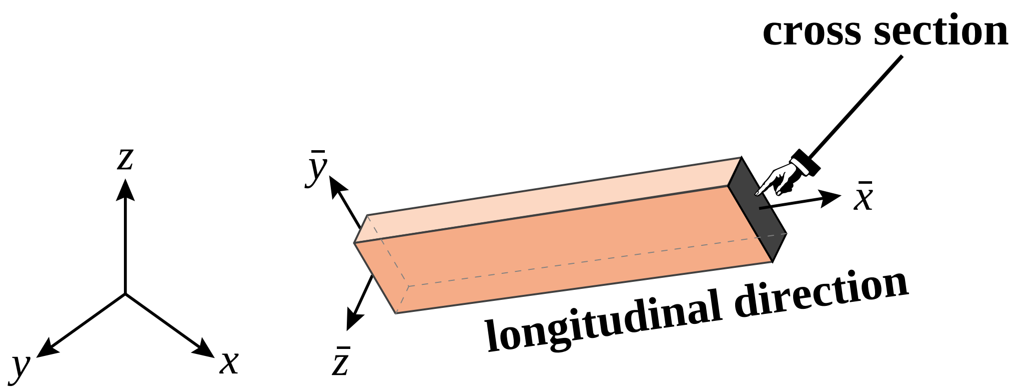
6
Constructing MoM Members

What Are MoM Members?

Skeletal structural members whose stiffness equations can be constructed by Mechanics of Materials (MoM) methods

Can be locally modeled as 1D elements

MoM Members Tend to Look Alike ...

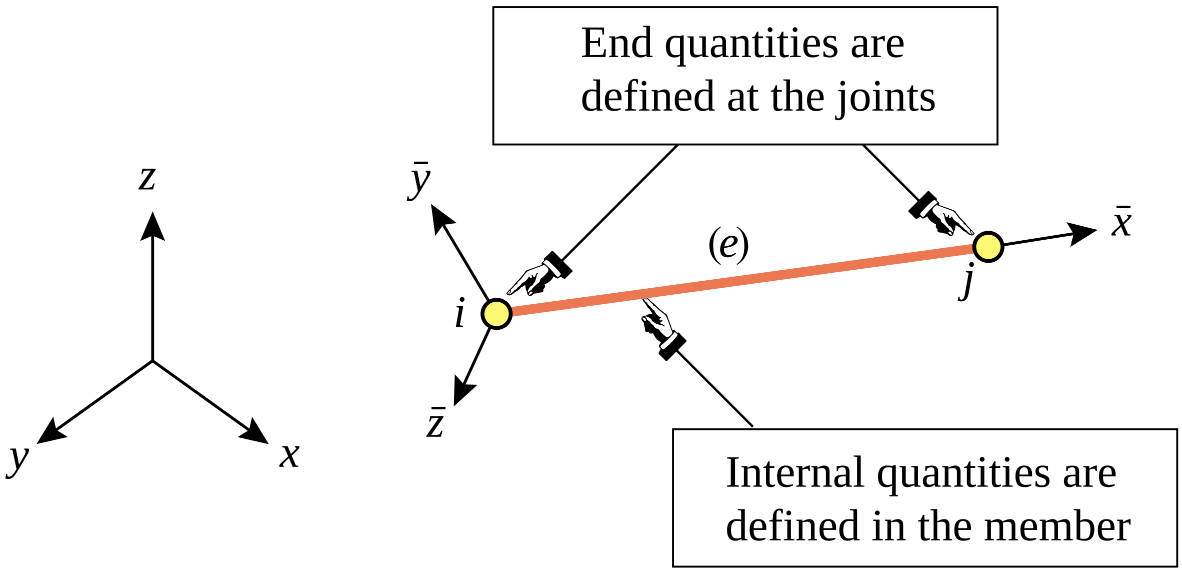


One dimension (longitudinal) much larger than the other two (transverse)

But Receive Different Names According to Function

Bars:	**transmit axial forces**
Beams:	**transmit bending**
Shafts:	**transmit torque**
Spars (=Webs):	**transmit shear**
Beam-column:	**transmit bending + compression**

Common Features of MoM Finite Element Models



Introduction to FEM

Governing Matrix Equations for Simplex MoM Element

From node displacements to internal deformations (strains)

$$\mathbf{v} = \mathbf{B}\,\bar{\mathbf{u}}$$

Kinematic

From deformations to internal forces

$$\mathbf{p} = \mathbf{S}\,\mathbf{v}$$

Constitutive

From internal forces to node forces

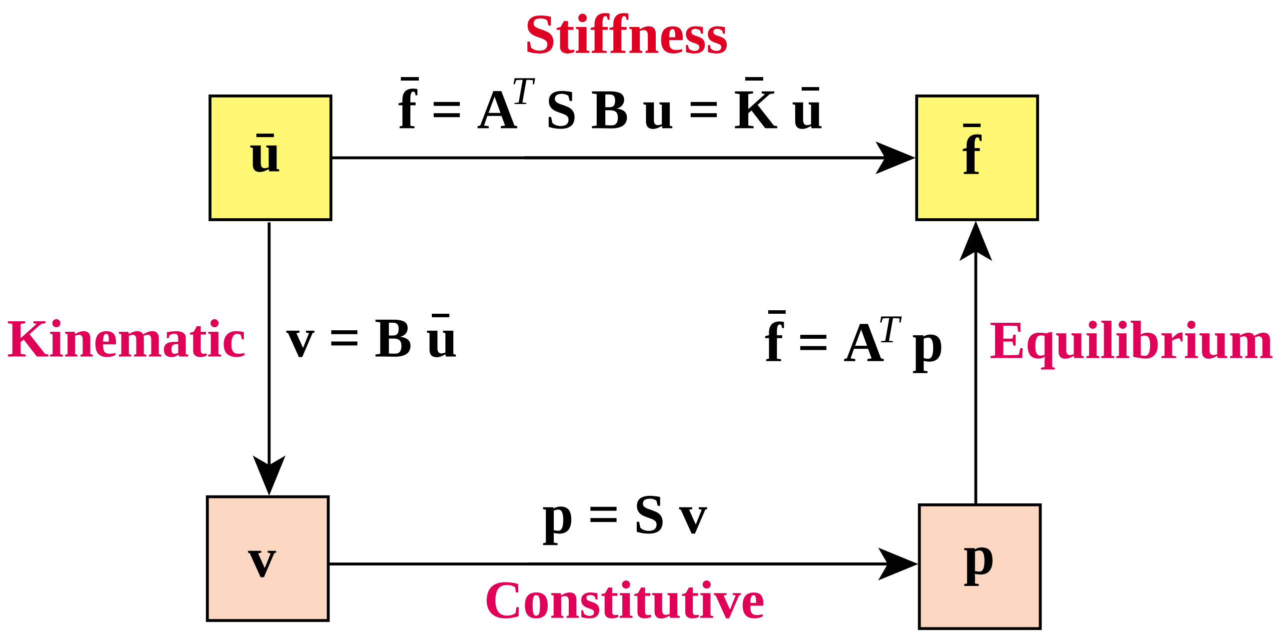
$$\bar{\mathbf{f}} = \mathbf{A}\,\mathbf{p}$$

Equilibrium

If $\bar{\mathbf{f}}$ and $\bar{\mathbf{u}}$ are PVW (Virtual Work) conjugate, $\mathbf{B} = \mathbf{A}$

Introduction to FEM

Tonti Diagram of Governing Matrix Equations for Simplex MoM Element



Elimination of the Internal Quantities v and p gives the Element Stiffness Equations through Simple Matrix Multiplications

$$\bar{\mathbf{f}} = \mathbf{A}^T \mathbf{S}\,\mathbf{B}\,\bar{\mathbf{u}} = \bar{\mathbf{K}}\bar{\mathbf{u}}$$

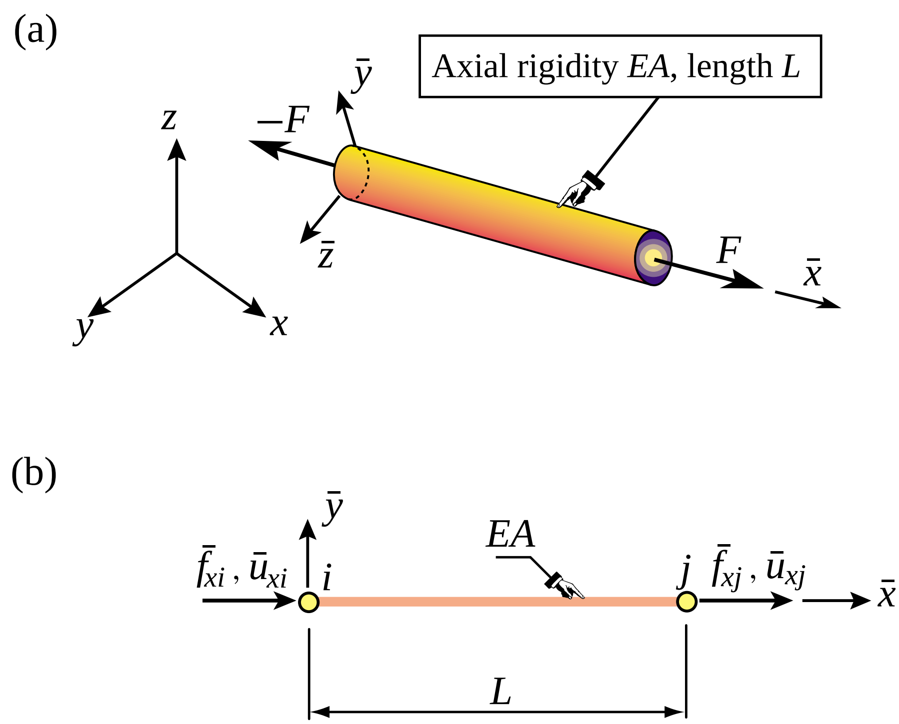
$$\downarrow$$

$$\bar{\mathbf{K}} = \mathbf{A}^T \mathbf{S}\,\mathbf{B}$$

If $\mathbf{B} = \mathbf{A}$ $\downarrow$

$$\bar{\mathbf{K}} = \mathbf{B}^T \mathbf{S}\,\mathbf{B}$$ symmetric if **S** is

The Bar Element Revisited



The Bar Element Revisited (cont'd)

$$d = [\,-1 \quad 1\,]\begin{bmatrix} \bar{u}_{xi} \\ \bar{u}_{xj} \end{bmatrix} = \mathbf{B}\bar{\mathbf{u}}$$

$$F = \frac{EA}{L} d = S\,d,$$

$$\bar{\mathbf{f}} = \begin{bmatrix} \bar{f}_{xi} \\ \bar{f}_{xj} \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} F = \mathbf{A}^T F$$

$$\bar{\mathbf{K}} = \mathbf{A}^T S\,\mathbf{B} = S\,\mathbf{B}^T\mathbf{B} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Can be expanded to the 4 x 4 of Chapter 2 by adding two zero rows and columns to accomodate $\bar{u}_{yi}$ **and** $\bar{u}_{yj}$

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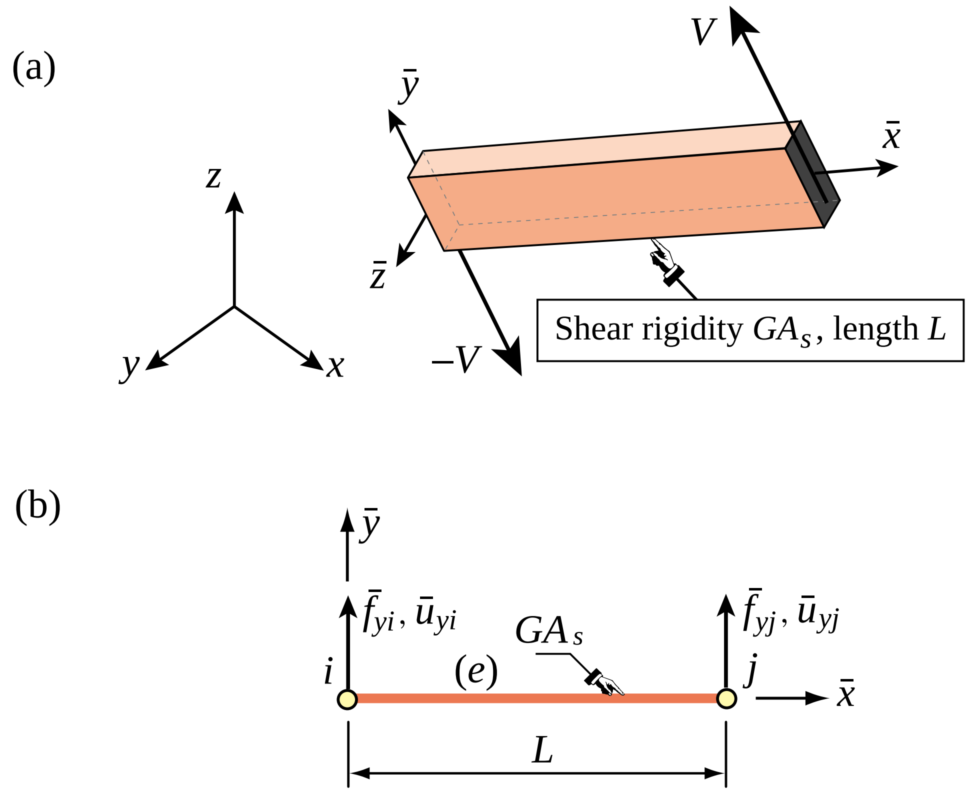
The Spar (a.k.a. Shear-Web) Element



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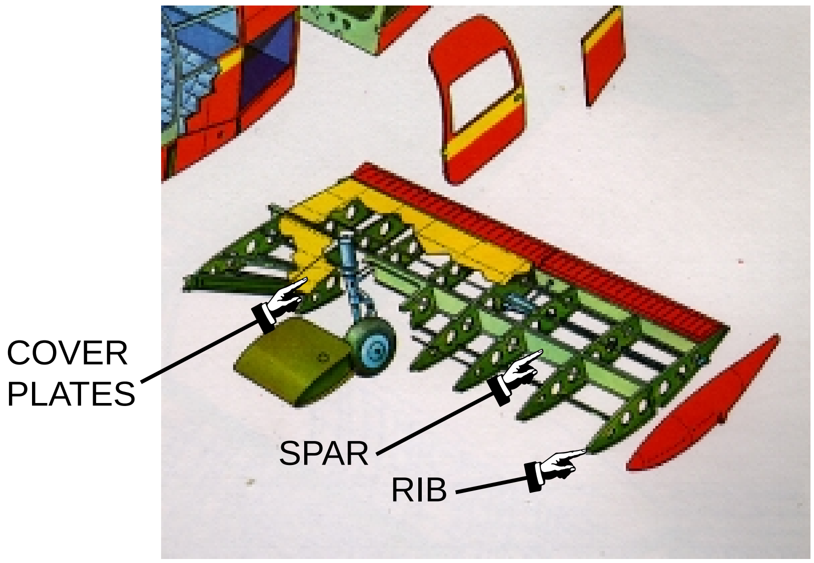
Spars used in Wing Structure

(Piper Cherokee)



The Spar Element (cont'd)

$$\gamma = \frac{1}{L}\begin{bmatrix} -1 & 1 \end{bmatrix}\begin{bmatrix} \bar{u}_{yi} \\ \bar{u}_{yj} \end{bmatrix} = \mathbf{B}\bar{\mathbf{u}}$$

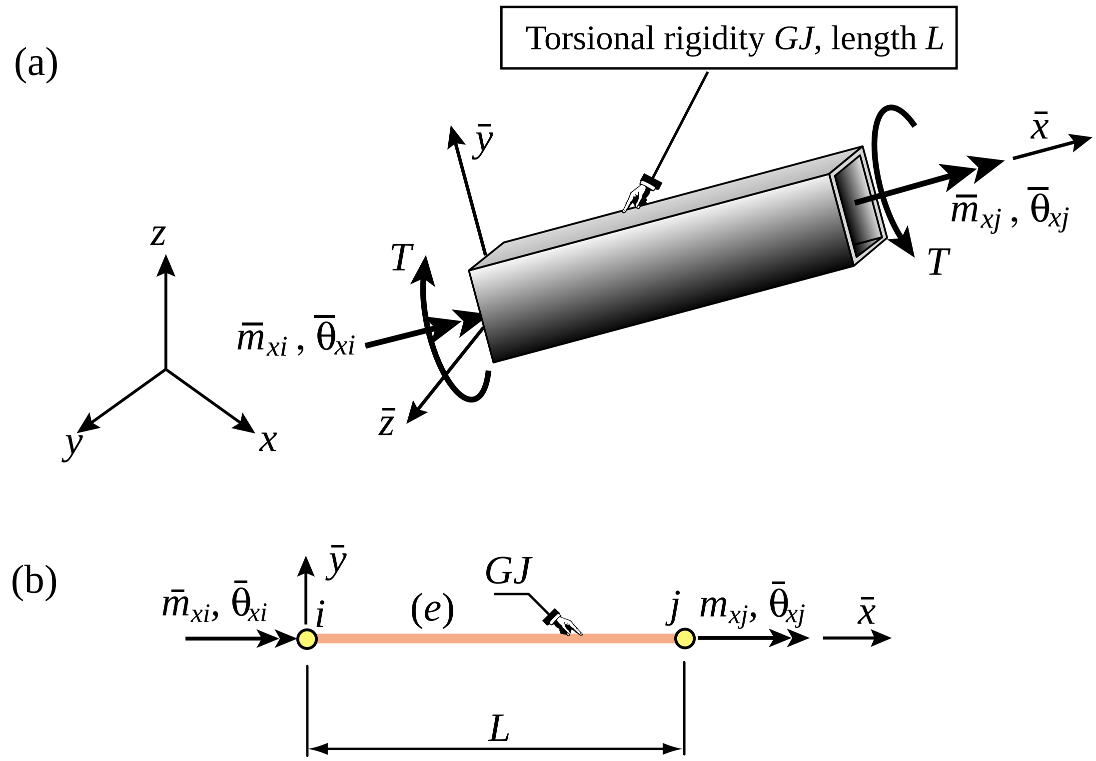
$$V = G A_s \gamma = S\,\gamma$$

$$\bar{\mathbf{f}} = \begin{bmatrix} \bar{f}_{yi} \\ \bar{f}_{yj} \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} V = \mathbf{A}^T V$$

$$\bar{\mathbf{f}} = \begin{bmatrix} \bar{f}_{yi} \\ \bar{f}_{yj} \end{bmatrix} = \mathbf{A}^T S\,\mathbf{B}\bar{\mathbf{u}} = \frac{G A_s}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \bar{u}_{yi} \\ \bar{u}_{yj} \end{bmatrix} = \bar{\mathbf{K}}\bar{\mathbf{u}}$$

$$\boxed{\bar{\mathbf{K}} = \frac{G A_s}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}$$

The Shaft Element



For stiffness derivation details see Notes

Matrix Equations for Non-Simplex MoM Element

From node displacements to internal deformations *at each section*

$$\mathbf{v} = \mathbf{B}\,\bar{\mathbf{u}}$$

Kinematic

From deformations to internal forces *at each section*

$$\mathbf{p} = \mathbf{R}\mathbf{v}$$

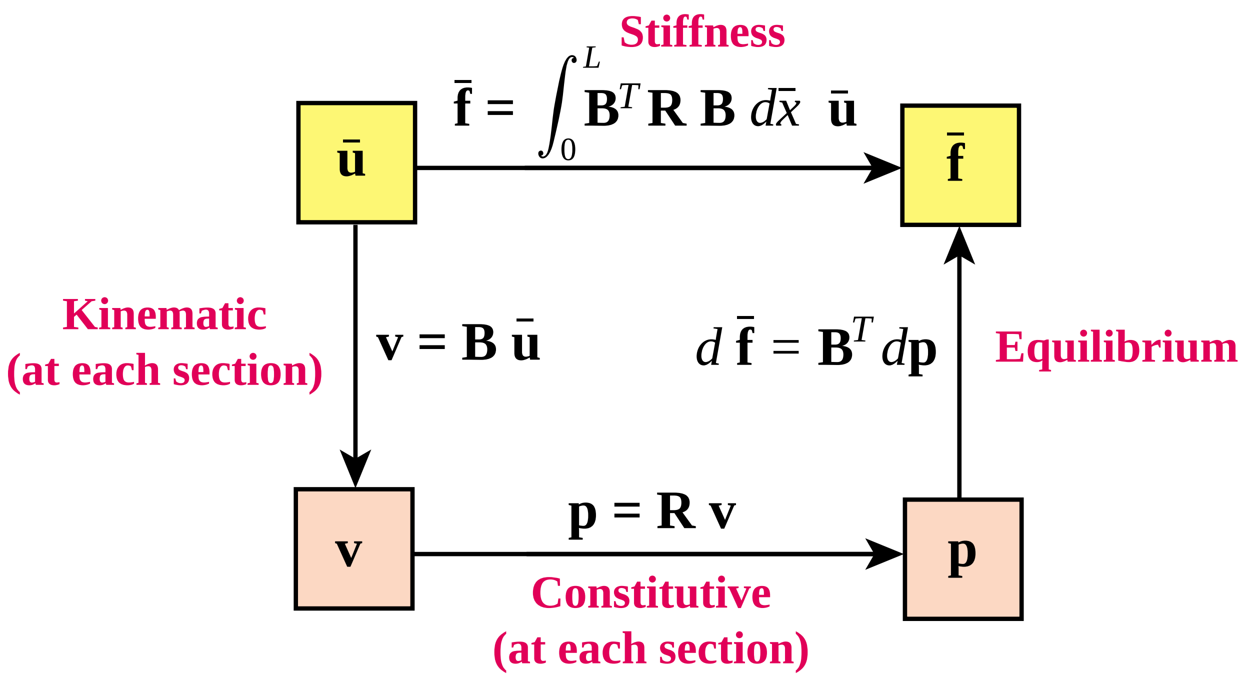
Constitutive

From internal forces to node forces

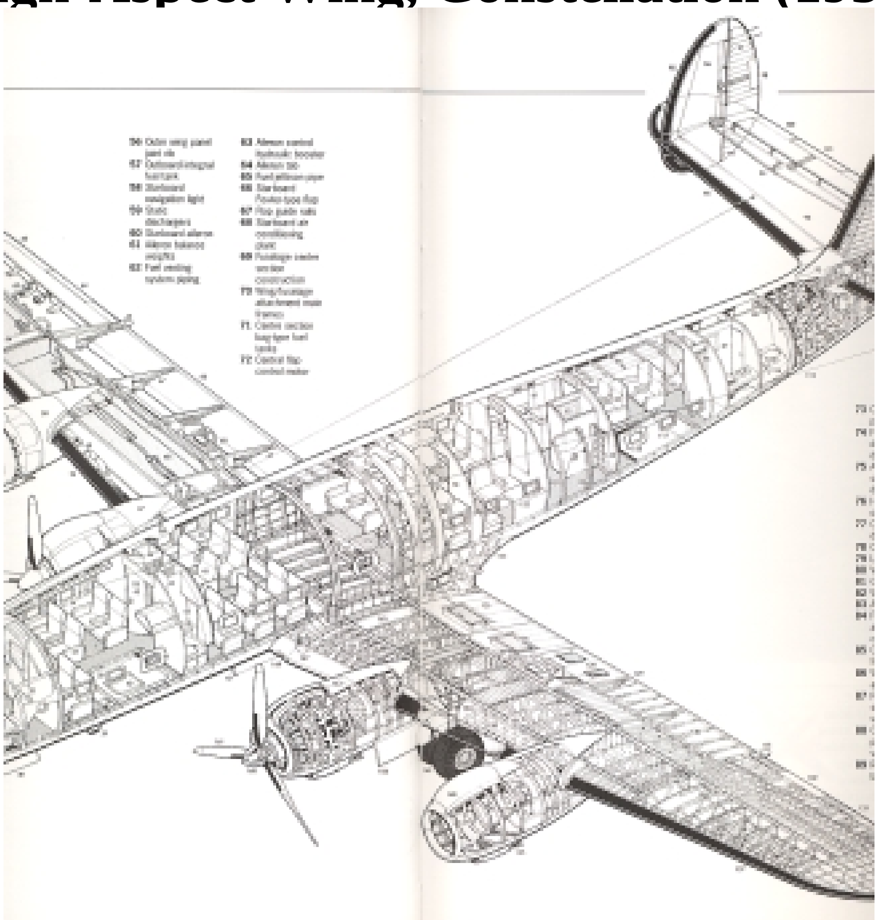
$$d\,\bar{\mathbf{f}} = \mathbf{A}^T d\mathbf{p}$$

Equilibrium

Tonti Diagram of Matrix Equations for Non-Simplex MoM Element (with A=B)



High-Aspect Wing, Constellation (1952)



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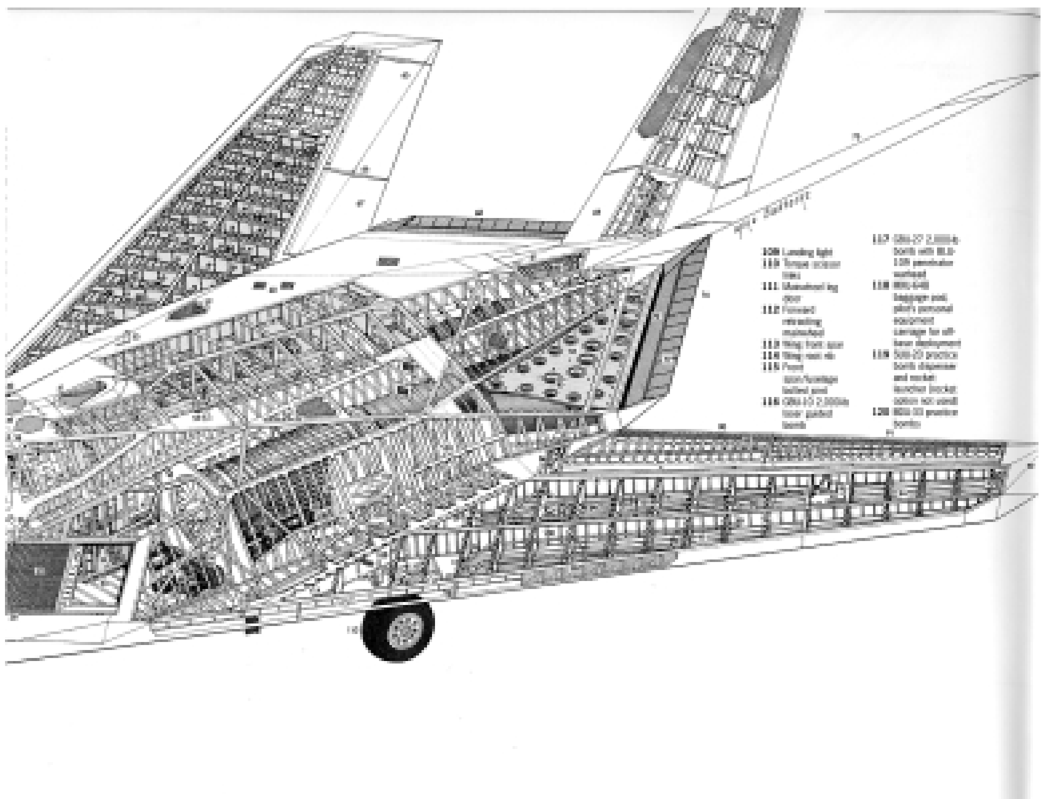
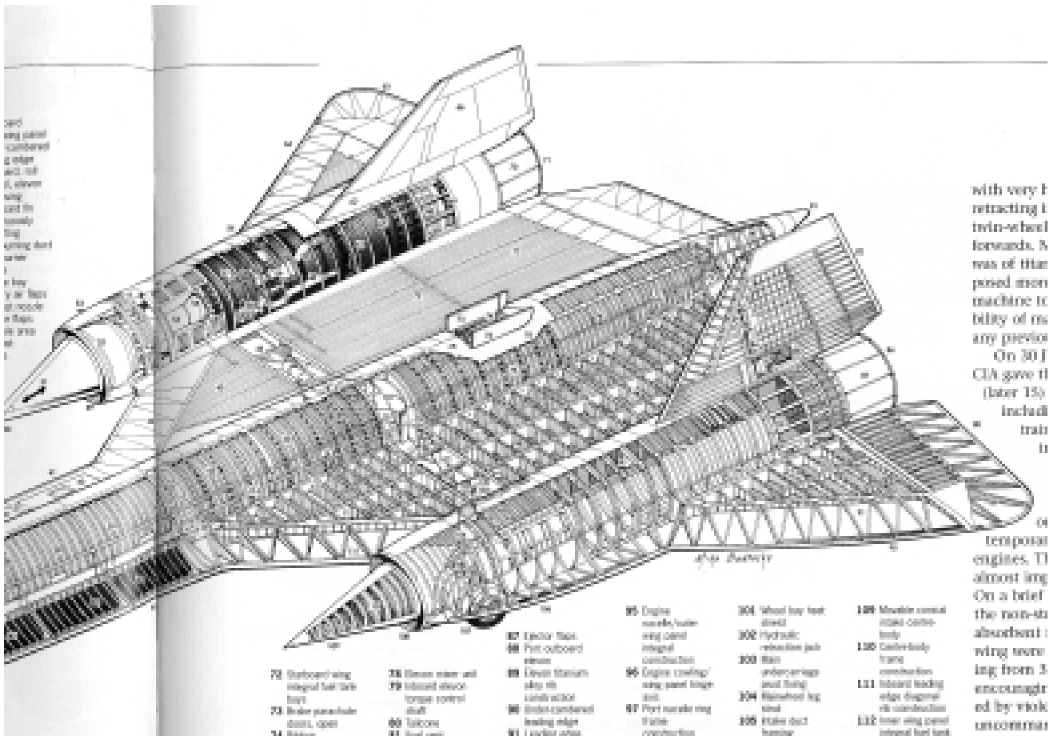
Low-Aspect Delta Wing, F-117 (1975)



Low-Aspect Delta Wing, Blackhawk (1972)



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Delta Wing Aircraft