Introduction to FEM

1
Overview

Course Coverage

This course consists of three Parts:

I. ***Finite Element Basic Concepts***

II. ***Formulation of Finite Elements***

III. ***Computer Implementation of FEM***

Where the Course Fits

The field of Mechanics can be subdivided into 3 major areas:

Mechanics
- *Theoretical*
- *Applied*
- ***Computational***

Computational Mechanics

Branches of ***Computational Mechanics*** can be distinguished according to the physical focus of attention

Computational Mechanics

- *Nano and Micromechanics*
- *Continuum Mechanics:*
  - ***Solids and Structures***
  - *Fluids*
  - *Multiphysics*
- *Systems*

Computational Solid and Structural Mechanics

A convenient subdivision of problems in Computational Solid and Structural Mechanics (CSM) is

Computational Solid and Structural Mechanics (CSM)
- ***Statics***
- *Dynamics*

CSM Statics

A further subdivision of problems in CSM Statics is

CSM Statics $\begin{cases} \textbf{\textit{Linear}} \\ \textit{Nonlinear} \end{cases}$

CSM Linear Statics

For the numerical simulation on the computer we must now chose a ***spatial discretization method:***

CSM Linear Statics

- ***Finite Element Method***
- *Finite Difference Method*
- *Boundary Element Method*
- *Finite Volume Method*
- *Spectral Method*
- *Mesh-Free Method*

CSM Linear Statics by FEM

Having selected the FEM for ***discretization,*** we must next pick a ***formulation and a solution method:***

Formulation of FEM Model
- ***Displacement***
- *Equilibrium*
- *Mixed*
- *Hybrid*

Solution of FEM Model
- ***Stiffness***
- *Flexibility*
- *Mixed*

Summarizing: This Course Covers

Computational structural mechanics

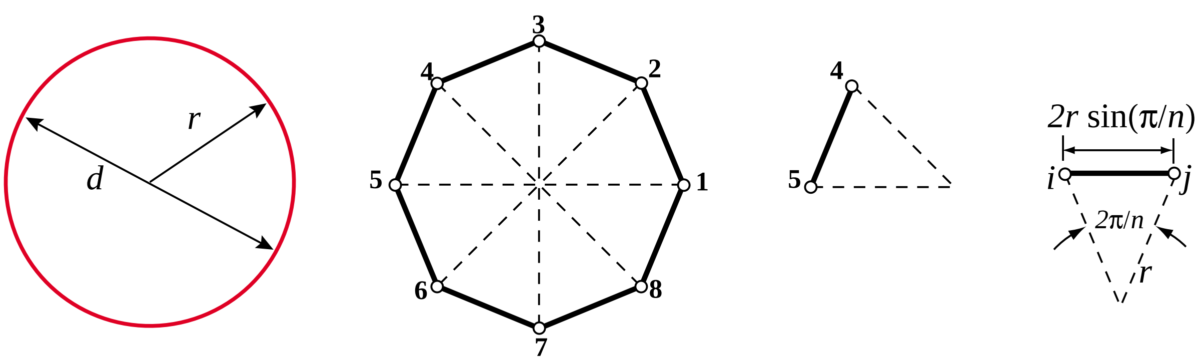
Linear static problems

Spatially discretized by displacement-formulated FEM

Solved by the stiffness method

What is a Finite Element?

Archimedes' problem (*circa* 250 B.C.): rectification of the circle as limit of inscribed regular polygons

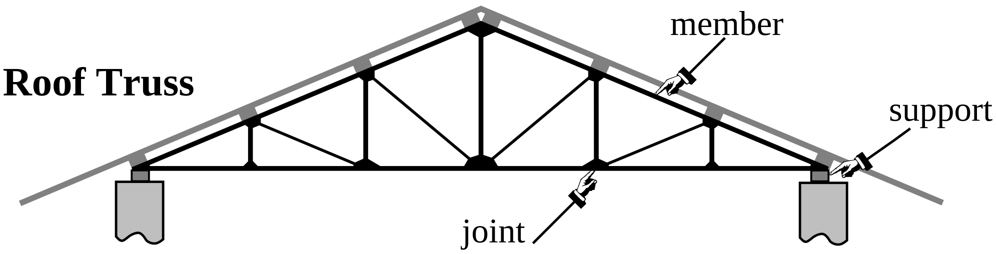


Computing π "by Archimedes FEM"

n	$\pi_n = n\sin(\pi/n)$	Extrapolated by Wynn-ϵ	Exact π to 16 places
1	0.000000000000000		
2	2.000000000000000		
4	2.828427124746190	3.414213562373096	
8	3.061467458920718		
16	3.121445152258052	3.141418327933211	
32	3.136548490545939		
64	3.140331156954753	3.141592658918053	
128	3.141277250932773		
256	3.141513801144301	3.141592653589786	3.141592653589793

The Idealization Process for a Simple Structure

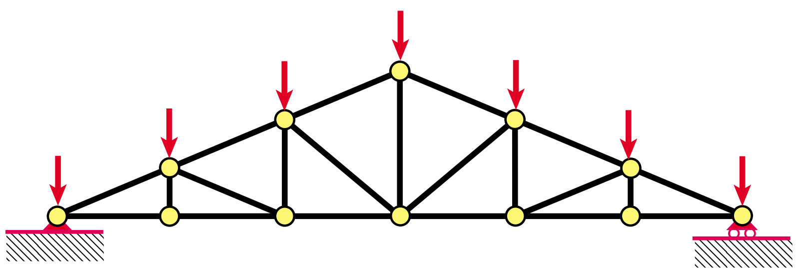
Roof Truss



Physical Model

IDEALIZATION & DISCRETIZATION

Mathematical *and* **Discrete Model**



Two Interpretations of FEM for Teaching

Physical

Breakdown of structural system into components (elements) and reconstruction by the assembly process

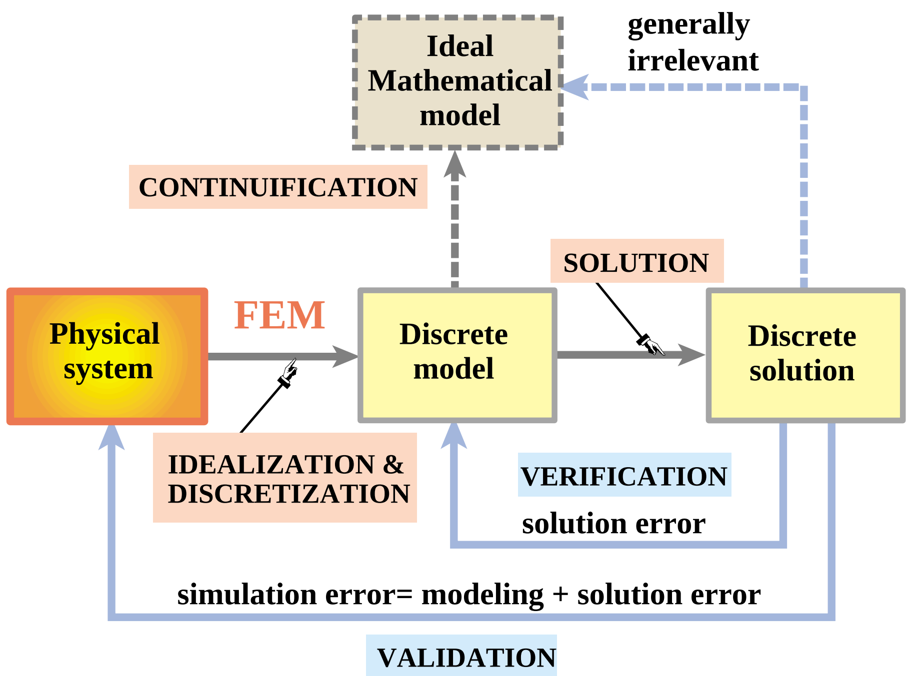
Emphasized in Part I

Mathematical

Numerical approximation of a Boundary Value Problem by Ritz-Galerkin discretization with functions of local support

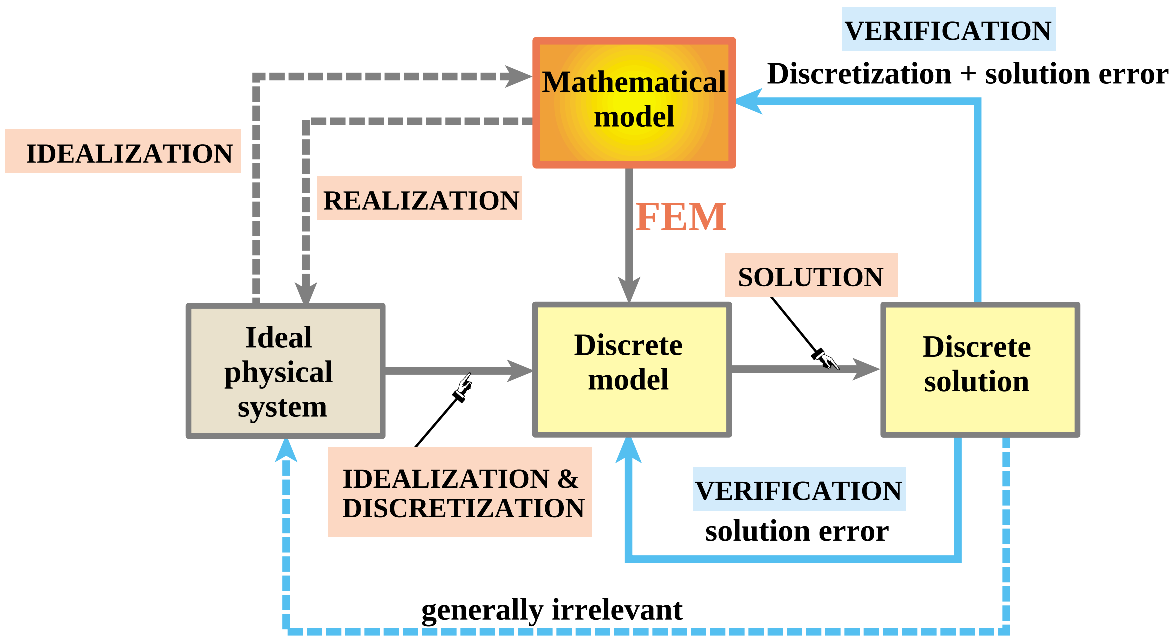
Emphasized in Part II

FEM in Modeling and Simulation: Physical FEM

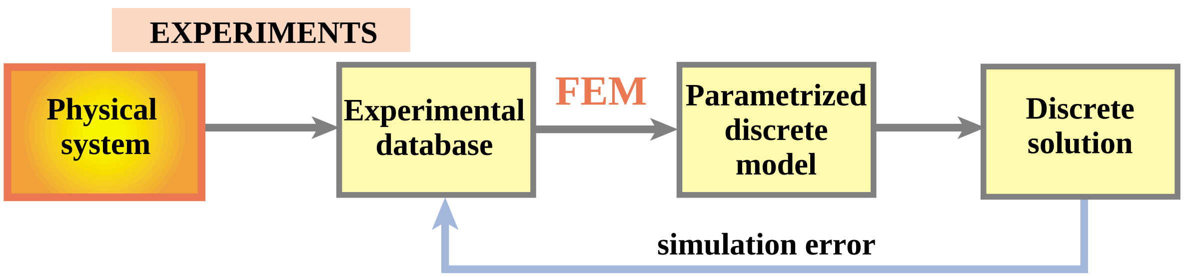


Introduction to FEM

FEM in Modeling and Simulation: Mathematical FEM

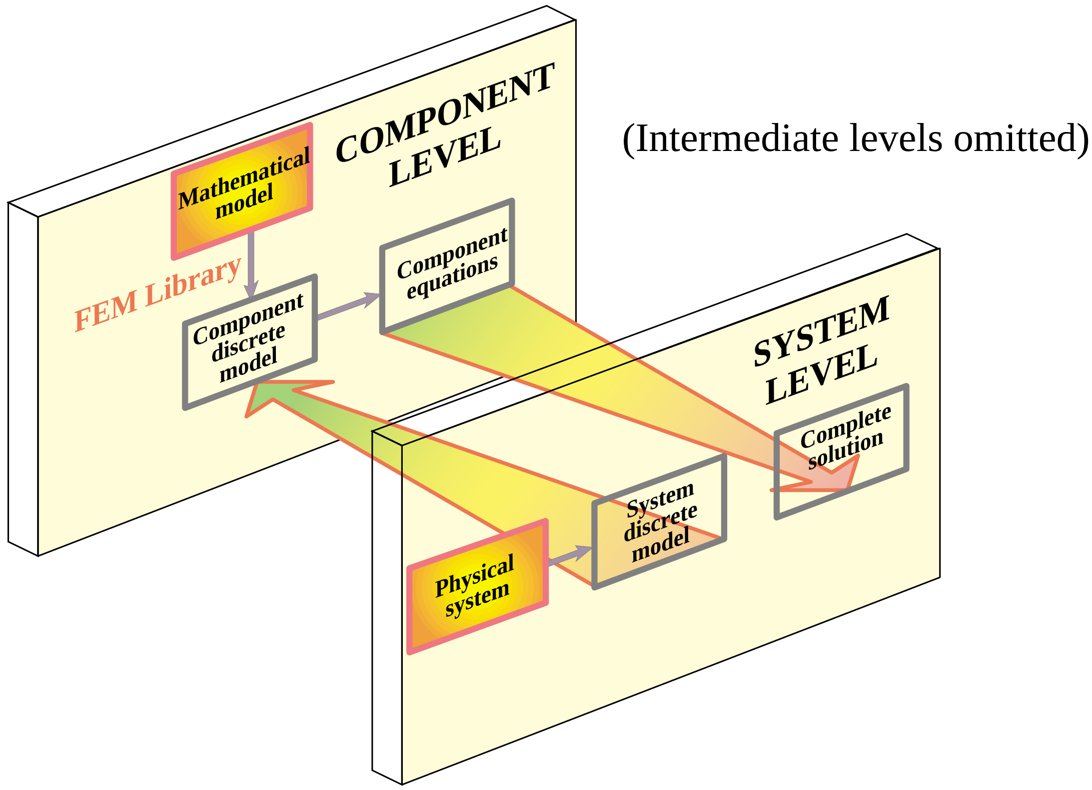


Model Updating in Physical FEM



Introduction to FEM

Synergy Between Mathematical and Physical FEM



Recommended Books for Linear FEM

Basic level (reference): Zienkiewicz & Taylor (1988), Vols I (1988), II (1993). A comprehensive upgrade of the 1977 edition. Primarily an encyclopedic reference work that provides a panoramic coverage of FEM, as well as a comprehensive list of references. Not a textbook.

Basic level (textbook): Cook, Malkus & Plesha (1989); this third edition is fairly comprehensive in scope and up to date although the coverage is more superficial than Zienkiewicz & Taylor.

Intermediate level: Hughes (1987). It requires substantial mathematical expertise on the part of the reader.

Mathematically oriented: Strang & Fix (1973). Most readable mathematical treatment although outdated in several subjects.

Most fun (if you like British "humor"): Irons & Ahmad (1980)

Best value for the $$$: Przemieniecki (Dover edition 1985, ~$16). Although outdated in many respects (e.g. the word "finite element" does not appear in this reprint of the original 1966 book), it is a valuable reference for programming simple elements.

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