

INSTRUMENT UNIT

THIRD STAGE
SIV-B



Static Condensation



Static Condensation by Matrix Algebra

Partition
$$\begin{bmatrix} \mathbf{K}_{bb} & \mathbf{K}_{bi} \\ \mathbf{K}_{ib} & \mathbf{K}_{ii} \end{bmatrix} \begin{bmatrix} \mathbf{u}_b \\ \mathbf{u}_i \end{bmatrix} = \begin{bmatrix} \mathbf{f}_b \\ \mathbf{f}_i \end{bmatrix}$$

Solve for interior displacements from 2nd matrix equation

$$\mathbf{u}_i = \mathbf{K}_{ii}^{-1} (\mathbf{f}_i - \mathbf{K}_{ib} \mathbf{u}_b),$$

replace into first matrix equation

$$\tilde{\mathbf{K}}_{bb} \mathbf{u}_b = \tilde{\mathbf{f}}_b \quad \text{condensed stiffness eqs}$$

where

$$\tilde{\mathbf{K}}_{bb} = \mathbf{K}_{bb} - \mathbf{K}_{bi} \mathbf{K}_{ii}^{-1} \mathbf{K}_{ib}$$

$$\tilde{\mathbf{f}}_b = \mathbf{f}_b - \mathbf{K}_{bi} \mathbf{K}_{ii}^{-1} \mathbf{f}_i$$

Static Condensation by Gauss Elimination

$$\begin{bmatrix} 6 & -2 & -1 & -3 \\ -2 & 5 & -2 & -1 \\ -1 & -2 & 7 & -4 \\ -3 & -1 & -4 & 8 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 4 \\ 0 \end{bmatrix}$$

Eliminate u_4 :

$$\begin{bmatrix} 6 - \frac{(-3) \times (-3)}{8} & -2 - \frac{(-1) \times (-3)}{8} & -1 - \frac{(-4) \times (-3)}{8} \\ -2 - \frac{(-3) \times (-1)}{8} & 5 - \frac{(-1) \times (-1)}{8} & -2 - \frac{(-4) \times (-1)}{8} \\ -1 - \frac{(-3) \times (-4)}{8} & -2 - \frac{(-1) \times (-4)}{8} & 7 - \frac{(-4) \times (-4)}{8} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 3 - \frac{0 \times (-3)}{8} \\ 6 - \frac{0 \times (-1)}{8} \\ 4 - \frac{0 \times (-4)}{8} \end{bmatrix}$$

$$\begin{bmatrix} \frac{39}{8} & -\frac{19}{8} & -\frac{5}{2} \\ -\frac{19}{8} & \frac{39}{8} & -\frac{5}{2} \\ -\frac{5}{2} & -\frac{5}{2} & 5 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 4 \end{bmatrix}$$

Static Condensation by Gauss Elimination (cont'd)

$$\begin{bmatrix} \frac{39}{8} & -\frac{19}{8} & -\frac{5}{2} \\ -\frac{19}{8} & \frac{39}{8} & -\frac{5}{2} \\ -\frac{5}{2} & -\frac{5}{2} & 5 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 4 \end{bmatrix}$$

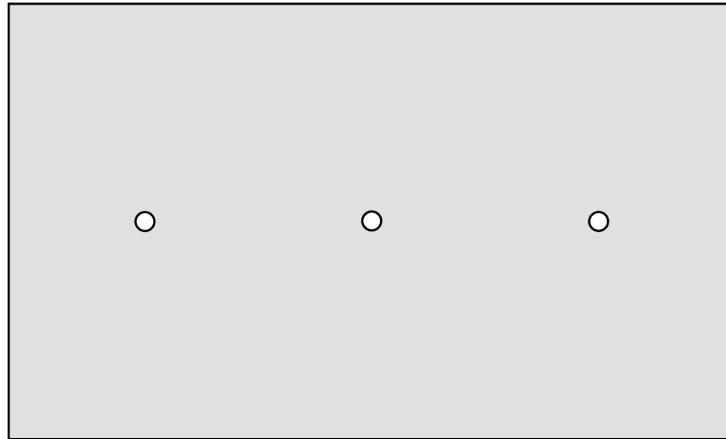
Now eliminate u_3

$$\begin{bmatrix} \frac{39}{8} - \frac{(-5/2) \times (-5/2)}{5} & -\frac{19}{8} - \frac{(-5/2) \times (-5/2)}{5} \\ -\frac{19}{8} - \frac{(-5/2) \times (-5/2)}{5} & \frac{39}{8} - \frac{(-5/2) \times (-5/2)}{5} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 3 - \frac{4 \times (-5/2)}{5} \\ 6 - \frac{4 \times (-5/2)}{5} \end{bmatrix}$$

$$\begin{bmatrix} \frac{29}{8} & -\frac{29}{8} \\ -\frac{29}{8} & \frac{29}{8} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

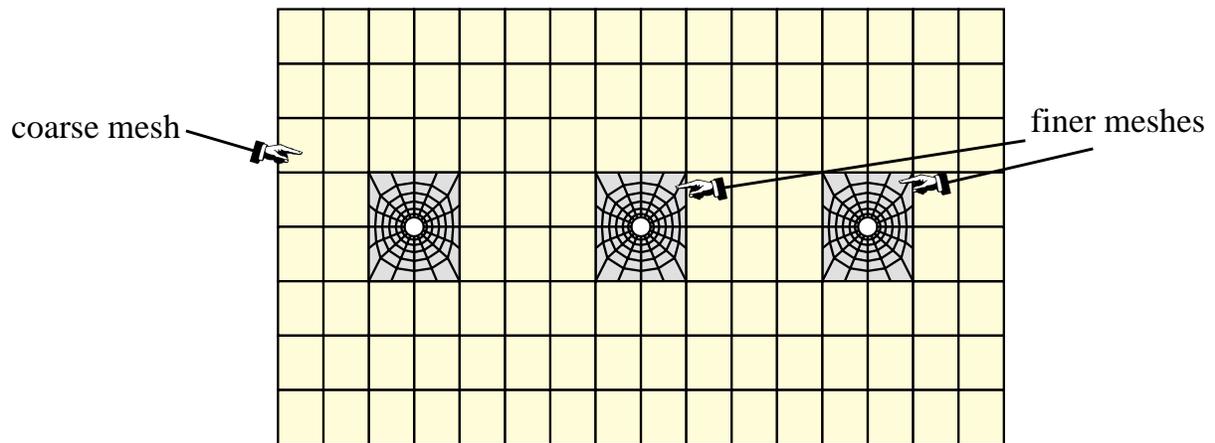
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Global-Local Analysis (an instance of Multiscale Analysis)

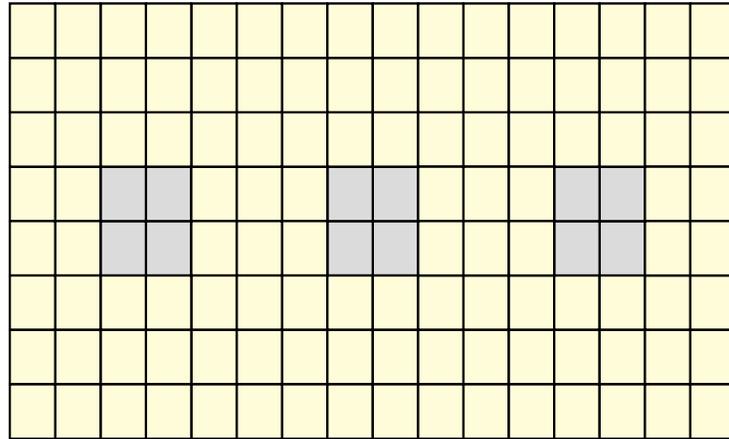


Example structure: panel with small holes

Standard (one-stage) FEM Analysis



Global-Local (two-stage) FEM Analysis



Global analysis with a coarse mesh, ignoring holes, followed by local analysis of the vicinity of the holes with finer meshes (next slide)

Local Analysis

BCs of displacement or (better) of force type
using extracted results from the global analysis

