CHAPTER 5

Synchronous Machines

As we have seen in Section 4.2.1, a synchronous machine is an ac machine whose speed under steady-state conditions is proportional to the frequency of the current in its armature. The rotor, along with the magnetic field created by the dc field current on the rotor, rotates at the same speed as, or in synchronism with, the rotating magnetic field produced by the armature currents, and a steady torque results. An elementary picture of how a synchronous machine works is given in Section 4.2.1 with emphasis on torque production in terms of the interactions between the machine's magnetic fields.

Analytical methods of examining the steady-state performance of polyphase synchronous machines will be developed in this chapter. Initial consideration will be given to cylindrical-rotor machines; the effects of salient poles are taken up in Sections 5.6 and 5.7.

5.1 INTRODUCTION TO POLYPHASE SYNCHRONOUS MACHINES

As indicated in Section 4.2.1, a synchronous machine is one in which alternating current flows in the armature winding, and dc excitation is supplied to the field winding. The armature winding is almost invariably on the stator and is usually a three-phase winding, as discussed in Chapter 4. The field winding is on the rotor. The cylindrical-rotor construction shown in Figs. 4.10 and 4.11 is used for two- and four-pole turbine generators. The salient-pole construction shown in Fig. 4.9 is best adapted to multipolar, slow-speed, hydroelectric generators and to most synchronous motors. The dc power required for excitation—approximately one to a few percent of the rating of the synchronous machine—is supplied by the *excitation system.*

In older machines, the excitation current was typically supplied through *slip rings* from a dc machine, referred to as the *exciter,* which was often mounted on the same shaft as the synchronous machine. In more modern systems, the excitation is supplied from ac exciters and solid-state rectifiers (either simple diode bridges or phase-controlled rectifiers). In some cases, the rectification occurs in the stationary frame, and the rectified excitation current is fed to the rotor via slip rings. In other systems, referred to as *brushless excitation systems,* the alternator of the ac exciter is on the rotor, as is the rectification system, and the current is supplied directly to the field-winding without the need for slip rings. One such system is described in Appendix D.

As is discussed in Chapter 4, a single synchronous generator supplying power to an impedance load acts as a voltage source whose frequency is determined by the speed of its mechanical drive (or *prime mover*), as can be seen from Eq. 4.2. From Eqs. 4.42, 4.44, and 4.50, the amplitude of the generated voltage is proportional to the frequency and the field current. The current and power factor are then determined by the generator field excitation and the impedance of the generator and load.

Synchronous generators can be readily operated in parallel, and, in fact, the electricity supply systems of industrialized countries typically have scores or even hundreds of them operating in parallel, interconnected by thousands of miles of transmission lines, and supplying electric energy to loads scattered over areas of many thousands of square miles. These huge systems have grown in spite of the necessity for designing the system so that synchronism is maintained following disturbances and the problems, both technical and administrative, which must be solved to coordinate the operation of such a complex system of machines and personnel. The principal reasons for these interconnected systems are reliability of service and economies in plant investment and operating costs.

When a synchronous generator is connected to a large interconnected system containing many other synchronous generators, the voltage and frequency at its armature terminals are substantially fixed by the system. As a result, armature currents will produce a component of the air-gap magnetic field which rotates at synchronous speed (Eq. 4.41) as determined by the system electrical frequency f_e. As is discussed in Chapter 4, for the production of a steady, unidirectional electromechanical torque, the fields of the stator and rotor must rotate at the same speed, and therefore the rotor must turn at precisely synchronous speed. Because any individual generator is a small fraction of the total system generation, it cannot significantly affect the system voltage or frequency. It is thus often useful, when studying the behavior of an individual generator or group of generators, to represent the remainder of the system as a constant-frequency, constant-voltage source, commonly referred to as an *infinite bus*.

Many important features of synchronous-machine behavior can be understood from the analysis of a single machine connected to an infinite bus. The steady-state behavior of a synchronous machine can be visualized in terms of the torque equation. From Eq. 4.81, with changes in notation appropriate to synchronous-machine theory,

$$T = \frac{\pi}{2}\left(\frac{\text{poles}}{2}\right)^2 \Phi_{\text{R}} F_{\text{f}} \sin \delta_{\text{RF}} \tag{5.1}$$

where

Φ_R = resultant air-gap flux per pole
F_f = mmf of the dc field winding
δ_{RF} = electrical phase angle between magnetic axes of Φ_R and F_f

The minus sign of Eq. 4.81 has been omitted with the understanding that the electromechanical torque acts in the direction to bring the interacting fields into alignment. In normal steady-state operation, the electromechanical torque balances the mechanical torque applied to the shaft. In a generator, the prime-mover torque acts in the direction of rotation of the rotor, pushing the rotor mmf wave ahead of the resultant air-gap flux. The electromechanical torque then opposes rotation. The opposite situation exists in a synchronous motor, where the electromechanical torque is in the direction of rotation, in opposition to the retarding torque of the mechanical load on the shaft.

Variations in the electromechanical torque result in corresponding variations in the *torque angle,* δ_{RF}, as seen from Eq. 5.1. The relationship is shown in the form of a *torque-angle curve* in Fig. 5.1, where the field current (rotor mmf) and resultant air-gap flux are assumed constant. Positive values of torque represent generator action, corresponding to positive values of δ_{RF} for which the rotor mmf wave leads the resultant air-gap flux.

As the prime-mover torque is increased, the magnitude of δ_{RF} must increase until the electromechanical torque balances the shaft torque. The readjustment process is actually a dynamic one, requiring a change in the mechanical speed of the rotor, typically accompanied by a damped mechanical oscillation of the rotor about its new steady-state torque angle. This oscillation is referred to as a *hunting transient.* In a practical machine undergoing such a transient, some changes in the amplitudes of the resultant flux-density and field-winding mmf wave may also occur because of various factors such as saturation effects, the effect of the machine leakage impedance, the response of the machine's excitation system, and so on. To emphasize the fundamental

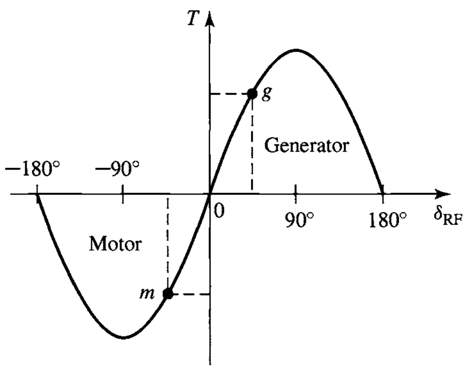


Figure 5.1 Torque-angle characteristic.

principles of synchronous-machine operation, such effects will be neglected in the present discussion.

The adjustment of the rotor to its new angular position following a load change can be observed experimentally in the laboratory by viewing the machine rotor with stroboscopic light triggered from the applied armature voltage (thus having a flashing frequency which causes the rotor to appear stationary when it is turning at its normal synchronous speed). Alternatively, electronic sensors can be used to determine the shaft position relative to the synchronous reference frame associated with the stator voltage. The resultant signal can be displayed on an oscilloscope or recorded with a data-acquisition system.

As can be seen from Fig. 5.1, an increase in prime-mover torque will result in a corresponding increase in the torque angle. When δ_{RF} becomes 90°, the electromechanical torque reaches its maximum value, known as the *pull-out torque*. Any further increase in prime-mover torque cannot be balanced by a corresponding increase in synchronous electromechanical torque, with the result that synchronism will no longer be maintained and the rotor will speed up. This phenomenon is known as *loss of synchronism* or *pulling out of step*. Under these conditions, the generator is usually disconnected from the external electrical system by the automatic operation of circuit breakers, and the prime mover is quickly shut down to prevent dangerous overspeed. Note from Eq. 5.1 that the value of the pull-out torque can be increased by increasing either the field current or the resultant air-gap flux. However, this cannot be done without limit; the field current is limited by the ability to cool the field winding, and the air-gap flux is limited by saturation of the machine iron.

As seen from Fig. 5.1, a similar situation occurs in a synchronous motor for which an increase in the shaft load torque beyond the pull-out torque will cause the rotor to lose synchronism and thus to slow down. Since a synchronous motor develops torque only at synchronous speed, it cannot be started simply by the application of armature voltages of rated frequency. In some cases, a squirrel-cage structure is included in the rotor, and the motor can be started as an induction motor and then synchronized when it is close to synchronous speed.

5.2 SYNCHRONOUS-MACHINE INDUCTANCES; EQUIVALENT CIRCUITS

In Section 5.1, synchronous-machine torque-angle characteristics are described in terms of the interacting air-gap flux and mmf waves. Our purpose now is to derive an equivalent circuit which represents the steady-state terminal volt-ampere characteristics.

A cross-sectional sketch of a three-phase cylindrical-rotor synchronous machine is shown schematically in Fig. 5.2. The figure shows a two-pole machine; alternatively, this can be considered as two poles of a multipole machine. The three-phase armature winding on the stator is of the same type used in the discussion of rotating magnetic fields in Section 4.5. Coils aa', bb', and cc' represent distributed windings producing sinusoidal mmf and flux-density waves in the air gap. The reference directions for

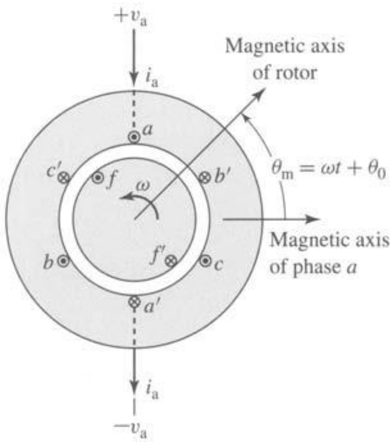


Figure 5.2 Schematic diagram of a two-pole, three-phase cylindrical-rotor synchronous machine.

the currents are shown by dots and crosses. The field winding ff' on the rotor also represents a distributed winding which produces a sinusoidal mmf and flux-density wave centered on its magnetic axis and rotating with the rotor.

When the flux linkages with armature phases a, b, c and field winding f are expressed in terms of the inductances and currents as follows,

$$\lambda_a = \mathcal{L}_{aa} i_a + \mathcal{L}_{ab} i_b + \mathcal{L}_{ac} i_c + \mathcal{L}_{af} i_f \tag{5.2}$$

$$\lambda_b = \mathcal{L}_{ba} i_a + \mathcal{L}_{bb} i_b + \mathcal{L}_{bc} i_c + \mathcal{L}_{bf} i_f \tag{5.3}$$

$$\lambda_c = \mathcal{L}_{ca} i_a + \mathcal{L}_{cb} i_b + \mathcal{L}_{cc} i_c + \mathcal{L}_{cf} i_f \tag{5.4}$$

$$\lambda_f = \mathcal{L}_{fa} i_a + \mathcal{L}_{fb} i_b + \mathcal{L}_{fc} i_c + \mathcal{L}_{ff} i_f \tag{5.5}$$

the induced voltages can be found from Faraday's law. Here, two like subscripts denote a self-inductance, and two unlike subscripts denote a mutual inductance between the two windings. The script $\mathcal{L}$ is used to indicate that, in general, both the self- and mutual inductances of a three-phase machine may vary with rotor angle, as is seen, for example, in Section C.2, where the effects of salient poles are analyzed.

Before we proceed, it is useful to investigate the nature of the various inductances. Each of these inductances can be expressed in terms of constants which can be computed from design data or measured by tests on an existing machine.

5.2.1 Rotor Self-Inductance

With a cylindrical stator, the self-inductance of the field winding is independent of the rotor position θ_m when the harmonic effects of stator slot openings are neglected. Hence

$$\mathcal{L}_{ff} = L_{ff} = L_{ff0} + L_{fl} \tag{5.6}$$

where the italic L is used for an inductance which is independent of θ_m. The component L_{ff0} corresponds to that portion of $\mathcal{L}_{ff}$ due to the space-fundamental component of air-gap flux. This component can be computed from air-gap dimensions and winding data, as shown in Appendix B. The additional component L_{fl} accounts for the field-winding leakage flux.

Under transient or unbalanced conditions, the flux linkages with the field winding, Eq. 5.5, vary with time, and the voltages induced in the rotor circuits have an important effect on machine performance. With balanced three-phase armature currents, however, the constant-amplitude magnetic field of the armature currents rotates in synchronism with the rotor. Thus the field-winding flux linkages produced by the armature currents do not vary with time, and the voltage induced in the field winding is therefore zero. With constant dc voltage V_f applied to the field-winding terminals, the field direct current I_f is given by Ohm's law, $I_f = V_f/R_f$.

5.2.2 Stator-to-Rotor Mutual Inductances

The stator-to-rotor mutual inductances vary periodically with θ_{me}, the electrical angle between the magnetic axes of the field winding and the armature phase a as shown in Fig. 5.2 and as defined by Eq. 4.54. With the space-mmf and air-gap flux distribution assumed sinusoidal, the mutual inductance between the field winding f and phase a varies as $\cos\theta_{me}$; thus

$$\mathcal{L}_{af} = \mathcal{L}_{fa} = L_{af}\cos\theta_{me} \tag{5.7}$$

Similar expressions apply to phases b and c, with θ_{me} replaced by $\theta_{me} - 120°$ and $\theta_{me} + 120°$, respectively. Attention will be focused on phase a. The inductance L_{af} can be calculated as discussed in Appendix B.

With the rotor rotating at synchronous speed ω_s (Eq. 4.40), the rotor angle will vary as

$$\theta_m = \omega_s t + \delta_0 \tag{5.8}$$

where δ_0 is the angle of the rotor at time $t = 0$. From Eq. 4.54

$$\theta_{me} = \left(\frac{\text{poles}}{2}\right)\theta_m = \omega_e t + \delta_{e0} \tag{5.9}$$

Here, $\omega_e = (\text{poles}/2)\omega_s$ is the electrical frequency and δ_{e0} is the electrical angle of the rotor at time $t = 0$.

Thus, substituting into Eq. 5.7 gives

$$\mathcal{L}_{af} = \mathcal{L}_{fa} = L_{af}\cos(\omega_e t + \delta_{e0}) \tag{5.10}$$

5.2.3 Stator Inductances; Synchronous Inductance

With a cylindrical rotor, the air-gap geometry is independent of θ_m if the effects of rotor slots are neglected. The stator self-inductances then are constant; thus

$$\mathcal{L}_{aa} = \mathcal{L}_{bb} = \mathcal{L}_{cc} = L_{aa} = L_{aa0} + L_{al} \tag{5.11}$$

has a wound rotor whose terminals are brought out through slip rings.

a. At what speed does the motor run?

b. What is the frequency of the voltages produced at the slip rings of the induction motor?

c. What will be the frequency of the voltages produced at the slip rings of the induction motor if two leads of the induction-motor stator are interchanged, reversing the direction of rotation of the resultant rotating field?

6.9 A three-phase, eight-pole, 60-Hz, 4160-V, 1000-kW squirrel-cage induction motor has the following equivalent-circuit parameters in ohms per phase Y referred to the stator:

$$R_1 = 0.220 \quad R_2 = 0.207 \quad X_1 = 1.95 \quad X_2 = 2.42 \quad X_m = 45.7$$

Determine the changes in these constants which will result from the following proposed design modifications. Consider each modification separately.

a. Replace the stator winding with an otherwise identical winding with a wire size whose cross-sectional area is increased by 4 percent.

b. Decrease the inner diameter of the stator laminations such that the air gap is decreased by 15 percent.

c. Replace the aluminum rotor bars (conductivity 3.5×10^7 mhos/m) with copper bars (conductivity 5.8×10^7 mhos/m).

d. Reconnect the stator winding, originally connected in Y for 4160-V operation, in Δ for 2.4 kV operation.

6.10 A three-phase, Y-connected, 460-V (line-line), 25-kW, 60-Hz, four-pole induction motor has the following equivalent-circuit parameters in ohms per phase referred to the stator:

$$R_1 = 0.103 \quad R_2 = 0.225 \quad X_1 = 1.10 \quad X_2 = 1.13 \quad X_m = 59.4$$

The total friction and windage losses may be assumed constant at 265 W, and the core loss may be assumed to be equal to 220 W. With the motor connected directly to a 460-V source, compute the speed, output shaft torque and power, input power and power factor and efficiency for slips of 1, 2 and 3 percent. You may choose either to represent the core loss by a resistance connected directly across the motor terminals or by resistance R_c connected in parallel with the magnetizing reactance X_m.

6.11 Consider the induction motor of Problem 6.10.

a. Find the motor speed in r/min corresponding to the rated shaft output power of 25 kW. (Hint: This can be easily done by writing a MATLAB script which searches over the motor slip.)

b. Similarly, find the speed in r/min at which the motor will operate with no external shaft load (assuming the motor load at that speed to consist only of the friction and windage losses).

c. Write a MATLAB script to plot motor efficiency versus output power as the motor output power varies from zero to full load.

air-gap component of the phase-a self-flux linkages. The second, L_{al}, known as the armature-winding *leakage inductance,* is due to the leakage component of phase-a flux linkages. The third component, $\frac{1}{2}L_{aa0}$, is due to the phase-a flux linkages from the space-fundamental component of air-gap flux produced by currents in phases b and c. Under balanced three-phase conditions, the phase-b and -c currents are related to the current in phase a by Eq. 5.15. Thus the synchronous inductance is an apparent inductance in that it accounts for the flux linkages of phase a in terms of the current in phase a, even though some of this flux linkage is due to currents in phases a and b. Hence, although synchronous inductance appears in that role in Eq. 5.18, it is not the self-inductance of phase a alone.

The significance of the synchronous inductance can be further appreciated with reference to the discussion of rotating magnetic fields in Section 4.5.2, where it was shown that under balanced three-phase conditions, the armature currents create a rotating magnetic flux wave in the air gap of magnitude equal to $\frac{3}{2}$ times the magnitude of that due to phase a alone, the additional component being due to the phase-b and -c currents. This corresponds directly to the $\frac{3}{2}L_{aa0}$ component of the synchronous inductance in Eq. 5.17; this component of the synchronous inductance accounts for the total space-fundamental air-gap component of phase-a flux linkages produced by the three armature currents under balanced three-phase conditions.

5.2.4 Equivalent Circuit

The phase-a terminal voltage is the sum of the armature-resistance voltage drop $R_a i_a$ and the induced voltage. The voltage e_{af} induced by the field winding flux (often referred to as the *generated voltage* or *internal voltage*) can be found from the time derivative of Eq. 5.18 with the armature current i_a set equal to zero. With dc excitation I_f in the field winding, substitution of Eq. 5.10 gives

$$e_{af} = \frac{d}{dt}(\mathcal{L}_{af} i_f) = -\omega_e L_{af} I_f \sin(\omega_e t + \delta_{e0}) \tag{5.19}$$

Using Eq. 5.18, the terminal voltage can then be expressed as

$$\begin{aligned} v_a &= R_a i_a + \frac{d\lambda_a}{dt} \\ &= R_a i_a + L_s \frac{di_a}{dt} + e_{af} \end{aligned} \tag{5.20}$$

The generated voltage e_{af} of Eq. 5.19 is at frequency ω_e, equal to the electrical frequency of the generator terminal voltage. Its rms amplitude is given by

$$E_{af} = \frac{\omega_e L_{af} I_f}{\sqrt{2}} \tag{5.21}$$

Under this synchronous operating condition, all machine armature quantities (current and flux linkage) will also vary sinusoidally in time at frequency ω_e. Thus, we can write the above equations in term of their rms, complex amplitudes. From Eq. 5.19 we can write the rms complex amplitude of the generated voltage as

$$\hat{E}_{\mathrm{af}} = j\left(\frac{\omega_{\mathrm{e}} L_{\mathrm{af}} I_{\mathrm{f}}}{\sqrt{2}}\right) e^{j\delta_{\mathrm{e0}}} \tag{5.22}$$

Similarly, the terminal-voltage equation, Eq. 5.20, can be written in terms of rms complex amplitudes as

$$\hat{V}_{\mathrm{a}} = R_{\mathrm{a}}\hat{I}_{\mathrm{a}} + jX_{\mathrm{s}}\hat{I}_{\mathrm{a}} + \hat{E}_{\mathrm{af}} \tag{5.23}$$

where $X_{\mathrm{s}} = \omega_{\mathrm{e}} L_{\mathrm{s}}$ is known as the *synchronous reactance.*

An equivalent circuit in complex form is shown in Fig. 5.3a. The reader should note that Eq. 5.23 and Fig. 5.3a are written with the reference direction for $\hat{I}_{\mathrm{a}}$ defined as positive into the machine terminals. This is known as the *motor reference direction* for the current.

Alternatively, the *generator reference direction* is defined with the reference direction for $\hat{I}_{\mathrm{a}}$ chosen as positive out of the machine terminals, as shown in Fig. 5.3b. Under this choice of current reference direction, Eq. 5.23 becomes

$$\hat{V}_{\mathrm{a}} = -R_{\mathrm{a}}\hat{I}_{\mathrm{a}} - jX_{\mathrm{s}}\hat{I}_{\mathrm{a}} + \hat{E}_{\mathrm{af}} \tag{5.24}$$

Note that these two representations are equivalent; when analyzing a particular synchronous-machine operating condition the actual current will be the same. The sign of I_{a} will simply be determined by the choice of reference direction. Either choice is acceptable, independent of whether the synchronous machine under investigation is operating as a motor or a generator. However, since power tends to flow into a motor, it is perhaps intuitively more satisfying to choose a reference direction with current flowing into the machine for the analysis of motor operation. The opposite is true for generator operation, for which power tends to flow out of the machine. Most of the synchronous-machine analysis techniques presented here were first developed to analyze the performance of synchronous generators in electric power systems. As a result, the generator reference direction is more common and is generally used from this point on in the text.

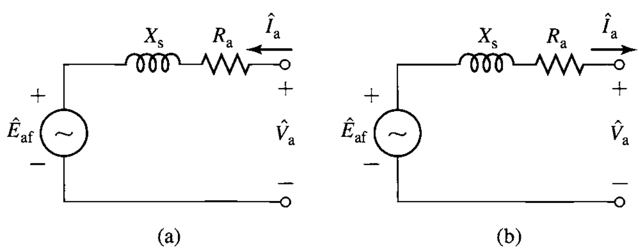


Figure 5.3 Synchronous-machine equivalent circuits: (a) motor reference direction and (b) generator reference direction.

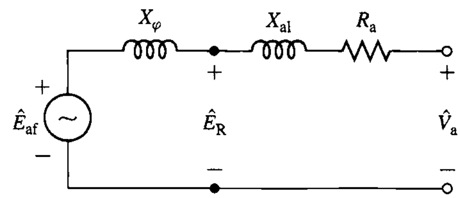


Figure 5.4 Synchronous-machine equivalent circuit showing air-gap and leakage components of synchronous reactance and air-gap voltage.

Figure 5.4 shows an alternative form of the equivalent circuit in which the synchronous reactance is shown in terms of its components. From Eq. 5.17

$$
\begin{aligned}
X_s = \omega_e L_s &= \omega_e L_{al} + \omega_e \left(\frac{3}{2} L_{aa0}\right) \\
&= X_{al} + X_\varphi
\end{aligned}
\tag{5.25}
$$

where $X_{al} = \omega L_{al}$ is the armature *leakage reactance* and $X_\varphi = \omega(\frac{3}{2}L_{aa0})$ is the reactance corresponding to the rotating space-fundamental air-gap flux produced by the three armature currents. The reactance X_φ is the effective *magnetizing reactance* of the armature winding under balanced three-phase conditions. The rms voltage $\hat{E}_R$ is the internal voltage generated by the resultant air-gap flux and is usually referred to as the *air-gap voltage* or the *voltage "behind" leakage reactance.*

It is important to recognize that the equivalent circuits of Figs. 5.3 and 5.4 are *single-phase, line-to-neutral equivalent circuits for a three-phase machine operating under balanced, three-phase conditions.* Thus, once the phase-*a* voltages and currents are found, either from the equivalent circuit or directly from the voltage equations (Eqs. 5.23 and 5.24), the currents and voltages for phases *b* and *c* can be found simply by phase-shifting those of phase *a* by $-120°$ and $120°$ respectively. Similarly, the total three-phase power of the machine can be found simply by multiplying that of phase *a* by three, unless the analysis is being performed in per unit (see Section 2.9), in which case the three-phase, per-unit power is equal to that found from solving for phase *a* alone and the factor of three is not needed.

EXAMPLE 5.1

A 60-Hz, three-phase synchronous motor is observed to have a terminal voltage of 460 V (line-line) and a terminal current of 120 A at a power factor of 0.95 lagging. The field-current under this operating condition is 47 A. The machine synchronous reactance is equal to 1.68 Ω (0.794 per unit on a 460-V, 100-kVA, 3-phase base). Assume the armature resistance to be negligible.

Calculate (*a*) the generated voltage E_{af} in volts, (*b*) the magnitude of the field-to-armature mutual inductance L_{af}, and (*c*) the electrical power input to the motor in kW and in horsepower.

■ Solution

a. Using the motor reference direction for the current and neglecting the armature resistance, the generated voltage can be found from the equivalent circuit of Fig. 5.3a or Eq. 5.23 as

$$\hat{E}_{af} = \hat{V}_a - jX_s\hat{I}_a$$

We will choose the terminal voltage as our phase reference. Because this is a line-to-neutral equivalent, the terminal voltage V_a must be expressed as a line-to-neutral voltage

$$\hat{V}_a = \frac{460}{\sqrt{3}} = 265.6 \text{ V, line-to-neutral}$$

A lagging power factor of 0.95 corresponds to a power factor angle $\theta = -\cos^{-1}(0.95) = -18.2°$. Thus, the phase-$a$ current is

$$\hat{I}_a = 120\, e^{-j18.2°} \text{ A}$$

Thus

$$\begin{aligned}\hat{E}_{af} &= 265.6 - j1.68(120\, e^{-j18.2°}) \\ &= 278.8\, e^{-j43.4°} \text{ V, line-to-neutral}\end{aligned}$$

and hence, the generated voltage E_{af} is equal to 278.8 V rms, line-to-neutral.

b. The field-to-armature mutual inductance can be found from Eq. 5.21. With $\omega_e = 120\pi$,

$$L_{af} = \frac{\sqrt{2}\, E_{af}}{\omega_e I_f} = \frac{\sqrt{2} \times 279}{120\pi \times 47} = 22.3 \text{ mH}$$

c. The three-phase power input to the motor P_{in} can be found as three times the power input to phase a. Hence,

$$\begin{aligned}P_{in} &= 3V_a I_a(\text{power factor}) = 3 \times 265.6 \times 120 \times 0.95 \\ &= 90.8 \text{ kW} = 122 \text{ hp}\end{aligned}$$

EXAMPLE 5.2

Assuming the input power and terminal voltage for the motor of Example 5.1 remain constant, calculate (a) the phase angle δ of the generated voltage and (b) the field current required to achieve unity power factor at the motor terminals.

■ Solution

a. For unity power factor at the motor terminals, the phase-a terminal current will be in phase with the phase-a line-to-neutral voltage $\hat{V}_a$. Thus

$$\hat{I}_a = \frac{P_{in}}{3V_a} = \frac{90.6 \text{ kW}}{3 \times 265.6 \text{ V}} = 114 \text{ A}$$

From Eq. 5.23,

$$\begin{aligned}\hat{E}_{af} &= \hat{V}_a - jX_s\hat{I}_a \\ &= 265.6 - j1.68 \times 114 = 328\, e^{-j35.8°} \text{ V, line-to-neutral}\end{aligned}$$

Thus, $E_{af} = 328$ V line-to-neutral and $\delta = -35.8°$.

b. Having found L_{af} in Example 5.1, we can find the required field current from Eq. 5.21.

$$I_f = \frac{\sqrt{2}\,E_{af}}{\omega_e L_{af}} = \frac{\sqrt{2} \times 328}{377 \times 0.0223} = 55.2\ \text{A}$$

Practice Problem 5.1

The synchronous machine of Examples 5.1 and 5.2 is to be operated as a synchronous generator. For operation at 60 Hz with a terminal voltage of 460 V line-to-line, calculate the field current required to supply a load of 85 kW, 0.95 power-factor leading.

Solution

46.3 A

It is helpful to have a rough idea of the order of magnitude of the impedance components. For machines with ratings above a few hundred kVA, the armature-resistance voltage drop at rated current usually is less than 0.01 times rated voltage; i.e., the armature resistance usually is less than 0.01 per unit on the machine rating as a base. (The per-unit system is described in Section 2.9.) The armature leakage reactance usually is in the range of 0.1 to 0.2 per unit, and the synchronous reactance is typically in the range of 1.0 to 2.0 per unit. In general, the per-unit armature resistance increases and the per-unit synchronous reactance decreases with decreasing size of the machine. In small machines, such as those in educational laboratories, the armature resistance may be in the vicinity of 0.05 per unit and the synchronous reactance in the vicinity of 0.5 per unit. In all but small machines, the armature resistance can usually be neglected in most analyses, except insofar as its effect on losses and heating is concerned.

5.3 OPEN- AND SHORT-CIRCUIT CHARACTERISTICS

The fundamental characteristics of a synchronous machine can be determined by a pair of tests, one made with the armature terminals open-circuited and the second with the armature terminals short-circuited. These tests are discussed here. Except for a few remarks on the degree of validity of certain assumptions, the discussions apply to both cylindrical-rotor and salient-pole machines.

5.3.1 Open-Circuit Saturation Characteristic and No-Load Rotational Losses

Like the magnetization curve for a dc machine, the *open-circuit characteristic* (also referred to as the *open-circuit saturation curve*) of a synchronous machine is a curve of the open-circuit armature terminal voltage (either in volts or in per unit) as a function of the field excitation when the machine is running at synchronous speed, as shown by curve *occ* in Fig. 5.5. Typically, the base voltage is chosen equal to the rated voltage of the machine.

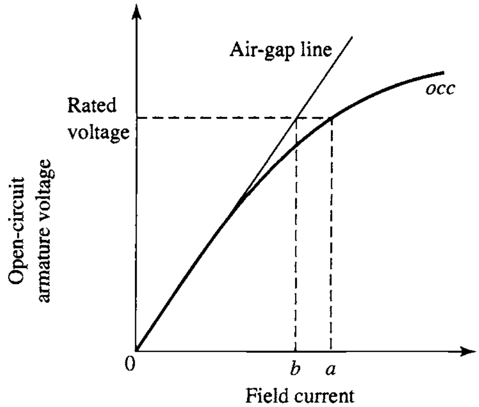


Figure 5.5 Open-circuit characteristic of a synchronous machine.

The open-circuit characteristic represents the relation between the space-fundamental component of the air-gap flux and the mmf acting on the magnetic circuit when the field winding constitutes the only mmf source. Note that the effects of magnetic saturation can be clearly seen; the characteristic bends downward with increasing field current as saturation of the magnetic material increases the reluctance of the flux paths in the machine and reduces the effectiveness of the field current in producing magnetic flux. As can be seen from Fig. 5.5, the open-circuit characteristic is initially linear as the field current is increased from zero. This portion of the curve (and its linear extension for higher values of field current) is known as the *air-gap line*. It represents the machine open-circuit voltage characteristic corresponding to unsaturated operation. Deviations of the actual open-circuit characteristic from this curve are a measure of the degree of saturation in the machine.

Note that with the machine armature winding open-circuited, the terminal voltage is equal to the generated voltage E_{af}. Thus the open-circuit characteristic is a measurement of the relationship between the field current I_{f} and E_{af}. It can therefore provide a direct measurement of the field-to-armature mutual inductance L_{af}.

EXAMPLE 5.3

An open-circuit test performed on a three-phase, 60-Hz synchronous generator shows that the rated open-circuit voltage of 13.8 kV is produced by a field current of 318 A. Extrapolation of the air-gap line from a complete set of measurements on the machine shows that the field-current corresponding to 13.8 kV on the air-gap line is 263 A. Calculate the saturated and unsaturated values of L_{af}.

■ Solution

From Eq. 5.21, L_{af} is found from

$$L_{\mathrm{af}} = \frac{\sqrt{2}\,E_{\mathrm{af}}}{\omega_{\mathrm{e}} I_{\mathrm{f}}}$$

Here, $E_{af} = 13.8\ \text{kV}/\sqrt{3} = 7.97\ \text{kV}$. Hence the saturated value of L_{af} is given by

$$(L_{af})_{sat} = \frac{\sqrt{2}(7.97 \times 10^3)}{377 \times 318} = 94\ \text{mH}$$

and the unsaturated value is

$$(L_{af})_{unsat} = \frac{\sqrt{2}(7.97 \times 10^3)}{377 \times 263} = 114\ \text{mH}$$

In this case, we see that saturation reduces the magnetic coupling between the field and armature windings by approximately 18 percent.

Practice Problem 5.2

If the synchronous generator of Example 5.3 is operated at a speed corresponding to a generated voltage of 50 Hz, calculate (*a*) the open-circuit line-to-line terminal voltage corresponding to a field current of 318 A and (*b*) the field-current corresponding to that same voltage on the 50-Hz air-gap line.

Solution

a. 11.5 kV
b. 263 A

When the machine is an existing one, the open-circuit characteristic is usually determined experimentally by driving the machine mechanically at synchronous speed with its armature terminals on open circuit and by reading the terminal voltage corresponding to a series of values of field current. If the mechanical power required to drive the synchronous machine during the open-circuit test is measured, the *no-load rotational losses* can be obtained. These losses consist of friction and windage losses associated with rotation as well as the core loss corresponding to the flux in the machine at no load. The friction and windage losses at synchronous speed are constant, while the open-circuit core loss is a function of the flux, which in turn is proportional to the open-circuit voltage.

The mechanical power required to drive the machine at synchronous speed and unexcited is its friction and windage loss. When the field is excited, the mechanical power equals the sum of the friction, windage, and open-circuit core loss. The open-circuit core loss therefore can be found from the difference between these two values of mechanical power. A typical curve of open-circuit core loss as a function of open-circuit voltage takes the form of that found in Fig. 5.6.

5.3.2 Short-Circuit Characteristic and Load Loss

A short-circuit characteristic can be obtained by applying a three-phase short circuit through suitable current sensors to the armature terminals of a synchronous machine. With the machine driven at synchronous speed, the field current can be increased

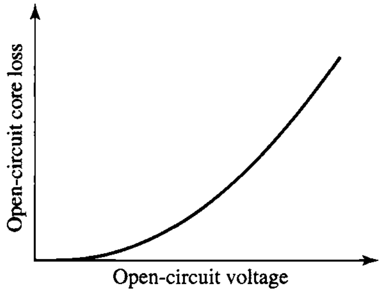


Figure 5.6 Typical form of an open-circuit core-loss curve.

and a plot of armature current versus field current can be obtained. This relation is known as the *short-circuit characteristic*. An open-circuit characteristic *occ* and a short-circuit characteristic *scc* are shown in Fig. 5.7.

With the armature short-circuited, $V_a = 0$ and, from Eq. 5.24 (using the generator reference direction for current)

$$\hat{E}_{af} = \hat{I}_a(R_a + jX_s) \tag{5.26}$$

The corresponding phasor diagram is shown in Fig. 5.8. Because the resistance is much smaller than the synchronous reactance, the armature current lags the excitation voltage by very nearly 90°. Consequently the armature-reaction-mmf wave is very nearly in line with the axis of the field poles and in opposition to the field mmf, as shown by phasors $\hat{A}$ and $\hat{F}$ representing the space waves of armature reaction and field mmf, respectively.

The resultant mmf creates the resultant air-gap flux wave which generates the air-gap voltage $\hat{E}_R$ (see Fig. 5.4) equal to the voltage consumed in armature resistance

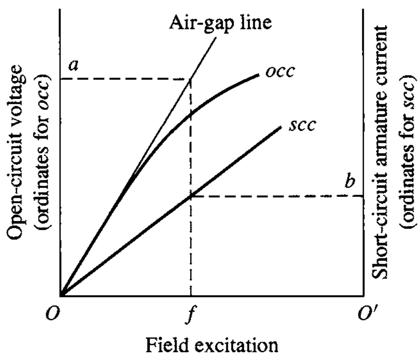


Figure 5.7 Open- and short-circuit characteristics of a synchronous machine.

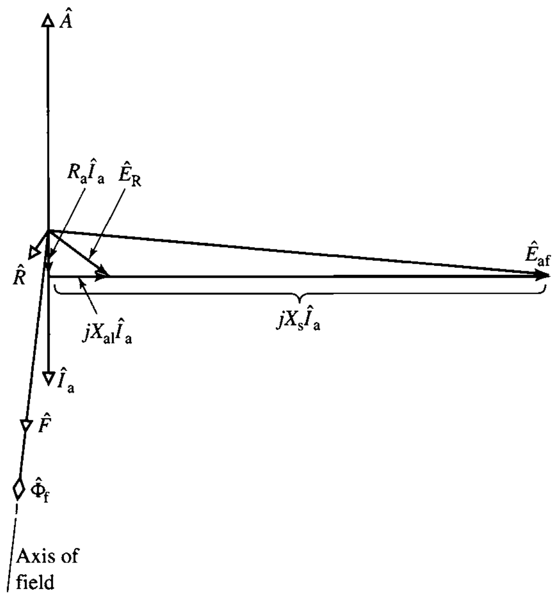


Figure 5.8 Phasor diagram for short-circuit conditions.

R_a and leakage reactance X_{al}; as an equation,

$$\hat{E}_R = \hat{I}_a(R_a + jX_{al}) \tag{5.27}$$

In many synchronous machines the armature resistance is negligible, and the leakage reactance is between 0.10 and 0.20 per unit; a representative value is about 0.15 per unit. That is, at rated armature current the leakage reactance voltage drop is about 0.15 per unit. From Eq. 5.27, therefore, the air-gap voltage at rated armature current on short circuit is about 0.15 per unit; i.e., the resultant air-gap flux is only about 0.15 times its normal voltage value. Consequently, the machine is operating in an unsaturated condition. The short-circuit armature current, therefore, is directly proportional to the field current over the range from zero to well above rated armature current; it is thus a straight line as can be seen in Fig. 5.7.

The *unsaturated synchronous reactance* (corresponding to unsaturated operating conditions within the machine) can be found from the open- and short-circuit characteristics. At any convenient field excitation, such as Of in Fig. 5.7, the armature current on short circuit is $O'b$, and the unsaturated generated voltage for the same field current corresponds to Oa, as read from the air-gap line. Note that the voltage on the air-gap line should be used because the machine is assumed to be operating in an unsaturated condition. If the line-to-neutral voltage corresponding to Oa is $V_{a,ag}$ and the armature current per phase corresponding to $O'b$ is $I_{a,sc}$, then from Eq. 5.26, with armature resistance neglected, the unsaturated synchronous reactance $X_{s,u}$ is

$$X_{s,u} = \frac{V_{a,ag}}{I_{a,sc}} \tag{5.28}$$

where the subscripts "ag" and "sc" indicate air-gap line conditions and short-circuit conditions, respectively. If $V_{a,ag}$ and $I_{a,sc}$ are expressed in per unit, the synchronous reactance will be in per unit. If $V_{a,ag}$ and $I_{a,sc}$ are expressed in rms line-to-neutral volts and rms amperes per phase, respectively, the synchronous reactance will be in ohms per phase.

Note that the synchronous reactance in ohms is calculated by using the phase or line-to-neutral voltage. Often the open-circuit saturation curve is given in terms of the line-to-line voltage, in which case the voltage must be converted to the line-to-neutral value by dividing by $\sqrt{3}$.

For operation at or near rated terminal voltage, it is sometimes assumed that the machine is equivalent to an unsaturated one whose magnetization line is a straight line through the origin and the rated-voltage point on the open-circuit characteristic, as shown by the dashed line Op in Fig. 5.9. According to this approximation, the *saturated value of the synchronous reactance* at rated voltage $V_{a,rated}$ is

$$X_s = \frac{V_{a,rated}}{I'_a} \tag{5.29}$$

where I'_a is the armature current $O'c$ read from the short-circuit characteristic at the field current Of' corresponding to $V_{a,rated}$ on the open-circuit characteristic, as shown in Fig. 5.9. As with the unsaturated synchronous reactance, if $V_{a,rated}$ and I'_a are expressed in per unit, the synchronous reactance will be in per unit. If $V_{a,rated}$ and I'_a are expressed in rms line-to-neutral volts and rms amperes per phase, respectively, the synchronous reactance will be in ohms per phase. This method of handling the

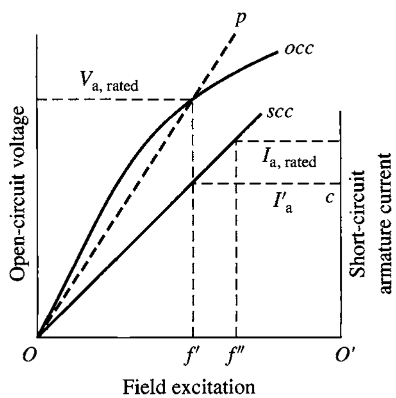


Figure 5.9 Open- and short-circuit characteristics showing equivalent magnetization line for saturated operating conditions.

effects of saturation, which assumes that the effects of saturation can be described by a single value of saturated reactance, usually gives satisfactory results when great accuracy is not required.

The *short-circuit ratio* (SCR) is defined as the ratio of the field current required for rated voltage on open circuit to the field current required for rated armature current on short circuit. That is, in Fig. 5.9

$$\mathrm{SCR} = \frac{Of'}{Of''} \tag{5.30}$$

It can be shown that the SCR is the reciprocal of the per-unit value of the saturated synchronous reactance found from Eq. 5.29. It is common to refer to the field current Of' required to achieve rated-open-circuit voltage as AFNL (*Amperes Field No Load*) and the field current Of'' required to achieve rated-short-circuit current as AFSC (*Amperes Field Short Circuit*). Thus, the short-circuit ratio can also be written as

$$\mathrm{SCR} = \frac{\mathrm{AFNL}}{\mathrm{AFSC}} \tag{5.31}$$

EXAMPLE 5.4

The following data are taken from the open- and short-circuit characteristics of a 45-kVA, three-phase, Y-connected, 220-V (line-to-line), six-pole, 60-Hz synchronous machine. From the open-circuit characteristic:

Line-to-line voltage = 220 V   Field current = 2.84 A

From the short-circuit characteristic:

Armature current, A	118	152
Field current, A	2.20	2.84

From the air-gap line:

Field current = 2.20 A   Line-to-line voltage = 202 V

Compute the unsaturated value of the synchronous reactance, its saturated value at rated voltage in accordance with Eq. 5.29, and the short-circuit ratio. Express the synchronous reactance in ohms per phase and in per unit on the machine rating as a base.

■ Solution

At a field current of 2.20 A the line-to-neutral voltage on the air-gap line is

$$V_{\mathrm{a,ag}} = \frac{202}{\sqrt{3}} = 116.7\ \mathrm{V}$$

and for the same field current the armature current on short circuit is

$$I_{\mathrm{a,sc}} = 118\ \mathrm{A}$$

From Eq. 5.28

$$X_{s,u} = \frac{116.7}{118} = 0.987\ \Omega/\text{phase}$$

Note that rated armature current is

$$I_{a,\text{rated}} = \frac{45{,}000}{\sqrt{3} \times 220} = 118\ \text{A}$$

Therefore, $I_{a,sc} = 1.00$ per unit. The corresponding air-gap-line voltage is

$$V_{a,ag} = \frac{202}{220} = 0.92 \text{ per unit}$$

From Eq. 5.28 in per unit

$$X_{s,u} = \frac{0.92}{1.00} = 0.92 \text{ per unit}$$

The saturated synchronous reactance can be found from the open- and short-circuit characteristics and Eq. 5.29

$$X_s = \frac{V_{a,\text{rated}}}{I'_a} = \frac{(220/\sqrt{3})}{152} = 0.836\ \Omega/\text{phase}$$

In per unit $I'_a = \frac{152}{118} = 1.29$, and from Eq. 5.29

$$X_s = \frac{1.00}{1.29} = 0.775 \text{ per unit}$$

Finally, from the open- and short-circuit characteristics and Eq. 5.30, the short-circuit ratio is given by

$$\text{SCR} = \frac{2.84}{2.20} = 1.29$$

Note that as was indicated following Eq. 5.30, the inverse of the short-circuit ratio is equal to the per-unit saturated synchronous reactance

$$X_s = \frac{1}{\text{SCR}} = \frac{1}{1.29} = 0.775 \text{ per unit}$$

Practice Problem 5.3

Calculate the saturated synchronous reactance (in Ω/phase and per unit) of a 85-kVA synchronous machine which achieves its rated open-circuit voltage of 460 V at a field current 8.7 A and which achieves rated short-circuit current at a field current of 11.2 A.

Solution

$$X_s = 3.21\ \Omega/\text{phase} = 1.29 \text{ per unit}$$

If the mechanical power required to drive the machine is measured while the short-circuit test is being made, information can be obtained regarding the losses

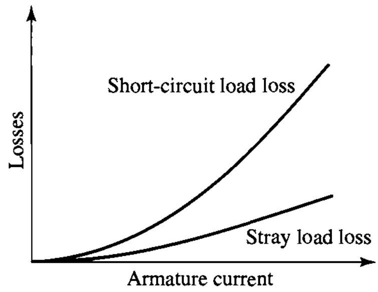


Figure 5.10 Typical form of short-circuit load loss and stray load loss curves.

caused by the armature current. Because the machine flux level is low under short-circuit conditions, the core loss under this condition is typically considered to be negligible. The mechanical power required to drive the synchronous machine during the short-circuit test then equals the sum of friction and windage loss (determined from the open-circuit test at zero field current) plus losses caused by the armature current. The losses caused by the armature current can then be found by subtracting friction and windage from the driving power. The losses caused by the short-circuit armature current are known collectively as the *short-circuit load loss*. A curve showing the typical form of short-circuit load loss plotted against armature current is shown in Fig. 5.10. Typically, it is approximately parabolic with armature current.

The short-circuit load loss consists of I^2R loss in the armature winding, local core losses caused by the armature leakage flux, and the very small core loss caused by the resultant flux. The dc resistance loss can be computed if the dc resistance is measured and corrected, when necessary, for the temperature of the windings during the short-circuit test. For copper conductors

$$\frac{R_T}{R_t} = \frac{234.5 + T}{234.5 + t} \tag{5.32}$$

where R_T and R_t are the resistances at Celsius temperatures T and t, respectively. If this dc resistance loss is subtracted from the short-circuit load loss, the difference will be the loss due to skin effect and eddy currents in the armature conductors plus the local core losses caused by the armature leakage flux. This difference between the short-circuit load loss and the dc resistance loss is the additional loss caused by the alternating current in the armature. It is the *stray-load loss* described in Appendix D, commonly considered to have the same value under normal load conditions as on short circuit. It is a function of the armature current, as shown by the curve in Fig. 5.10.

As with any ac device, the *effective resistance of the armature* $R_{a,\text{eff}}$ can be computed as the power loss attributable to the armature current divided by the square of the current. On the assumption that the stray load loss is a function of only the armature current, the effective resistance of the armature can be determined from the short-circuit load loss:

$$R_{\mathrm{a,eff}} = \frac{\text{short-circuit load loss}}{(\text{short-circuit armature current})^2} \tag{5.33}$$

If the short-circuit load loss and armature current are in per unit, the effective resistance will be in per unit. If they are in watts per phase and amperes per phase, respectively, the effective resistance will be in ohms per phase. Usually it is sufficiently accurate to find the value of $R_{\mathrm{a,eff}}$ at rated current and then to assume it to be constant.

EXAMPLE 5.5

For the 45-kVA, three-phase, Y-connected synchronous machine of Example 5.4, at rated armature current (118 A) the short-circuit load loss (total for three phases) is 1.80 kW at a temperature of 25°C. The dc resistance of the armature at this temperature is 0.0335 Ω/phase. Compute the effective armature resistance in per unit and in ohms per phase at 25°C.

■ Solution

The short-circuit load loss is $1.80/45 = 0.040$ per unit at $I_{\mathrm{a}} = 1.00$ per unit. Therefore,

$$R_{\mathrm{a,eff}} = \frac{0.040}{(1.00)^2} = 0.040 \text{ per unit}$$

On a per-phase basis the short-circuit load loss is $1800/3 = 600$ W/phase and consequently the effective resistance is

$$R_{\mathrm{a,eff}} = \frac{600}{(118)^2} = 0.043\ \Omega/\text{phase}$$

The ratio of ac-to-dc resistance is

$$\frac{R_{\mathrm{a,eff}}}{R_{\mathrm{a,dc}}} = \frac{0.043}{0.0335} = 1.28$$

Because this is a small machine, its per-unit resistance is relatively high. The effective armature resistance of machines with ratings above a few hundred kilovoltamperes usually is less than 0.01 per unit.

Practice Problem 5.4

Consider a three-phase 13.8 kV 25-MVA synchronous generator whose three-phase short-circuit loss is 52.8 kW at rated armature current. Calculate (*a*) its rated armature current and (*b*) its effective armature resistance in Ω/phase and in per unit.

Solution

a. 1046 A
b. $R_{\mathrm{a,eff}} = 0.0161\ \Omega/\text{phase} = 0.0021$ per unit

5.4 STEADY-STATE POWER-ANGLE CHARACTERISTICS

The maximum power a synchronous machine can deliver is determined by the maximum torque which can be applied without loss of synchronism with the external system to which it is connected. The purpose of this section is to derive expressions for the steady-state power limits of synchronous machines in simple situations for which the external system can be represented as an impedance in series with a voltage source.

Since both the external system and the machine itself can be represented as an impedance in series with a voltage source, the study of power limits becomes merely a special case of the more general problem of the limitations on power flow through a series impedance. The impedance will include the synchronous impedance of the synchronous machine as well as an equivalent impedance of the external system (which may consist of transmission lines and transformer banks as well as additional synchronous machines).

Consider the simple circuit of Fig. 5.11a, consisting of two ac voltages $\hat{E}_1$ and $\hat{E}_2$ connected by an impedance $Z = R + jX$ through which the current is $\hat{I}$. The phasor diagram is shown in Fig. 5.11b. Note that in this phasor diagram, the reference direction for positive angles is counter-clockwise. Thus, in Fig. 5.11b, the angle δ is positive while the angle ϕ can be seen to be negative.

The power P_2 delivered through the impedance to the load-end voltage source $\hat{E}_2$ is

$$P_2 = E_2 I \cos\phi \tag{5.34}$$

where ϕ is the phase angle of $\hat{I}$ with respect to $\hat{E}_2$. The phasor current is

$$\hat{I} = \frac{\hat{E}_1 - \hat{E}_2}{Z} \tag{5.35}$$

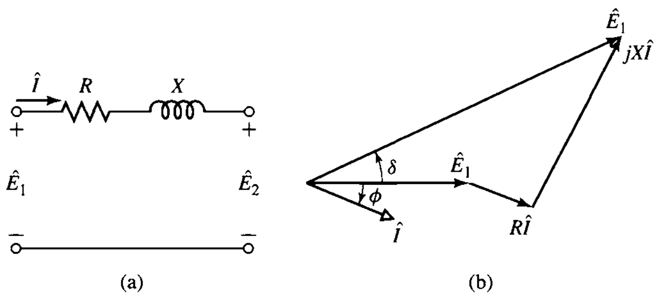


Figure 5.11 (a) Impedance interconnecting two voltages; (b) phasor diagram.

If the phasor voltages and the impedance are expressed in polar form,

$$\hat{E}_1 = E_1\, e^{j\delta} \tag{5.36}$$

$$\hat{E}_2 = E_2 \tag{5.37}$$

$$Z = R + jX = |Z|\, e^{j\phi_Z} \tag{5.38}$$

where δ is the phase angle by which $\hat{E}_1$ leads $\hat{E}_2$ and $\phi_Z = \tan^{-1}(X/R)$ is the phase angle of the impedance Z, then

$$\hat{I} = I\, e^{j\phi} = \frac{E_1\, e^{j\delta} - E_2}{|Z|\, e^{j\phi_Z}} = \frac{E_1}{|Z|}\, e^{j(\delta-\phi_Z)} - \frac{E_2}{|Z|}\, e^{-j\phi_Z} \tag{5.39}$$

Taking the real part of Eq. 5.39 gives

$$I \cos\phi = \frac{E_1}{|Z|} \cos(\delta - \phi_Z) - \frac{E_2}{|Z|} \cos(-\phi_Z) \tag{5.40}$$

Noting that $\cos(-\phi_Z) = \cos\phi_Z = R/|Z|$ we see that substitution of Eq. 5.40 in Eq. 5.34 gives

$$P_2 = \frac{E_1 E_2}{|Z|} \cos(\delta - \phi_Z) - \frac{E_2^2 R}{|Z|^2} \tag{5.41}$$

or

$$P_2 = \frac{E_1 E_2}{|Z|} \sin(\delta + \alpha_Z) - \frac{E_2^2 R}{|Z|^2} \tag{5.42}$$

where

$$\alpha_Z = 90^\circ - \phi_Z = \tan^{-1}\left(\frac{R}{X}\right) \tag{5.43}$$

Similarly power P_1 at source end $\hat{E}_1$ of the impedance can be expressed as

$$P_1 = \frac{E_1 E_2}{|Z|} \sin(\delta - \alpha_Z) + \frac{E_1^2 R}{|Z|^2} \tag{5.44}$$

If, as is frequently the case, the resistance is negligible, then $R \ll |Z|$, $|Z| \approx X$ and $\alpha_Z \approx 0$ and hence

$$P_1 = P_2 = \frac{E_1 E_2}{X} \sin\delta \tag{5.45}$$

Equation 5.45 is a very important equation in the study of synchronous machines and indeed in the study of ac power systems in general. When applied to the situation of a synchronous machine connected to an ac system, Eq. 5.45 is commonly referred to as the *power-angle characteristic* for a synchronous machine, and the angle δ is known as the *power angle*. If the resistance is negligible and the voltages are constant, then from Eq. 5.45 the maximum power transfer

$$P_{1,\max} = P_{2,\max} = \frac{E_1 E_2}{X} \tag{5.46}$$

occurs when $\delta = \pm 90°$. Note that if δ is positive, $\hat{E}_1$ leads $\hat{E}_2$ and, from Eq. 5.45, power flows from source $\hat{E}_1$ to $\hat{E}_2$. Similarly, when δ is negative, $\hat{E}_1$ lags $\hat{E}_2$ and power flows from source $\hat{E}_2$ to $\hat{E}_1$.

Equation 5.45 is valid for any voltage sources $\hat{E}_1$ and $\hat{E}_2$ separated by a reactive impedance jX. Thus for a synchronous machine with generated voltage $\hat{E}_{\mathrm{af}}$ and synchronous reactance X_{s} connected to a system whose Thevenin equivalent is a voltage source $\hat{V}_{\mathrm{EQ}}$ in series with a reactive impedance jX_{EQ}, as shown in Fig. 5.12, the power-angle characteristic can be written

$$P = \frac{E_{\mathrm{af}} V_{\mathrm{EQ}}}{X_{\mathrm{s}} + X_{\mathrm{EQ}}} \sin \delta \tag{5.47}$$

where P is the power transferred from the synchronous machine to the system and δ is the phase angle of $\hat{E}_{\mathrm{af}}$ with respect to $\hat{V}_{\mathrm{EQ}}$.

In a similar fashion, it is possible to write a power-angle characteristic in terms of X_{s}, E_{af}, the terminal voltage V_{a}, and the relative angle between them, or alternatively X_{EQ}, V_{a} and V_{EQ} and their relative angle. Although these various expressions are equally valid, they are not equally useful. For example, while E_{af} and V_{EQ} remain constant as P is varied, V_{a} will not. Thus, while Eq. 5.47 gives an easily solved relation between P and δ, a power-angle characteristic based upon V_{a} cannot be solved without an additional expression relating V_{a} to P.

It should be emphasized that the derivation of Eqs. 5.34 to 5.47 is based on a single-phase ac circuit. For a balanced three-phase system, if E_1 and E_2 are the line-neutral voltages, the results must be multiplied by three to get the total three-phase power; alternatively E_1 and E_2 can be expressed in terms of the line-to-line voltage (equal to $\sqrt{3}$ times the line-neutral voltage), in which case the equations give three-phase power directly.

When the power expression of Eq. 5.45 is compared with the expression of Eq. 5.1 for torque in terms of interacting flux and mmf waves, they are seen to

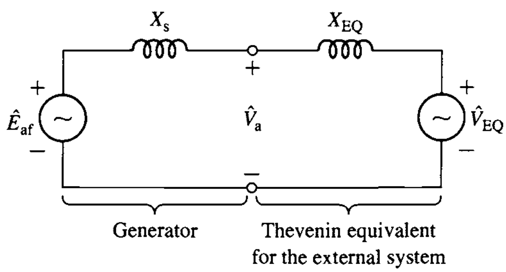


Figure 5.12 Equivalent-circuit representation of a synchronous machine connected to an external system.

be of the same form. This is no coincidence. Remember that torque and power are proportional when, as here, speed is constant. What we are really saying is that Eq. 5.1, applied specifically to an idealized cylindrical-rotor machine and translated to circuit terms, becomes Eq. 5.45. A quick mental review of the background of each relation should show that they stem from the same fundamental considerations.

From Eq. 5.47 we see that the maximum power transfer associated with synchronous-machine operation is proportional to the magnitude of the system voltage, corresponding to V_{EQ}, as well as to that of the generator internal voltage E_{af}. Thus, for constant system voltage, the maximum power transfer can be increased by increasing the synchronous-machine field current and thus the internal voltage. Of course, this cannot be done without limit; neither the field current nor the machine fluxes can be raised past the point where cooling requirements fail to be met.

In general, stability considerations dictate that a synchronous machine achieve steady-state operation for a power angle considerably less than 90°. Thus, for a given system configuration, it is necessary to ensure that the machine will be able to achieve its rated operation and that this operating condition will be within acceptable operating limits for both the machine and the system.

EXAMPLE 5.6

A three-phase, 75-MVA, 13.8-kV synchronous generator with saturated synchronous reactance $X_s = 1.35$ per unit and unsaturated synchronous reactance $X_{s,u} = 1.56$ per unit is connected to an external system with equivalent reactance $X_{EQ} = 0.23$ per unit and voltage $V_{EQ} = 1.0$ per unit, both on the generator base. It achieves rated open-circuit voltage at a field current of 297 amperes.

a. Find the maximum power P_{max} (in MW and per unit) that can be supplied to the external system if the internal voltage of the generator is held equal to 1.0 per unit.
b. Using MATLAB,† plot the terminal voltage of the generator as the generator output is varied from zero to P_{max} under the conditions of part (a).
c. Now assume that the generator is equipped with an *automatic voltage regulator* which controls the field current to maintain constant terminal voltage. If the generator is loaded to its rated value, calculate the corresponding power angle, per-unit internal voltage, and field current. Using MATLAB, plot per-unit E_{af} as a function of per-unit power.

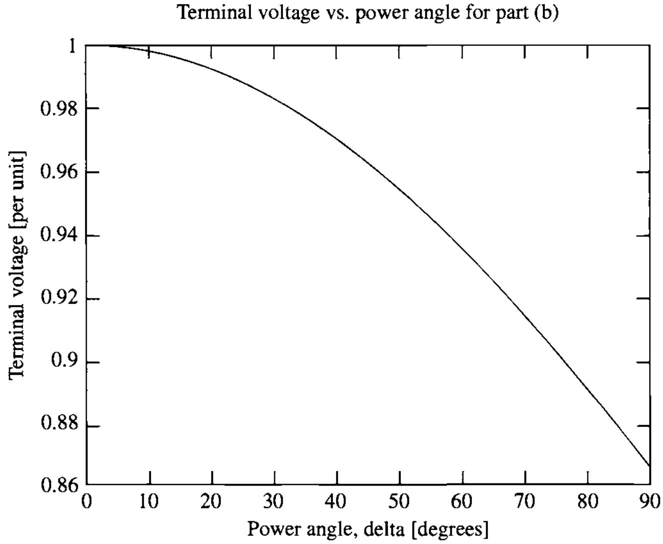
■ Solution

a. From Eq. 5.47

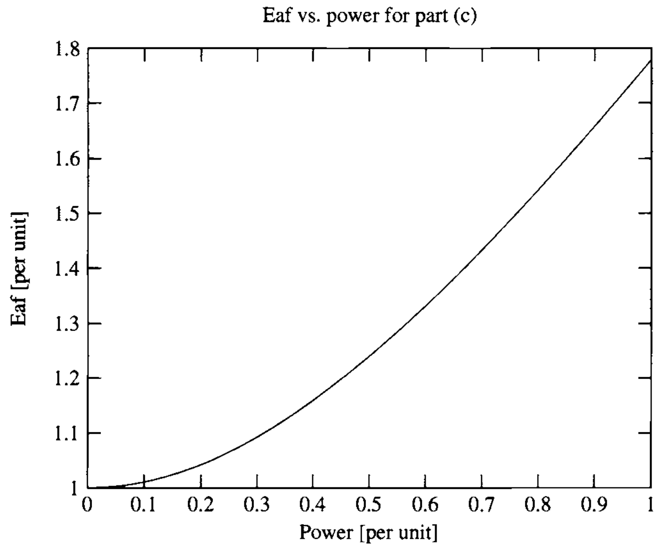
$$P_{max} = \frac{E_{af} V_{EQ}}{X_s + X_{EQ}}$$

Note that although this is a three-phase generator, no factor of 3 is required because we are working in per unit.

† MATLAB is a registered trademark of The MathWorks, Inc.



(a)



(b)

Figure 5.13 Example 5.6. (a) MATLAB plot of terminal voltage vs. δ for part (b). (b) MATLAB plot of E_{af} vs. power for part (c).

Because the machine is operating with a terminal voltage near its rated value, we should express P_{max} in terms of the saturated synchronous reactance. Thus

$$P_{max} = \frac{1}{1.35 + 0.23} = 0.633 \text{ per unit} = 47.5 \text{ MW}$$

b. From Fig 5.12, the generator terminal current is given by

$$\hat{I}_a = \frac{\hat{E}_{af} - \hat{V}_{EQ}}{j(X_s + X_{EQ})} = \frac{E_{af}\, e^{j\delta} - V_{EQ}}{j(X_s + X_{EQ})} = \frac{e^{j\delta} - 1.0}{j1.58}$$

The generator terminal voltage is then given by

$$\hat{V}_a = \hat{V}_{EQ} + jX_{EQ}\hat{I}_a = 1.0 + \frac{.23}{1.58}(e^{j\delta} - 1.0)$$

Figure 5.13a is the desired MATLAB plot. The terminal voltage can be seen to vary from 1.0 at $\delta = 0°$ to approximately 0.87 at $\delta = 90°$.

c. With the terminal voltage held constant at $V_a = 1.0$ per unit, the power can be expressed as

$$P = \frac{V_a V_{EQ}}{X_{EQ}} \sin\delta_t = \frac{1}{0.23}\sin\delta_t = 4.35 \sin\delta_t$$

where δ_t is the angle of the terminal voltage with respect to $\hat{V}_{EQ}$.

For $P = 1.0$ per unit, $\delta_t = 13.3°$ and hence $\hat{I}$ is equal to

$$\hat{I}_a = \frac{V_a\, e^{j\delta_t} - V_{EQ}}{jX_{EQ}} = 1.007\, e^{j6.65°}$$

and

$$\hat{E}_{af} = \hat{V}_{EQ} + j(X_{EQ} + X_s)\hat{I}_a = 1.78\, e^{j62.7°}$$

or $E_{af} = 1.78$ per unit, corresponding to a field current of $I_f = 1.78 \times 297 = 529$ amperes. The corresponding power angle is 62.7°.

Figure 5.13b is the desired MATLAB plot. E_{af} can be seen to vary from 1.0 at $P = 0$ to 1.78 at $P = 1.0$.

Here is the MATLAB script:

```
clc
clear

% Solution for part (b)

%System parameters

Veq = 1.0;
Eaf = 1.0;
Xeq = .23;
Xs = 1.35;

% Solve for Va as delta varies from 0 to 90 degrees
for n = 1:101
delta(n) = (pi/2.)*(n-1)/100;
```

```
Ia(n) = (Eaf *exp(j*delta(n)) - Veq)/(j*(Xs + Xeq));
Va(n) = abs(Veq + j*Xeq*Ia(n));
degrees(n) = 180*delta(n)/pi;
end

%Now plot the results
plot(degrees,Va)
xlabel('Power angle, delta [degrees]')
ylabel('Terminal voltage [per unit]')
title('Terminal voltage vs. power angle for part (b)')

fprintf('\n\nHit any key to continue\n')
pause

% Solution for part (c)

%Set terminal voltage to unity

Vterm = 1.0;

for n = 1:101
P(n) = (n-1)/100;
deltat(n) = asin(P(n)*Xeq/(Vterm*Veq));
Ia(n) = (Vterm *exp(j*deltat(n)) - Veq)/(j*Xeq);
Eaf(n) = abs(Vterm + j*(Xs+Xeq)*Ia(n));
end

%Now plot the results
plot(P,Eaf)
xlabel('Power [per unit]')
ylabel('Eaf [per unit]')
title('Eaf vs. power for part (c)')
```

Practice Problem 5.5

Consider the 75-MVA, 13.8 kV machine of Example 5.6. It is observed to be operating at terminal voltage of 13.7 kV and an output power of 53 MW at 0.87 pf lagging. Find (*a*) the phase current in kA, (*b*) the internal voltage in per unit, and (*c*) the corresponding field current in amperes.

Solution

a. $I_a = 2.57$ kA
b. $E_{af} = 1.81$ per unit
c. $I_f = 538$ amperes

EXAMPLE 5.7

A 2000-hp, 2300-V, unity-power-factor, three-phase, Y-connected, 30-pole, 60-Hz synchronous motor has a synchronous reactance of 1.95 Ω/phase. For this problem all losses may be neglected.

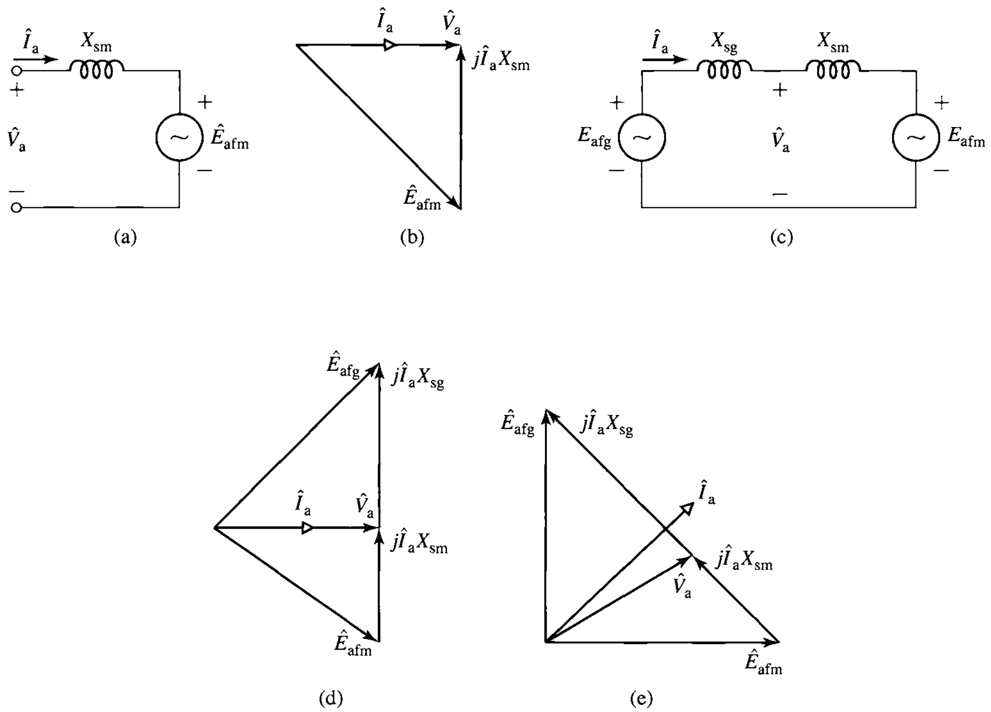


Figure 5.14 Equivalent circuits and phasor diagrams for Example 5.7.

a. Compute the maximum power and torque which this motor can deliver if it is supplied with power directly from a 60-Hz, 2300-V infinite bus. Assume its field excitation is maintained constant at the value which would result in unity power factor at rated load.
b. Instead of the infinite bus of part (a), suppose that the motor is supplied with power from a three-phase, Y-connected, 2300-V, 1500-kVA, two-pole, 3600 r/min turbine generator whose synchronous reactance is 2.65 Ω/phase. The generator is driven at rated speed, and the field excitations of generator and motor are adjusted so that the motor runs at unity power factor and rated terminal voltage at full load. Calculate the maximum power and torque which could be supplied corresponding to these values of field excitation.

■ Solution

Although this machine is undoubtedly of the salient-pole type, we will solve the problem by simple cylindrical-rotor theory. The solution accordingly neglects reluctance torque. The machine actually would develop a maximum torque somewhat greater than our computed value, as discussed in Section 5.7.

a. The equivalent circuit is shown in Fig. 5.14a and the phasor diagram at full load in Fig. 5.14b, where $\hat{E}_{afm}$ is the generated voltage of the motor and X_{sm} is its synchronous reactance. From the motor rating with losses neglected,

$$\text{Rated kVA} = 2000 \times 0.746 = 1492 \text{ kVA, three-phase}$$

$$= 497 \text{ kVA/phase}$$

$$\text{Rated voltage} = \frac{2300}{\sqrt{3}} = 1328 \text{ V line-to-neutral}$$

$$\text{Rated current} = \frac{497{,}000}{1328} = 374 \text{ A/phase-Y}$$

From the phasor diagram at full load

$$E_{\text{afm}} = \sqrt{V_{\text{a}}^2 + (I_{\text{a}}X_{\text{sm}})^2} = 1515 \text{ V}$$

When the power source is an infinite bus and the field excitation is constant, V_{a} and E_{afm} are constant. Substitution of V_{a} for E_1, E_{afm} for E_2, and X_{sm} for X in Eq. 5.46 then gives

$$P_{\text{max}} = \frac{V_{\text{a}}E_{\text{afm}}}{X_{\text{sm}}} = \frac{1328 \times 1515}{1.95} = 1032 \text{ kW/phase}$$
$$= 3096 \text{ kW, three-phase}$$

In per unit, $P_{\text{max}} = 3096/1492 = 2.07$ per unit. Because this power exceeds the motor rating, the motor cannot deliver this power for any extended period of time.

With 30 poles at 60 Hz, the synchronous angular velocity is found from Eq. 4.40

$$\omega_{\text{s}} = \left(\frac{2}{\text{poles}}\right)\omega_{\text{e}} = \left(\frac{2}{30}\right)(2\pi 60) = 8\pi \text{ rad/sec}$$

and hence

$$T_{\text{max}} = \frac{P_{\text{max}}}{\omega_{\text{s}}} = \frac{3096 \times 10^3}{8\pi} = 123.2 \text{ kN} \cdot \text{m}$$

b. When the power source is the turbine generator, the equivalent circuit becomes that shown in Fig. 5.14c, where $\hat{E}_{\text{afg}}$ is the generated voltage of the generator and X_{sg} is its synchronous reactance. Here the synchronous generator is equivalent to an external voltage $\hat{V}_{\text{EQ}}$ and reactance X_{EQ} as in Fig. 5.12. The phasor diagram at full motor load, unity power factor, is shown in Fig. 5.14d. As before, $V_{\text{a}} = 1330$ V/phase at full load and $E_{\text{afm}} = 1515$ V/phase.

From the phasor diagram

$$E_{\text{afg}} = \sqrt{V_{\text{ta}}^2 + (I_{\text{a}}X_{\text{sg}})^2} = 1657 \text{ V}$$

Since the field excitations and speeds of both machines are constant, E_{afg} and E_{afm} are constant. Substitution of E_{afg} for E_1, E_{afm} for E_2, and $X_{\text{sg}} + X_{\text{sm}}$ for X in Eq. 5.46 then gives

$$P_{\text{max}} = \frac{E_{\text{afg}}E_{\text{afm}}}{X_{\text{sg}} + X_{\text{sm}}} = \frac{1657 \times 1515}{4.60} = 546 \text{ kW/phase}$$
$$= 1638 \text{ kW, three-phase}$$

In per unit, $P_{\text{max}} = 1638/1492 = 1.10$ per unit.

$$T_{\text{max}} = \frac{P_{\text{max}}}{\omega_{\text{s}}} = \frac{1635 \times 10^3}{8\pi} = 65.2 \text{ kN} \cdot \text{m}$$

Synchronism would be lost if a load torque greater than this value were applied to the motor shaft. Of course, as in part (a), this loading exceeds the rating of the motor and could not be sustained under steady-state operating conditions.

Practice Problem 5.6

If the excitation system of the generator of Example 5.7 becomes damaged and must be limited to supplying only one half of the field excitation of part (b) of the example, calculate the maximum power which can be supplied to the motor.

Solution

819 kW

5.5 STEADY-STATE OPERATING CHARACTERISTICS

The principal steady-state operating characteristics of a synchronous machine are described by the interrelations between terminal voltage, field current, armature current, power factor, and efficiency. A selection of performance curves of importance in practical application of synchronous machines are presented here.

Consider a synchronous generator delivering power at constant frequency and rated terminal voltage to a load whose power factor is constant. The curve showing the field current required to maintain rated terminal voltage as the constant-power-factor load is varied is known as a *compounding curve*. The characteristic form of three compounding curves at various constant power factors are shown in Fig. 5.15.

Synchronous generators are usually rated in terms of the maximum apparent power (kVA or MVA) load at a specific voltage and power factor (often 80, 85, or 90 percent lagging) which they can carry continuously without overheating. The real power output of the generator is usually limited to a value within the apparent

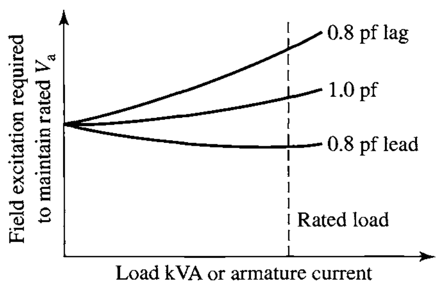


Figure 5.15 Characteristic form of synchronous-generator compounding curves.

power rating by the capability of its prime mover. By virtue of its *voltage-regulating system* (which controls the field current in response to the measured value of terminal voltage), the machine normally operates at a constant terminal voltage whose value is within ±5 percent of rated voltage. When the real-power loading and voltage are fixed, the allowable reactive-power loading is limited by either armature- or field-winding heating. A typical set of *capability curves* for a large, hydrogen-cooled turbine generator is shown in Fig. 5.16. They give the maximum reactive-power loadings corresponding to various real power loadings with operation at rated terminal voltage. Note that the three-curves seen in the figure correspond to differing pressure of the hydrogen cooling gas. As can be seen, increasing the hydrogen pressure improves cooling and permits a larger overall loading of the machine.

Armature-winding heating is the limiting factor in the region from unity to rated power factor (0.85 lagging power factor in Fig. 5.16). For example, for a given real-power loading, increasing the reactive power past a point on the armature-heating limited portion of the capability curve will result in an armature current in excess of that which can be succesfully cooled, resulting in armature-winding temperatures which will damage the armature-winding insulation and degrade its life. Similarly, for lower power factors, field-winding heating is the limiting factor.

Capability curves provide a valuable guide both to power system planners and to operators. As system planners consider modifications and additions to power systems,

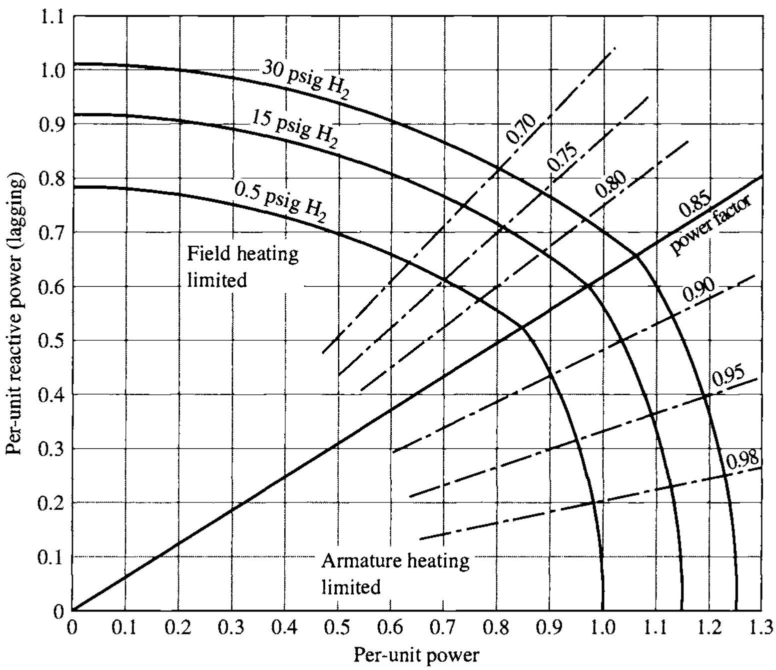


Figure 5.16 Capability curves of an 0.85 power factor, 0.80 short-circuit ratio, hydrogen-cooled turbine generator. Base MVA is rated MVA at 0.5 psig hydrogen.

they can readily see whether the various existing or proposed generators can safely supply their required loadings. Similarly, power system operators can quickly see whether individual generators can safely respond to changes in system loadings which occur during the normal course of system operation.

The derivation of capability curves such as those in Fig. 5.16 can be seen as follows. Operation under conditions of constant terminal voltage and armature current (at the maximum value permitted by heating limitations) corresponds to a constant value of apparent output power determined by the product of terminal voltage and current. Since the per-unit apparent power is given by

$$\text{Apparent power} = \sqrt{P^2 + Q^2} = V_a I_a \tag{5.48}$$

where P represents the per-unit real power and Q represents the per-unit reactive power, we see that a constant apparent power corresponds to a circle centered on the origin on a plot of reactive power versus real power. Note also from Eq. 5.48, that, for constant terminal voltage, constant apparent power corresponds to constant armature-winding current and hence constant armature-winding heating. Such a circle, corresponding to the maximum acceptable level of armature heating, is shown in Fig. 5.17.

Similarly, consider operation when terminal voltage is constant and field current (and hence E_{af}) is limited to a maximum value, also determined by heating limitations.

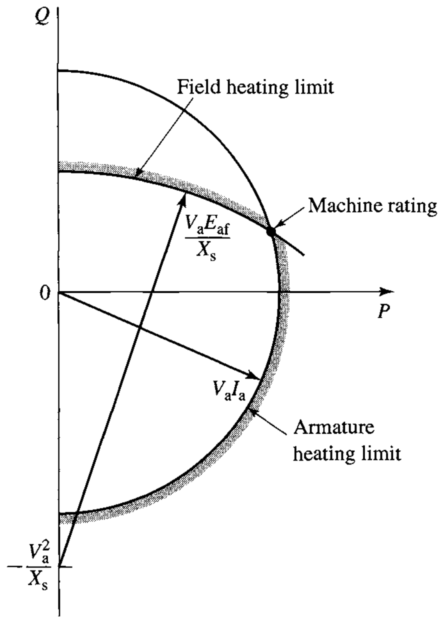


Figure 5.17 Construction used for the derivation of a synchronous generator capability curve.

In per unit,

$$P - jQ = \hat{V}_a \hat{I}_a \tag{5.49}$$

From Eq. 5.24 (with $R_a = 0$)

$$\hat{E}_{af} = \hat{V}_a + jX_s \hat{I}_a \tag{5.50}$$

Equations 5.49 and 5.50 can be solved to yield

$$P^2 + \left(Q + \frac{V_a^2}{X_s}\right)^2 = \left(\frac{V_a E_{af}}{X_s}\right)^2 \tag{5.51}$$

This equation corresponds to a circle centered at $Q = -(V_a^2/X_s)$ in Fig. 5.17 and determines the field-heating limitation on machine operation shown in Fig. 5.16. It is common to specify the rating (apparent power and power factor) of the machine as the point of intersection of the armature- and field-heating limitation curves.

For a given real-power loading, the power factor at which a synchronous machine operates, and hence its armature current, can be controlled by adjusting its field excitation. The curve showing the relation between armature current and field current at a constant terminal voltage and with a constant real power is known as a *V curve* because of its characteristic shape. A family of V curves for a synchronous generator takes the form of those shown in Fig. 5.18.

For constant power output, the armature current is minimum at unity power factor and increases as the power factor decreases. The dashed lines are loci of constant power factor; they are the synchronous-generator compounding curves (see Fig. 5.15) showing how the field current must be varied as the load is changed to maintain constant power factor. Points to the right of the unity-power-factor compounding curve correspond to overexcitation and lagging power factor; points to the left correspond

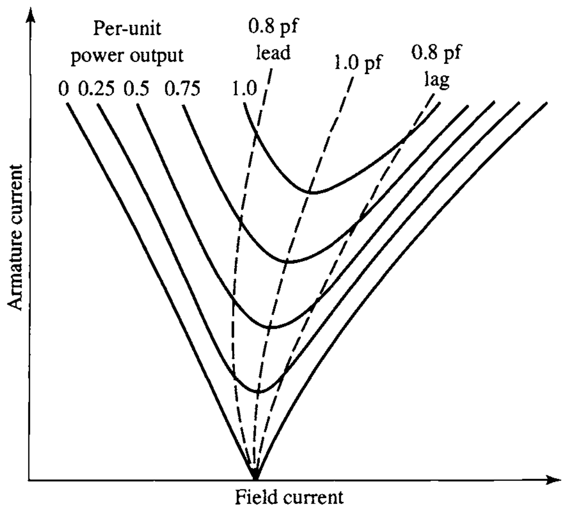


Figure 5.18 Typical form of synchronous-generator V curves.

to underexcitation and leading power factor. Synchronous-motor V curves and compounding curves are very similar to those of synchronous generators. In fact, if it were not for the small effects of armature resistance, motor and generator compounding curves would be identical except that the lagging- and leading-power-factor curves would be interchanged.

As in all electromechanical machines, the efficiency of a synchronous machine at any particular operating point is determined by the losses which consist of I^2R losses in the windings, core losses, stray-load losses, and mechanical losses. Because these losses change with operating condition and are somewhat difficult to measure accurately, various standard procedures have been developed to calculate the efficiency of synchronous machines.[2] The general principles for these calculations are described in Appendix D.

EXAMPLE 5.8

Data are given in Fig. 5.19 with respect to the losses of the 45-kVA synchronous machine of Examples 5.4 and 5.5. Compute its efficiency when it is running as a synchronous motor at a terminal voltage of 220 V and with a power input to its armature of 45 kVA at 0.80 lagging power factor. The field current measured in a load test taken under these conditions is I_f (test) = 5.50 A. Assume the armature and field windings to be at a temperature of 75°C.

■ Solution

For the specified operating conditions, the armature current is

$$I_a = \frac{45 \times 10^3}{\sqrt{3} \times 230} = 113 \text{ A}$$

The I^2R losses must be computed on the basis of the dc resistances of the windings at 75°C. Correcting the winding resistances by means of Eq. 5.32 gives

$$\text{Field-winding resistance} R_f \text{ at } 75°\text{C} = 35.5\ \Omega$$

$$\text{Armature dc resistance} R_a \text{ at } 75°\text{C} = 0.0399\ \Omega/\text{phase}$$

The field I^2R loss is therefore

$$I_f^2 R_f = 5.50^2 \times 35.5 = 1.07 \text{ kW}$$

According to ANSI standards, losses in the excitation system, including those in any field-rheostat, are not charged against the machine.

The armature I^2R loss is

$$3I_a^2 R_a = 3 \times 113^2 \times 0.0399 = 1.53 \text{ kW}$$

and from Fig. 5.19 at $I_a = 113$ A the stray-load loss = 0.37 kW. The stray-load loss is considered to account for the losses caused by the armature leakage flux. According to ANSI standards, no temperature correction is to be applied to the stray load loss.

[2] See, for example, IEEE Std. 115-1995, "IEEE Guide: Test Procedures for Synchronous Machines," Institute of Electrical and Electronic Engineers, Inc., 345 East 47th Street, New York, New York, 10017 and NEMA Standards Publication No. MG-1-1998, "Motors and Generators," National Electrical Manufacturers Association, 1300 North 17th Street, Suite 1847, Rosslyn, Virginia, 22209.

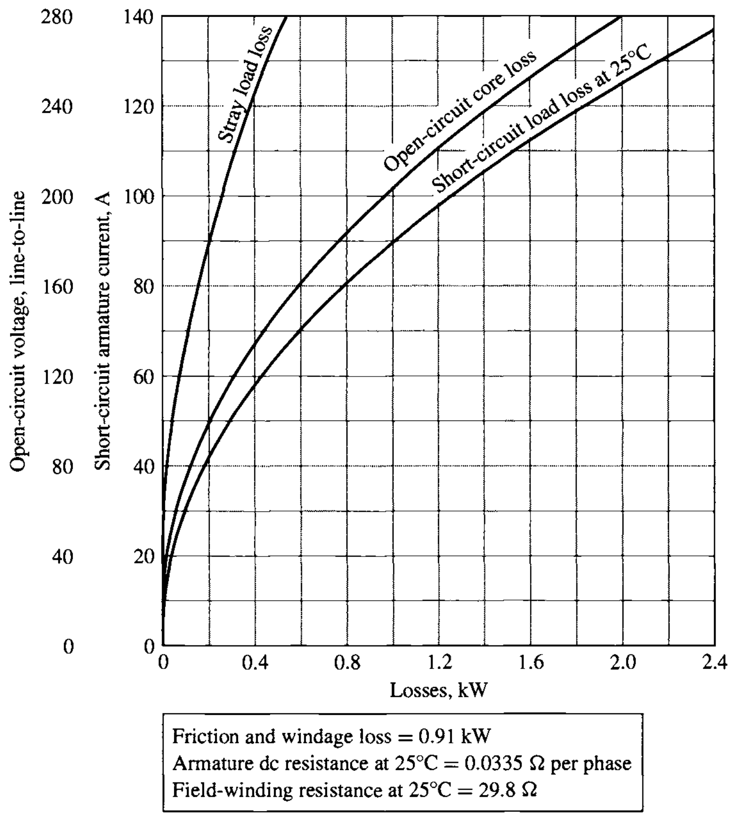


Figure 5.19 Losses in a three-phase, 45-kVA, Y-connected, 220-V, 60-Hz, six-pole synchronous machine (Example 5.8).

Core loss under load is primarily a function of the main core flux in the motor. As is discussed in Chapter 2, the voltage across the magnetizing branch in a transformer (corresponding to the transformer core flux) is calculated by subtracting the leakage impedance drop from the terminal voltage. In a directly analogous fashion, the main core flux in a synchronous machine (i.e., the air-gap flux) can be calculated as the voltage behind the leakage impedance of the machine. Typically the armature resistance is small, and hence it is common to ignore the resistance and to calculate the voltage behind the leakage reactance. The core loss can then be estimated from the open-circuit core-loss curve at the voltage behind leakage reactance.

In this case, we do not know the machine leakage reactance. Thus, one approach would be simply to assume that the air-gap voltage is equal to the terminal voltage and to determine the core-loss under load from the core-loss curve at the value equal to terminal voltage.[3] In this case, the motor terminal voltage is 230 V line-to-line and thus from Fig. 5.19, the open-circuit core loss is 1.30 kW.

[3] Although not rigorously correct, it has become common practice to ignore the leakage impedance drop when determining the under-load core loss.

To estimate the effect of ignoring the leakage reactance drop, let us assume that the leakage reactance of this motor is 0.20 per unit or

$$X_{\mathrm{al}} = 0.2\left(\frac{220^2}{45 \times 10^3}\right) = 0.215\ \Omega$$

Under this assumption, the air-gap voltage is equal to

$$\begin{aligned}\hat{V}_{\mathrm{a}} - jX_{\mathrm{al}}\hat{I}_{\mathrm{a}} &= \frac{230}{\sqrt{3}} - j0.215 \times 141(0.8 + j0.6) \\ &= 151 - j24.2 = 153\, e^{-j9.1^\circ}\ \text{V, line-to-neutral}\end{aligned}$$

which corresponds to a line-to-line voltage of $\sqrt{3}\,(153) = 265$ V. From Fig. 5.19, the corresponding core-loss is 1.8 kW, 500 W higher than the value determined using the terminal voltage. We will use this value for the purposes of this example.

Including the friction and windage loss of 0.91 kW, all losses have now been found:

$$\text{Total losses} = 1.07 + 1.53 + 0.37 + 1.80 + 0.91 = 5.68\ \text{kW}$$

The total motor input power is the input power to the armature plus that to the field.

$$\text{Input power} = 0.8 \times 45 + 1.07 = 37.1\ \text{kW}$$

and the output power is equal to the total input power minus the total losses

$$\text{Output power} = 37.1 - 5.68 = 31.4\ \text{kW}$$

Therefore

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} = 1 - \frac{31.4}{37.1} = 0.846 = 84.6\%$$

Practice Problem 5.7

Calculate the efficiency of the motor of Example 5.8 if the motor is operating at a power input of 45 kW, unity power factor. You may assume that the motor stray-load losses remain unchanged and that the motor field current is 4.40 A.

Solution

$$\text{Efficiency} = 88.4\%$$

5.6 EFFECTS OF SALIENT POLES; INTRODUCTION TO DIRECT- AND QUADRATURE-AXIS THEORY

The essential features of salient-pole machines are developed in this section based on physical reasoning. A mathematical treatment, based on an inductance formulation like that presented in Section 5.2, is given in Appendix C, where the dq0 transformation is developed.

5.6.1 Flux and MMF Waves

The flux produced by an mmf wave in a uniform-air-gap machine is independent of the spatial alignment of the wave with respect to the field poles. In a salient-pole machine, such as that shown schematically in Fig. 5.20, however, the preferred direction of magnetization is determined by the protruding field poles. The permeance along the polar axis, commonly referred to as the rotor *direct axis,* is appreciably greater than that along the interpolar axis, commonly referred to as the rotor *quadrature axis.*

Note that, by definition, the field winding produces flux which is oriented along the rotor direct axis. Thus, when phasor diagrams are drawn, the field-winding mmf and its corresponding flux $\hat{\Phi}_f$ are found along the rotor direct axis. The generated internal voltage is proportional to the time derivative of the field-winding flux, and thus its phasor $\hat{E}_{af}$ leads the flux $\hat{\Phi}_f$ by 90°. Since by convention the quadrature axis leads the direct axis by 90°, we see that *the generated-voltage phasor $\hat{E}_{af}$ lies along the quadrature axis.* Thus a key point in the analysis of synchronous-machine phasor diagrams is that, by locating the phasor $\hat{E}_{af}$, the location of both the quadrature axis and the direct axis is immediately determined. This forms the basis of the direct- and quadrature-axis formulation for the analysis of salient-pole machines in which all machine voltages and currents can be resolved into their *direct-* and *quadrature-axis components.*

The armature-reaction flux wave $\hat{\Phi}_{ar}$ lags the field flux wave by a space angle of $90° + \phi_{lag}$, where ϕ_{lag} is the time-phase angle by which the armature current lags the generated voltage. If the armature current $\hat{I}_a$ lags the generated voltage $\hat{E}_{af}$ by 90°, the armature-reaction flux wave is directly opposite the field poles and in the opposite direction to the field flux $\hat{\Phi}_f$, as shown in the phasor diagram of Fig. 5.20a.

The corresponding component flux-density waves at the armature surface produced by the field current and by the synchronously-rotating space-fundamental component of armature-reaction mmf are shown in Fig. 5.20b, in which the effects of slots

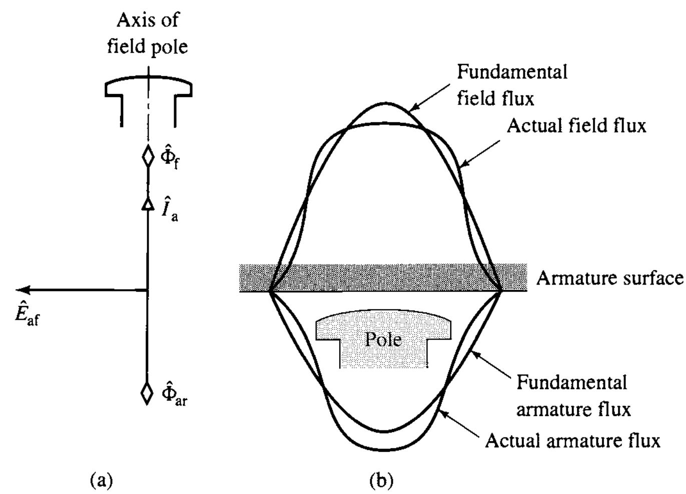


Figure 5.20 Direct-axis air-gap fluxes in a salient-pole synchronous machine.

are neglected. The waves consist of a space fundamental and a family of odd-harmonic components. In a well-designed machine the harmonic effects are usually small. Accordingly, only the space-fundamental components will be considered. It is the fundamental components which are represented by the flux-per-pole phasors $\hat{\Phi}_f$ and $\hat{\Phi}_{ar}$ in Fig. 5.20a.

Conditions are quite different when the armature current is in phase with the generated voltage, as illustrated in the phasor diagram of Fig. 5.21a. The axis of the armature-reaction wave then is opposite an interpolar space, as shown in Fig. 5.21b. The armature-reaction flux wave is badly distorted, comprising principally a fundamental and a prominent third space harmonic. The third-harmonic flux wave generates third-harmonic emf's in the armature phase (line-to-neutral) voltages. They will be of the form

$$E_{3,a} = \sqrt{2} V_3 \cos(3\omega_e t + \phi_3) \tag{5.52}$$

$$E_{3,b} = \sqrt{2} V_3 \cos(3(\omega_e t - 120^\circ) + \phi_3) = \sqrt{2} V_3 \cos(3\omega_e t + \phi_3) \tag{5.53}$$

$$E_{3,c} = \sqrt{2} V_3 \cos(3(\omega_e t - 120^\circ) + \phi_3) = \sqrt{2} V_3 \cos(3\omega_e t + \phi_3) \tag{5.54}$$

Note that these third-harmonic phase voltages are equal in magnitude and in phase. Hence they do not appear as components of the line-to-line voltages, which are equal to the differences between the various phase voltages.

Because of the longer air gap between the poles and the correspondingly larger reluctance, the space-fundamental armature-reaction flux when the armature reaction is along the quadrature axis (Fig. 5.21) is less than the space fundamental armature-reaction flux which would be created by the same armature current if the armature flux wave were directed along the direct axis (Fig. 5.20). Hence, the quadrature-axis magnetizing reactance is less than that of the direct axis.

Focusing our attention on the space-fundamental components of the air-gap flux and mmf, the effects of salient poles can be taken into account by resolving the

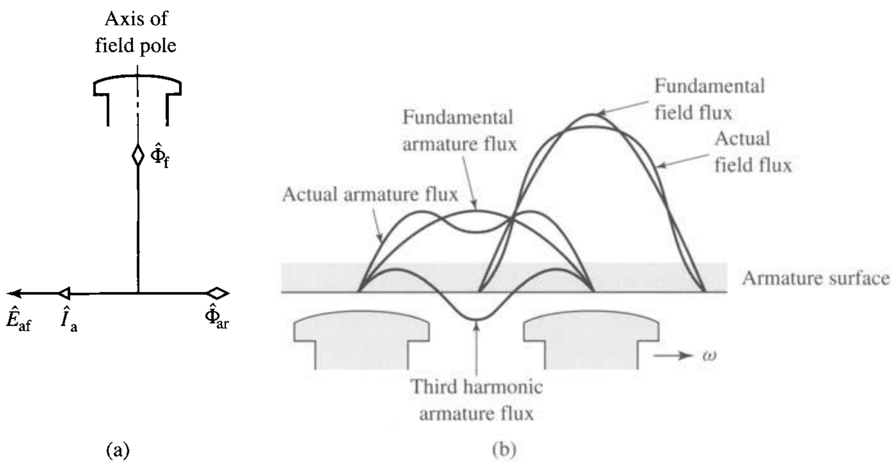


Figure 5.21 Quadrature-axis air-gap fluxes in a salient-pole synchronous machine.

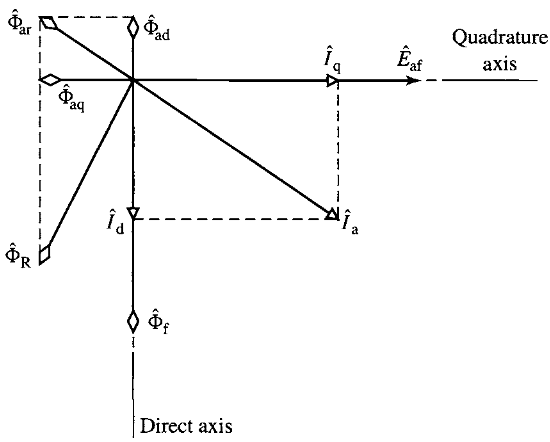


Figure 5.22 Phasor diagram of a salient-pole synchronous generator.

armature current $\hat{I}_a$ into two components, one along the direct axis and the other along the quadrature axis as shown in the phasor diagram of Fig. 5.22. This diagram is drawn for a unsaturated salient-pole generator operating at a lagging power factor. The direct-axis component $\hat{I}_d$ of the armature current, in time-quadrature with the generated voltage $\hat{E}_{af}$, produces a component of the space-fundamental armature-reaction flux $\hat{\Phi}_{ad}$ along the axis of the field poles (the direct axis), as in Fig. 5.20. The quadrature-axis component $\hat{I}_q$, in phase with the generated voltage, produces a component of the space-fundamental armature-reaction flux $\hat{\Phi}_{aq}$ in space-quadrature with the field poles, as in Fig. 5.21. Note that the subscripts d ("direct") and q ("quadrature") on the armature-reaction fluxes refer to their space phase and not to the time phase of the component currents producing them.

Thus a *direct-axis quantity* is one whose magnetic effect is aligned with the axes of the field poles; direct-axis mmf's produce flux along these axes. A *quadrature-axis quantity* is one whose magnetic effect is centered on the interpolar space. For an unsaturated machine, the armature-reaction flux $\hat{\Phi}_{ar}$ is the sum of the components $\hat{\Phi}_{ad}$ and $\hat{\Phi}_{aq}$. The resultant flux $\hat{\Phi}_R$ is the sum of $\hat{\Phi}_{ar}$ and the field flux $\hat{\Phi}_f$.

5.6.2 Phasor Diagrams for Salient-Pole Machines

With each of the component currents $\hat{I}_d$ and $\hat{I}_q$ there is associated a component synchronous-reactance voltage drop, $j\hat{I}_dX_d$ and $j\hat{I}_qX_q$ respectively. The reactances X_d and X_q are, respectively, the *direct-* and *quadrature-axis synchronous reactances;* they account for the inductive effects of all the space-fundamental fluxes created by the armature currents along the direct and quadrature axes, including both armature-leakage and armature-reaction fluxes. Thus, the inductive effects of the direct- and quadrature-axis armature-reaction flux waves can be accounted for by *direct-* and *quadrature-axis magnetizing reactances,* $X_{\varphi d}$ and $X_{\varphi q}$ respectively, similar to the

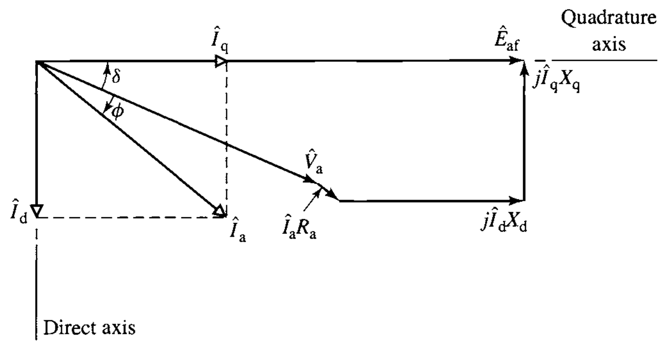


Figure 5.23 Phasor diagram for a synchronous generator showing the relationship between the voltages and the currents.

magnetizing reactance X_φ of cylindrical-rotor theory. The direct- and quadrature-axis synchronous reactances are then given by

$$X_d = X_{al} + X_{\varphi d} \tag{5.55}$$

$$X_q = X_{al} + X_{\varphi q} \tag{5.56}$$

where X_{al} is the armature leakage reactance, assumed to be the same for direct- and quadrature-axis currents. Compare Eqs. 5.55 and 5.56 with Eq. 5.25 for the nonsalient-pole case. As shown in the generator phasor diagram of Fig. 5.23, the generated voltage $\hat{E}_{af}$ equals the phasor sum of the terminal voltage $\hat{V}_a$ plus the armature-resistance drop $\hat{I}_a R_a$ and the component synchronous-reactance drops $j\hat{I}_d X_d + j\hat{I}_q X_q$.

As we have discussed, the quadrature-axis synchronous reactance X_q is less than of the direct axis X_d because of the greater reluctance of the air gap in the quadrature axis. Typically, X_q is between $0.6X_d$ and $0.7X_d$. Note that a small salient-pole effect is also present in turbo-alternators, even though they are cylindrical-rotor machines, because of the effect of the rotor slots on the quadrature-axis reluctance.

Just as for the synchronous reactance X_s of a cylindrical-rotor machine, these reactances are not constant with flux density but rather saturate as the machine flux density increases. It is common to find both unsaturated and saturated values specified for each of these parameters.[4] The saturated values apply to typical machine operating conditions where the terminal voltage is near its rated value. For our purposes in this chapter and elsewhere in this book, we will not focus attention on this issue and, unless specifically stated, the reader may assume that the values of X_d and X_q given are the saturated values.

In using the phasor diagram of Fig. 5.23, the armature current must be resolved into its direct- and quadrature-axis components. This resolution assumes that the phase angle $\phi + \delta$ of the armature current with respect to the generated voltage is known.

[4] See, for example, IEEE Std. 115-1995, "IEEE Guide: Test Procedures for Synchronous Machines," Institute of Electrical and Electronic Engineers, Inc., 345 East 47th Street, New York, New York, 10017.

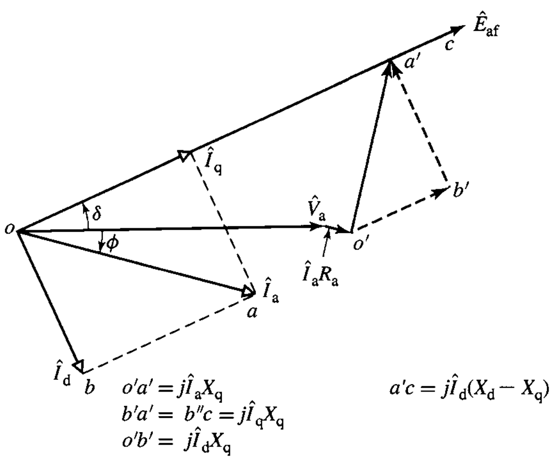


Figure 5.24 Relationships between component voltages in a phasor diagram.

Often, however, the power-factor angle ϕ at the machine terminals is explicitly known, rather than the angle $\phi + \delta$. It then becomes necessary to locate the quadrature axis and to compute the power angle δ. This can be done with the aid of the construction of Fig. 5.24.

The phasor diagram of Fig. 5.23 is repeated by the solid-line phasors in Fig. 5.24. Study of this phasor diagram shows that the dashed phasor $o'a'$, perpendicular to $\hat{I}_a$, equals $j\hat{I}_a X_q$. This result follows geometrically from the fact that triangles $o'a'b'$ and oab are similar because their corresponding sides are perpendicular. Thus

$$\frac{o'a'}{oa} = \frac{b'a'}{ba} \tag{5.57}$$

or

$$o'a' = \left(\frac{b'a'}{\mathrm{ba}}\right) oa = \frac{|\hat{I}_q| X_q}{|\hat{I}_q|} |\hat{I}_a| = X_q |\hat{I}_a| \tag{5.58}$$

Thus, line $o'a'$ is the phasor $jX_q\hat{I}_a$ and the phasor sum $\hat{V}_a + \hat{I}_a R_a + j\hat{I}_a X_q$ then locates the angular position of the generated voltage $\hat{E}_{af}$ (which in turn lies along the quadrature axis) and therefore the direct and quadrature axes. Physically this must be so, because all the field excitation in a normal machine is in the direct axis. Once the quadrature axis (and hence δ) is known, $\hat{E}_{af}$ can be found as shown in Fig. 5.23

$$\hat{E}_{af} = \hat{V}_a + R_a\hat{I}_a + jX_d\hat{I}_d + jX_q\hat{I}_q \tag{5.59}$$

One use of these relations in determining the excitation requirements for specified operating conditions at the terminals of a salient-pole machine is illustrated in Example 5.9.

EXAMPLE 5.9

The reactances X_d and X_q of a salient-pole synchronous generator are 1.00 and 0.60 per unit, respectively. The armature resistance may be considered to be negligible. Compute the generated voltage when the generator delivers its rated kVA at 0.80 lagging power factor and rated terminal voltage.

■ Solution

First, the phase of $\hat{E}_{af}$ must be found so that $\hat{I}_a$ can be resolved into its direct- and quadrature-axis components. The phasor diagram is shown in Fig. 5.25. As is commonly done for such problems, the terminal voltage $\hat{V}_a$ will be used as the reference phasor, i.e., $\hat{V}_a = V_a e^{j0.0^\circ} = V_a$. In per unit

$$\hat{I}_a = I_a e^{j\phi} = 0.80 - j0.60 = 1.0\, e^{-j36.9^\circ}$$

The quadrature axis is located by the phasor

$$\hat{E}' = \hat{V}_a + jX_q\hat{I}_a = 1.0 + j0.60(1.0\, e^{-j36.9^\circ}) = 1.44\, e^{j19.4^\circ}$$

Thus, $\delta = 19.4^\circ$, and the phase angle between $\hat{E}_{af}$ and $\hat{I}_a$ is $\delta - \phi = 19.4^\circ - (-36.9^\circ) = 56.3^\circ$. Note, that although it would appear from Fig. 5.25 that the appropriate angle is $\delta + \phi$, this is not correct because the angle ϕ as drawn in Fig. 5.25 is a negative angle. In general, the desired angle is equal to the difference between the power angle and the phase angle of the terminal current.

The armature current can now be resolved into its direct- and quadrature-axis components. Their magnitudes are

$$I_d = |\hat{I}_a| \sin(\delta - \phi) = 1.00 \sin(56.3^\circ) = 0.832$$

and

$$I_q = |\hat{I}_a| \cos(\delta - \phi) = 1.00 \cos(56.3^\circ) = 0.555$$

As phasors,

$$\hat{I}_d = 0.832\, e^{j(-90^\circ + 19.4^\circ)} = 0.832\, e^{j70.6^\circ}$$

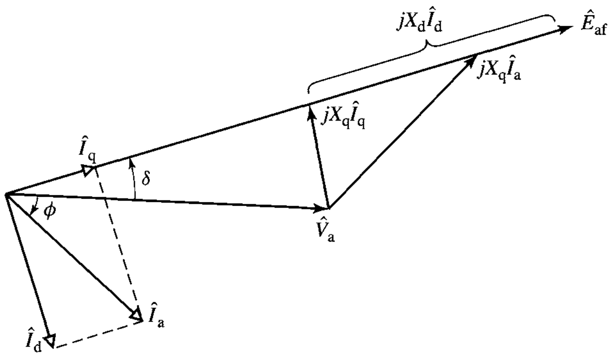


Figure 5.25 Generator phasor diagram for Example 5.9.

and

$$\hat{I}_q = 0.555\, e^{j19.4^\circ}$$

We can now find E_{af} from Eq. 5.59

$$\begin{aligned}\hat{E}_{af} &= \hat{V}_a + jX_d\hat{I}_d + jX_q j\hat{I}_q \\ &= 1.0 + j1.0(0.832\, e^{j70.6^\circ}) + j0.6(0.555\, e^{j19.4^\circ}) \\ &= 1.77\, e^{j19.4^\circ}\end{aligned}$$

and we see that $E_{af} = 1.77$ per unit. Note that, as expected, $\angle \hat{E}_{af} = 19.4^\circ = \delta$, thus confirming that $\hat{E}_{af}$ lies along the quadrature axis.

Practice Problem 5.8

Find the generated voltage for the generator of Example 5.9 if it is loaded to (*a*) 0.73 per-unit kVA, unity power factor at a terminal voltage of 0.98 per unit and (*b*) 0.99 per-unit kVA, 0.94 leading power factor and rated terminal voltage.

Solution

a. $\hat{E}_{af} = 1.20\, e^{j24.5^\circ}$ per unit
b. $\hat{E}_{af} = 1.08\, e^{j35.0^\circ}$ per unit

EXAMPLE 5.10

In the simplified theory of Section 5.2, the synchronous machine is assumed to be representable by a single reactance, the synchronous reactance X_s of Eq. 5.25. The question naturally arises: How serious an approximation is involved if a salient-pole machine is treated in this simple fashion? Suppose that a salient-pole machine were treated by cylindrical-rotor theory as if it had a single synchronous reactance equal to its direct-axis value X_d? To investigate this question, we will repeat Example 5.9 under this assumption.

■ Solution

In this case, under the assumption that

$$X_q = X_d = X_s = 1.0 \text{ per unit}$$

the generated voltage can be found simply as

$$\begin{aligned}\hat{E}_{af} &= V_a + jX_s\hat{I}_a \\ &= 1.0 + j1.0(1.0\, e^{-j36.9^\circ}) = 1.79\, e^{j26.6^\circ} \text{ per unit}\end{aligned}$$

Comparing this result with that of Example 5.9 (in which we found that $E_{af} = 1.77\, e^{j19.4^\circ}$), we see that the magnitude of the predicted generated voltage is relatively close to the correct value. As a result, we see that the calculation of the field current required for this operating condition will be relatively accurate under the simplifying assumption that the effects of saliency can be neglected.

However, the calculation of the power angle δ (19.4° versus a value of 26.6° if saliency is neglected) shows a considerably larger error. In general, such errors in the calculation of generator steady-state power angles may be of significance when studying the transient behavior of a system including a number of synchronous machines. Thus, although saliency can perhaps be ignored when doing "back-of-the-envelope" system calculations, it is rarely ignored in large-scale, computer-based system studies.

5.7 POWER-ANGLE CHARACTERISTICS OF SALIENT-POLE MACHINES

For the purposes of this discussion, it is sufficient to limit our discussion to the simple system shown in the schematic diagram of Fig. 5.26a, consisting of a salient-pole synchronous machine *SM* connected to an infinite bus of voltage $\hat{V}_{\mathrm{EQ}}$ through a series impedance of reactance X_{EQ}. Resistance will be neglected because it is usually small. Consider that the synchronous machine is acting as a generator. The phasor diagram is shown by the solid-line phasors in Fig. 5.26b. The dashed phasors show the external reactance drop resolved into components due to $\hat{I}_{\mathrm{d}}$ and $\hat{I}_{\mathrm{q}}$. The effect of the external impedance is merely to add its reactance to the reactances of the machine; the total values of the reactance between the excitation voltage $\hat{E}_{\mathrm{af}}$ and the bus voltage $\hat{V}_{\mathrm{EQ}}$ is therefore

$$X_{\mathrm{dT}} = X_{\mathrm{d}} + X_{\mathrm{EQ}} \tag{5.60}$$

$$X_{\mathrm{qT}} = X_{\mathrm{q}} + X_{\mathrm{EQ}} \tag{5.61}$$

If the bus voltage $\hat{V}_{\mathrm{EQ}}$ is resolved into components its direct-axis component $V_{\mathrm{d}} = V_{\mathrm{EQ}} \sin\delta$ and quadrature-axis component $V_{\mathrm{q}} = V_{\mathrm{EQ}} \cos\delta$ in phase with $\hat{I}_{\mathrm{d}}$ and $\hat{I}_{\mathrm{q}}$, respectively, the power P delivered to the bus per phase (or in per unit) is

$$P = I_{\mathrm{d}} V_{\mathrm{d}} + I_{\mathrm{q}} V_{\mathrm{q}} = I_{\mathrm{d}} V_{\mathrm{EQ}} \sin\delta + I_{\mathrm{q}} V_{\mathrm{EQ}} \cos\delta \tag{5.62}$$

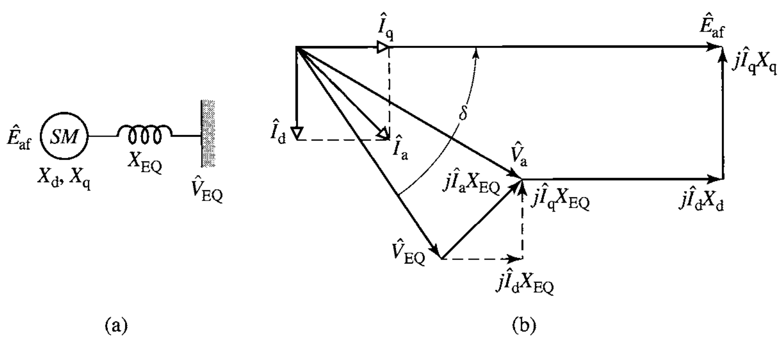


Figure 5.26 Salient-pole synchronous machine and series impedance: (a) single-line diagram and (b) phasor diagram.

Also, from Fig. 5.26b,

$$I_d = \frac{E_{af} - V_{EQ}\cos\delta}{X_{dT}} \tag{5.63}$$

and

$$I_q = \frac{V_{EQ}\sin\delta}{X_{qT}} \tag{5.64}$$

Substitution of Eqs. 5.63 and 5.64 in Eq. 5.62 gives

$$P = \frac{E_{af}V_{EQ}}{X_{dT}}\sin\delta + \frac{V_{EQ}^2(X_{dT} - X_{qT})}{2X_{dT}X_{qT}}\sin 2\delta \tag{5.65}$$

Equation 5.65 is directly analogous to Eq. 5.47 which applies to the case of a nonsalient machine. It gives the power per phase when E_{af} and V_{EQ} are expressed as line-neutral voltages and the reactances are in Ω/phase, in which case the result must be multiplied by three to get three-phase power. Alternatively, expressing E_{af} and V_{EQ} as line-to-line voltages will result in three-phase power directly. Similarly, Eq. 5.65 can be applied directly if the various quantites are expressed in per unit.

The general form of this power-angle characteristic is shown in Fig. 5.27. The first term is the same as the expression obtained for a cylindrical-rotor machine (Eq. 5.47). The second term includes the effect of salient poles. It represents the fact that the air-gap flux wave creates torque, tending to align the field poles in the position of minimum

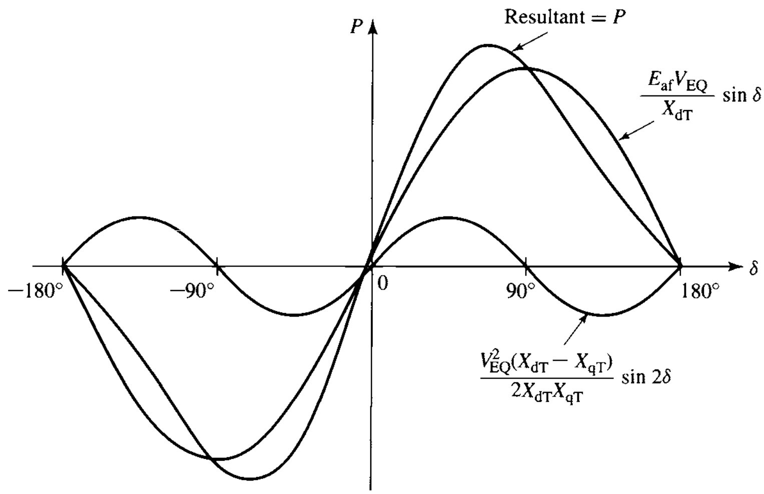


Figure 5.27 Power-angle characteristic of a salient-pole synchronous machine showing the fundamental component due to field excitation and the second-harmonic component due to reluctance torque.

reluctance. This term is the power corresponding to the *reluctance torque* and is of the same general nature as the reluctance torque discussed in Section 3.5. Note that the reluctance torque is independent of field excitation. Also note that, if $X_{dT} = X_{qT}$ as in a uniform-air-gap machine, there is no preferential direction of magnetization, the reluctance torque is zero and Eq. 5.65 reduces to the power-angle equation for a cylindrical-rotor machine (Eq. 5.47).

Notice that the characteristic for negative values of δ is the same except for a reversal in the sign of P. That is, the generator and motor regions are alike if the effects of resistance are negligible. For generator action $\hat{E}_{af}$ leads $\hat{V}_{EQ}$; for motor action $\hat{E}_{af}$ lags $\hat{V}_{EQ}$. Steady-state operation is stable over the range where the slope of the power-angle characteristic is positive. Because of the reluctance torque, a salient-pole machine is "stiffer" than one with a cylindrical rotor; i.e., for equal voltages and equal values of X_{dT}, a salient-pole machine develops a given torque at a smaller value of δ, and the maximum torque which can be developed is somewhat greater.

EXAMPLE 5.11

The 2000-hp, 2300-V synchronous motor of Example 5.7 is assumed to have a synchronous reactance $X_s = 1.95\ \Omega$/phase. In actual fact, it is a salient-pole machine with reactances $X_d = 1.95\ \Omega$/phase and $X_q = 1.40\ \Omega$/phase. Neglecting all losses, compute the maximum mechanical power in kilowatts which this motor can deliver if it is supplied with electric power from an infinite bus (Fig. 5.28a) at rated voltage and frequency and if its field excitation is held constant at that value which would result in unity-power-factor operation at rated load. The shaft load is assumed to be increased gradually so that transient swings are negligible and the steady-state power limit applies. Also, compute the value of δ corresponding to this maximum power operation.

■ Solution

The first step is to compute the synchronous motor excitation at rated voltage, full load, and unity power factor. As in Example 5.7, the full-load terminal voltage and current are 1330 V

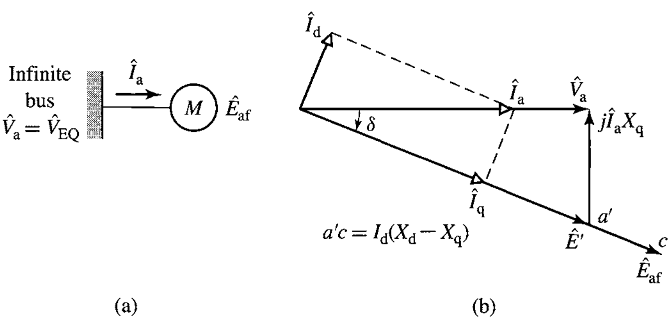


Figure 5.28 (a) Single-line diagram and (b) phasor diagram for motor of Example 5.11.

line-to-neutral and 374 A/phase, respectively. The phasor diagram for the specified full-load conditions is shown in Fig. 5.28b. The only essential difference between this phasor diagram and the generator phasor diagram of Fig. 5.25 is that $\hat{I}_a$ in Fig. 5.28 represents motor input current; i.e., we have switched to the motor reference direction for $\hat{I}_a$. Thus, switching the sign of the current to account for the choice of the motor reference direction and neglecting the effects of armature resistance, Eq. 5.59 becomes

$$\hat{E}_{af} = \hat{V}_a - j\hat{I}_d X_d - j\hat{I}_q X_q$$

As in Fig. 5.28b, the quadrature axis can now be located by the phasor

$$\hat{E}' = \hat{V}_a - j\hat{I}_a X_q = 1330 - j374(1.40) = 1429\, e^{-21.5^\circ}$$

That is, $\delta = -21.5^\circ$, with $\hat{E}_{af}$ lagging $\hat{V}_a$. The magnitude of $\hat{I}_d$ is

$$I_d = I_a \sin|\delta| = 374 \sin(21.5^\circ) = 137 \text{ A}$$

With reference to the phasor element labeled $a'c$ in Fig. 5.28b, the magnitude of $\hat{E}_{af}$ can be found by adding the length $a'c = I_d(X_d - X_q)$ numerically to the magnitude of $\hat{E}'$; thus

$$E_{af} = E' + I_d(X_d - X_q) = 1429 + 137(0.55) = 1504 \text{ V line-to-neutral}$$

(Alternatively, E_{af} could have been determined as $\hat{E}_{af} = \hat{V}_a - j\hat{I}_d X_d - j\hat{I}_q X_q$.)

From Eq. 5.65 the power-angle characteristic for this motor is

$$\begin{aligned} P &= \frac{E_{af} V_{EQ}}{X_d} \sin|\delta| + V_{EQ}^2 \frac{X_d - X_q}{2X_d X_q} \sin(2|\delta|) \\ &= 1030 \sin|\delta| + 178 \sin(2|\delta|) \quad \text{kW/phase} \end{aligned}$$

Note that we have used $|\delta|$ in this equation. That is because Eq. 5.65 as written applies to a generator and calculates the electrical power output from the generator. For our motor, δ is negative and direct use of Eq. 5.65 will give a value of power $P < 0$ which is of course correct for motor operation. Since we know that this is a motor and that we are calculating electric power into the motor terminals, we ignore the sign issue here entirely and calculate the motor power directly as a positive number.

The maximum motor input power occurs when $dP/d\delta = 0$

$$\frac{dP}{d\delta} = 1030 \cos\delta + 356 \cos 2\delta$$

Setting this equal to zero and using the trigonometric identity

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

permit us to solve for the angle δ at which the maximum power occurs:

$$\delta = 73.2^\circ$$

Therefore the maximum power is

$$P_{max} = 1080 \text{ kW/phase} = 3240 \text{ kW, three-phase}$$

We can compare this value with $P_{max} = 3090$ kW found in part (a) of Example 5.7, where the effects of salient poles were neglected. We see that the error caused by neglecting saliency is slightly less than five percent in this case.

Practice Problem 5.9

A 325-MVA, 26-kV, 60-Hz, three-phase, salient-pole synchronous generator is observed to operating at a power output of 250-MW and a lagging power factor of 0.89 at a terminal voltage of 26 kV. The generator synchronous reactances are $X_d = 1.95$ and $X_q = 1.18$, both in per unit. The generator achieves rated-open-circuit voltage at a field current AFNL = 342 A.

Calculate (a) the angle δ between the generator terminal voltage and the generated voltage, (b) the magnitude of the generated voltage E_{af} in per unit, and (c) the required field current in amperes.

Solution

a. 31.8°
b. $E_{af} = 2.29$ per unit
c. $I_f = 783$ A

The effect, as seen in Example 5.11, of salient poles on the maximum power capability of a synchronous machine increases as the excitation voltage is decreased, as can be seen from Eq. 5.65. Under typical operating conditions, the effect of salient poles usually amounts to a few percent at most. Only at small excitations does the reluctance torque become important. Thus, except at small excitations or when very accurate results are required, a salient-pole machine usually can be adequately treated by simple cylindrical-rotor theory.

5.8 PERMANENT-MAGNET AC MOTORS

Permanent-magnet ac motors are polyphase synchronous motors with permanent-magnet rotors. Thus they are similar to the synchronous machines discussed up to this point in this chapter with the exception that the field windings are replaced by permanent magnets.

Figure 5.29 is a schematic diagram of a three-phase permanent-magnet ac machine. Comparison of this figure with Fig. 5.2 emphasizes the similarities between the permanent-magnet ac machine and the conventional synchronous machine. In fact, the permanent-magnet ac machine can be readily analyzed with the techniques of this chapter simply by assuming that the machine is excited by a field current of constant value, making sure to calculate the various machine inductances based on the effective permeability of the permanent-magnet rotor.

Figure 5.30 shows a cutaway view of a typical permanent-magnet ac motor. This figure also shows a speed and position sensor mounted on the rotor shaft. This sensor is used for control of the motor, as is discussed in Section 11.2. A number

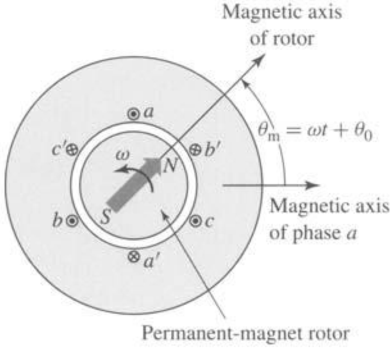


Figure 5.29 Schematic diagram of a three-phase permanent-magnet ac machine. The arrow indicates the direction of rotor magnetization.

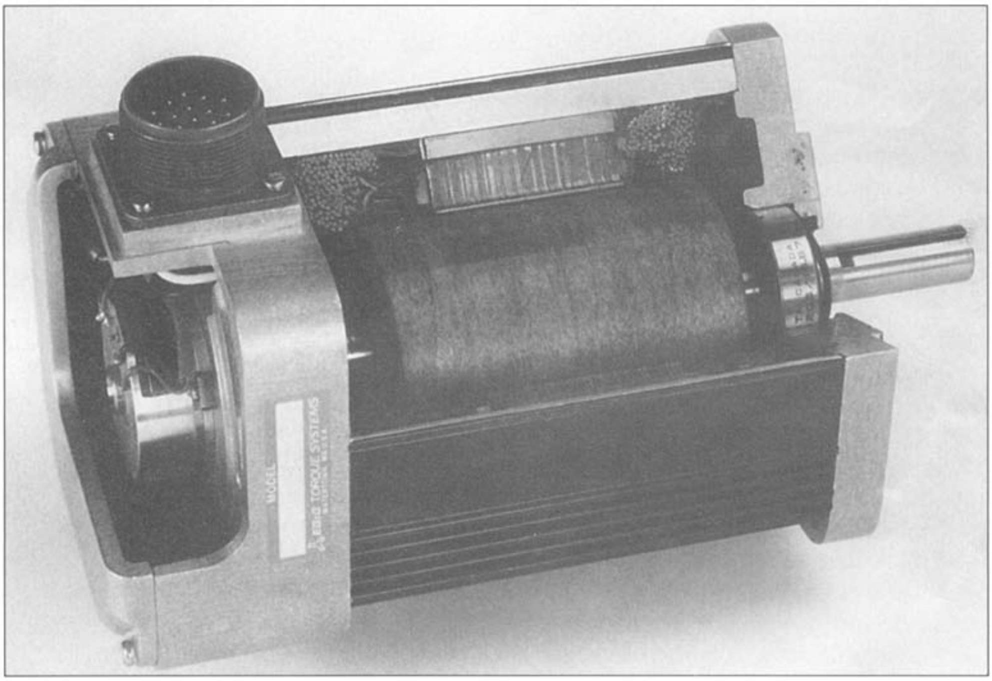


Figure 5.30 Cutaway view of a permanent-magnet ac motor. Also shown is the shaft speed and position sensor used to control the motor. (*EG&G Torque Systems.*)

of techniques may be used for shaft-position sensing, including Hall-effect devices, light-emitting diodes and phototransistors in combination with a pulsed wheel, and inductance pickups.

Permanent-magnet ac motors are typically operated from variable-frequency motor drives. Under conditions of constant-frequency, sinusiodal polyphase excitation, a permanent-magnet ac motor behaves similarly to a conventional ac synchronous machine with constant field excitation.

An alternate viewpoint of a permanent-magnet ac motor is that it is a form of permanent-magnet stepping motor with a nonsalient stator (see Section 8.5). Under this viewpoint, the only difference between the two is that there will be little, if any, saliency (cogging) torque in the permanent-magnet ac motor. In the simplest operation, the phases can be simply excited with stepped waveforms so as to cause the rotor to step sequentially from one equilibrium position to the next. Alternatively, using rotor-position feedback from a shaft-position sensor, the motor phase windings can be continuously excited in such a fashion as to control the torque and speed of the motor. As with the stepping motor, the frequency of the excitation determines the motor speed, and the angular position between the rotor magnetic axis and a given phase and the level of excitation in that phase determines the torque which will be produced.

Permanent-magnet ac motors are frequently referred to as *brushless motors* or *brushless dc motors*. This terminology comes about both because of the similarity, when combined with a variable-frequency, variable-voltage drive system, of their speed-torque characteristics to those of dc motors and because of the fact that one can view these motors as inside-out dc motors, with their field winding on the rotor and with their armature electronically commutated by the shaft-position sensor and by switches connected to the armature windings.

5.9 SUMMARY

Under steady-state operating conditions, the physical picture of the operation of a polyphase synchronous machine is simply seen in terms of the interaction of two magnetic fields as discussed in Section 4.7.2. Polyphase currents on the stator produce a rotating magnetic flux wave while dc currents on the rotor produce a flux wave which is stationary with respect to the rotor. Constant torque is produced only when the rotor rotates in synchronism with the stator flux wave. Under these conditions, there is a constant angular displacement between the rotor and stator flux waves and the result is a torque which is proportional to the sine of the displacement angle.

We have seen that a simple set of tests can be used to determine the significant parameters of a synchronous machine including the synchronous reactance X_s or X_d. Two such tests are an open-circuit test, in which the machine terminal voltage is measured as a function of field current, and a short-circuit test, in which the armature is short-circuited and the short-circuit armature current is measured as a function of field current. These test methods are a variation of a testing technique applicable not only to synchronous machines but also to any electrical system whose behavior can be approximated by a linear equivalent circuit to which Thevenin's theorem applies. From a Thevenin-theorem viewpoint, an open-circuit test gives the internal voltage, and a short-circuit test gives information regarding the internal impedance. From the more specific viewpoint of electromechanical machinery, an open-circuit test gives information regarding excitation requirements, core losses, and (for rotating machines) friction and windage losses; a short-circuit test gives information regarding the magnetic reactions of the load current, leakage impedances, and losses associated with the load current such as I^2R and stray load losses. The only real complication arises from the effects of magnetic nonlinearity, effects which can be taken into account approximately by considering the machine to be equivalent to an unsaturated one whose magnetization curve is the straight line *Op* of Fig. 5.9 and whose synchronous reactance is empirically adjusted for saturation as in Eq. 5.29.

In many cases, synchronous machines are operated in conjunction with an external system which can be represented as a constant-frequency, constant-voltage source known as an *infinite bus*. Under these conditions, the synchronous speed is determined by the frequency of the infinite bus, and the machine output power is proportional to the product of the bus voltage, the machine internal voltage (which is, in turn, proportional to the field excitation), and the sine of the phase angle between them (the power angle), and it is inversely proportional to the net reactance between them.

While the real power at the machine terminals is determined by the shaft power input to the machine (if it is acting as a generator) or the shaft load (if it is a motor), varying the field excitation varies the reactive power. For low values of field current, the machine will absorb reactive power from the system and the power angle will be large. Increasing the field current will reduce the reactive power absorbed by the machine as well as the power angle. At some value of field current, the machine power factor will be unity and any further increase in field current will cause the machine to supply reactive power to the system.

Once brought up to synchronous speed, synchronous motors can be operated quite efficiently when connected to a constant-frequency source. However, as we have seen, a synchronous motor develops torque only at synchronous speed and hence has no starting torque. To make a synchronous motor self-starting, a squirrel-cage winding, called an *amortisseur* or *damper winding,* can be inserted in the rotor pole faces, as shown in Fig. 5.31. The rotor then comes up almost to synchronous speed by induction-motor action with the field winding unexcited. If the load and inertia are not too great, the motor will pull into synchronism when the field winding is energized from a dc source.

Alternatively, as we will see in Chapter 11, synchronous motors can be operated from polyphase variable-frequency drive systems. In this case they can be easily started and operated quite flexibly. Small permanent-magnet synchronous machines operated under such conditions are frequently referred to as *brushless motors* or *brushless-dc motors,* both because of the similarity of their speed-torque characteristics to those of dc motors and because of the fact that one can view these motors as inside-out dc motors, with the commutation of the stator windings produced electronically by the drive electronics.

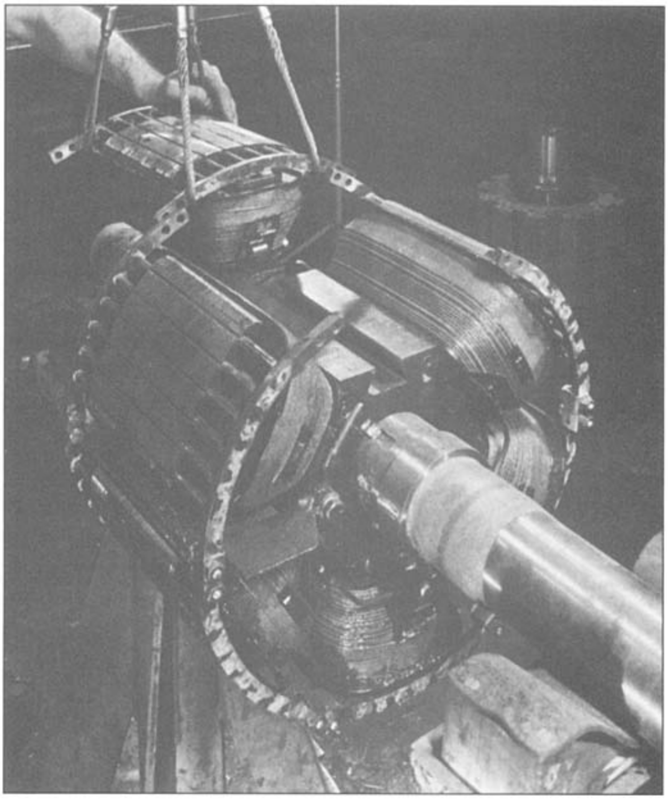


Figure 5.31 Rotor of a six-pole 1200 r/min synchronous motor showing field coils, pole-face damper winding, and construction. (*General Electric Company.*)

5.10 PROBLEMS

5.1 The full-load torque angle of a synchronous motor at rated voltage and frequency is 35 electrical degrees. Neglect the effects of armature resistance and leakage reactance. If the field current is held constant, how would the full-load torque angle be affected by the following changes in operating condition?

a. Frequency reduced 10 percent, load torque and applied voltage constant.

b. Frequency reduced 10 percent, load power and applied voltage constant.

c. Both frequency and applied voltage reduced 10 percent, load torque constant.

d. Both frequency and applied voltage reduced 10 percent, load power constant.

5.2 The armature phase windings of a two-phase synchronous machine are displaced by 90 electrical degrees in space.

a. What is the mutual inductance between these two windings?

b. Repeat the derivation leading to Eq. 5.17 and show that the synchronous inductance is simply equal to the armature phase inductance; that is, $L_s = L_{aa0} + L_{al}$, where L_{aa0} is the component of the armature phase inductance due to space-fundamental air-gap flux and L_{al} is the armature leakage inductance.

5.3 Design calculations show the following parameters for a three-phase, cylindrical-rotor synchronous generator:

$$\text{Phase-}a\text{ self-inductance } L_{aa} = 4.83 \text{ mH}$$

$$\text{Armature leakage inductance } L_{al} = 0.33 \text{ mH}$$

Calculate the phase-phase mutual inductance and the machine synchronous inductance.

5.4 The open-circuit terminal voltage of a three-phase, 60-Hz synchronous generator is found to be 15.4 kV rms line-to-line when the field current is 420 A.

a. Calculate the stator-to-rotor mutual inductance L_{af}.

b. Calculate the open-circuit terminal voltage if the field current is held constant while the generator speed is reduced so that the frequency of the generated voltage is 50 Hz.

5.5 A 460-V, 50-kW, 60-Hz, three-phase synchronous motor has a synchronous reactance of $X_s = 4.15\ \Omega$ and an armature-to-field mutual inductance, $L_{af} = 83$ mH. The motor is operating at rated terminal voltage and an input power of 40 kW. Calculate the magnitude and phase angle of the line-to-neutral generated voltage $\hat{E}_{af}$ and the field current I_f if the motor is operating at (*a*) 0.85 power factor lagging, (*b*) unity power factor, and (*c*) 0.85 power factor leading.

5.6 The motor of Problem 5.5 is supplied from a 460-V, three-phase source through a feeder whose impedance is $Z_f = 0.084 + j0.82\ \Omega$. Assuming the system (as measured at the source) to be operating at an input power of 40 kW, calculate the magnitude and phase angle of the line-to-neutral generated voltage $\hat{E}_{af}$ and the field current I_f for power factors of (*a*) 0.85 lagging, (*b*) unity, and (*c*) 0.85 leading.

5.7 A 50-Hz, two-pole, 750 kVA, 2300 V, three-phase synchronous machine has a synchronous reactance of 7.75 Ω and achieves rated open-circuit terminal voltage at a field current of 120 A.

a. Calculate the armature-to-field mutual inductance.

b. The machine is to be operated as a motor supplying a 600 kW load at its rated terminal voltage. Calculate the internal voltage E_{af} and the corresponding field current if the motor is operating at unity power factor.

c. For a constant load power of 600 kW, write a MATLAB script to plot the terminal current as a function of field current. For your plot, let the field current vary between a minimum value corresponding to a machine loading of 750 kVA at leading power factor and a maximum value corresponding to a machine loading of 750 kVA at lagging power factor. What value of field current produces the minimum terminal current? Why?

5.8 The manufacturer's data sheet for a 26-kV, 750-MVA, 60-Hz, three-phase synchronous generator indicates that it has a synchronous reactance $X_s = 2.04$ and a leakage reactance $X_{al} = 0.18$, both in per unit on the generator base. Calculate (*a*) the synchronous inductance in mH, (*b*) the armature leakage inductance in mH, and (*c*) the armature phase inductance L_{aa} in mH and per unit.

5.9 The following readings are taken from the results of an open- and a short-circuit test on an 800-MVA, three-phase, Y-connected, 26-kV, two-pole, 60-Hz turbine generator driven at synchronous speed:

Field current, A	1540	2960
Armature current, short-circuit test, kA	9.26	17.8
Line voltage, open-circuit characteristic, kV	26.0	(31.8)
Line voltage, air-gap line, kV	29.6	(56.9)

The number in parentheses are extrapolations based upon the measured data. Find (*a*) the short-circuit ratio, (*b*) the unsaturated value of the synchronous reactance in ohms per phase and per unit, and (*c*) the saturated synchronous reactance in per unit and in ohms per phase.

5.10 The following readings are taken from the results of an open- and a short-circuit test on a 5000-kW, 4160-V, three-phase, four-pole, 1800-rpm synchronous motor driven at rated speed:

Field current, A	169	192
Armature current, short-circuit test, A	694	790
Line voltage, open-circuit characteristic, V	3920	4160
Line voltage, air-gap line, V	4640	5270

The armature resistance is 11 mΩ/phase. The armature leakage reactance is estimated to be 0.12 per unit on the motor rating as base. Find (*a*) the short-circuit ratio, (*b*) the unsaturated value of the synchronous reactance in ohms per phase and per unit, and (*c*) the saturated synchronous reactance in per unit and in ohms per phase.

5.11 Write a MATLAB script which automates the calculations of Problems 5.9 and 5.10. The following minimum set of data is required:

- AFNL: The field current required to achieve rated open-circuit terminal voltage.
- The corresponding terminal voltage on the air gap line.
- AFSC: The field current required to achieve rated short-circuit current on the short-circuit characteristic.

Your script should calculate (*a*) the short-circuit ratio, (*b*) the unsaturated value of the synchronous reactance in ohms per phase and per unit, and (*c*) the saturated synchronous reactance in per unit and in ohms per phase.

5.12 Consider the motor of Problem 5.10.

a. Compute the field current required when the motor is operating at rated voltage, 4200 kW input power at 0.87 power factor leading. Account for saturation under load by the method described in the paragraph relating to Eq. 5.29.

b. In addition to the data given in Problem 5.10, additional points on the open-circuit characteristic are given below:

Field current, A	200	250	300	350
Line voltage, V	4250	4580	4820	5000

If the circuit breaker supplying the motor of part (a) is tripped, leaving the motor suddenly open-circuited, estimate the value of the motor terminal voltage following the trip (before the motor begins to slow down and before any protection circuitry reduces the field current).

5.13 Using MATLAB, plot the field current required to achieve unity-power-factor operation for the motor of Problem 5.10 as the motor load varies from zero to full load. Assume the motor to be operating at rated terminal voltage.

5.14 Loss data for the motor of Problem 5.10 are as follows:

Open-circuit core loss at 4160 V = 37 kW
Friction and windage loss = 46 kW
Field-winding resistance at 75°C = 0.279 Ω

Compute the output power and efficiency when the motor is operating at rated input power, unity power factor, and rated voltage. Assume the field-winding to be operating at a temperature of 125°C.

5.15 The following data are obtained from tests on a 145-MVA, 13.8-kV, three-phase, 60-Hz, 72-pole hydroelectric generator.

Open-circuit characteristic:

I_f, A	100	200	300	400	500	600	700	775	800
Voltage, kV	2.27	4.44	6.68	8.67	10.4	11.9	13.4	14.3	14.5

Short-circuit test:

$I_f = 710$ A, $I_a = 6070$ A

a. Draw (or plot using MATLAB) the open-circuit saturation curve, the air-gap line, and the short-circuit characteristic.

b. Find AFNL and AFSC. (Note that if you use MATLAB for part (a), you can use the MATLAB function 'polyfit' to fit a second-order polynomial to the open-circuit saturation curve. You can then use this fit to find AFNL.)

c. Find (*i*) the short-circuit ratio, (*ii*) the unsaturated value of the synchronous reactance in ohms per phase and per unit and (*iii*) the saturated synchronous reactance in per unit and in ohms per phase.

5.16 What is the maximum per-unit reactive power that can be supplied by a synchronous machine operating at its rated terminal voltage whose synchronous reactance is 1.6 per unit and whose maximum field current is limited to 2.4 times that required to achieve rated terminal voltage under open-circuit conditions?

5.17 A 25-MVA, 11.5 kV synchronous machine is operating as a synchronous condenser, as discussed in Appendix D (section D.4.1). The generator short-circuit ratio is 1.68 and the field current at rated voltage, no load is 420 A. Assume the generator to be connected directly to an 11.5 kV source.

a. What is the saturated synchronous reactance of the generator in per unit and in ohms per phase?

The generator field current is adjusted to 150 A.

b. Draw a phasor diagram, indicating the terminal voltage, internal voltage, and armature current.

c. Calculate the armature current magnitude (per unit and amperes) and its relative phase angle with respect to the terminal voltage.

d. Under these conditions, does the synchronous condenser appear inductive or capacitive to the 11.5 kV system?

e. Repeat parts (b) through (d) for a field current of 700 A.

5.18 The synchronous condenser of Problem 5.17 is connected to a 11.5 kV system through a feeder whose series reactance is 0.12 per unit on the machine base. Using MATLAB, plot the voltage (kV) at the synchronous-condenser terminals as the synchronous-condenser field current is varied between 150 A and 700 A.

5.19 A synchronous machine with a synchronous reactance of 1.28 per unit is operating as a generator at a real power loading of 0.6 per unit connected to a system with a series reactance of 0.07 per unit. An increase in its field current is observed to cause a decrease in armature current.

a. Before the increase, was the generator supplying or absorbing reactive power from the power system?

b. As a result of this increase in excitation, did the generator terminal voltage increase or decrease?

c. Repeat parts (a) and (b) if the synchronous machine is operating as a motor.

5.20 Superconducting synchronous machines are designed with superconducting fields windings which can support large current densities and create large magnetic flux densities. Since typical operating magnetic flux densities exceed the saturation flux densities of iron, these machines are typically designed without iron in the magnetic circuit; as a result, these machines exhibit no saturation effects and have low synchronous reactances.

Consider a two-pole, 60-Hz, 13.8-kV, 10-MVA superconducting generator which achieves rated open-circuit armature voltage at a field current of 842 A. It achieves rated armature current into a three-phase terminal short circuit for a field current of 226 A.

a. Calculate the per-unit synchronous reactance.

Consider the situation in which this generator is connected to a 13.8 kV distribution feeder of negligible impedance and operating at an output power of 8.75 MW at 0.9 pf lagging. Calculate:

b. the field current in amperes, the reactive-power output in MVA, and the rotor angle for this operating condition.

c. the resultant rotor angle and reactive-power output in MVA if the field current is reduced to 842 A while the shaft-power supplied by the prime mover to the generator remains constant.

5.21 For a synchronous machine with constant synchronous reactance X_s operating at a constant terminal voltage V_t and a constant excitation voltage E_{af}, show that the locus of the tip of the armature-current phasor is a circle. On a phasor diagram with terminal voltage shown as the reference phasor, indicate the position of the center of this circle and its radius. Express the coordinates of the center and the radius of the circle in terms of V_t, E_{af} and X_s.

5.22 A four-pole, 60-Hz, 24-kV, 650-MVA synchronous generator with a synchronous reactance of 1.82 per unit is operating on a power system which can be represented by a 24-kV infinite bus in series with a reactive impedance of $j0.21\ \Omega$. The generator is equipped with a voltage regulator that adjusts the field excitation such that the generator terminal voltage remains at 24 kV independent of the generator loading.

a. The generator output power is adjusted to 375 MW.
   - (i) Draw a phasor diagram for this operating condition.
   - (ii) Find the magnitude (in kA) and phase angle (with respect to the generator terminal voltage) of the terminal current.
   - (iii) Determine the generator terminal power factor.
   - (iv) Find the magnitude (in per unit and kV) of the generator excitation voltage E_{af}.

b. Repeat part (a) if the generator output power is increased to 600 MW.

5.23 The generator of Problem 5.22 achieves rated open-circuit armature voltage at a field current of 850 A. It is operating on the system of Problem 5.22 with its voltage regulator set to maintain the terminal voltage at 0.99 per unit (23.8 kV).

a. Use MATLAB to plot the generator field current (in A) as a function of load (in MW) as the load on the generator output power is varied from zero to full load.

b. Plot the corresponding reactive output power in MVAR as a function of output load.

c. Repeat the plots of parts (a) and (b) if the voltage regulator is set to regulate the terminal voltage to 1.01 per unit (24.2 kV).

5.24 The 145 MW hydroelectric generator of Problem 5.15 is operating on a 13.8-kV power system. Under normal operating procedures, the generator is operated under automatic voltage regulation set to maintain its terminal voltage at 13.8 kV. In this problem you will investigate the possible consequences should the operator forget to switch over to the automatic voltage regulator and instead leave the field excitation constant at AFNL, the value corresponding to rated open-circuit voltage. For the purposes of this problem, neglect the effects of saliency and assume that the generator can be represented by the saturated synchronous reactance found in Problem 5.15.

a. If the power system is represented simply by a 13.8 kV infinite (ignoring the effects of any equivalent impedance), can the generator be loaded to full load? If so, what is the power angle δ corresponding to full load? If not, what is the maximum load that can be achieved?

b. Repeat part (a) with the power system now represented by a 13.8 kV infinite bus in series with a reactive impedance of $j0.14\ \Omega$.

5.25 Repeat Example 5.9 assuming the generator is operating at one-half of its rated kVA at a lagging power factor of 0.8 and rated terminal voltage.

5.26 Repeat Problem 5.24 assuming that the saturated direct-axis synchronous inductance X_d is equal to that found in Problem 5.15 and that the saturated quadrature-axis synchronous reactance X_q is equal to 75 percent of this value. Compare your answers to those found in Problem 5.24.

5.27 Write a MATLAB script to plot a set of per-unit power-angle curves for a salient-pole synchronous generator connected to an infinite bus ($V_{bus} = 1.0$ per unit). The generator reactances are $X_d = 1.27$ per unit and $X_q = 0.95$ per unit. Assuming $E_{af} = 1.0$ per unit, plot the following curves:

a. Generator connected directly to the infinite bus.

b. Generator connected to the infinite bus through a reactance $X_{bus} =$ 0.1 per unit.

c. Generator connected directly to the infinite bus. Neglect saliency effects, setting $X_q = X_d$.

d. Generator connected to the infinite bus through a reactance $X_{bus} =$ 0.1 per unit. Neglect saliency effects, setting $X_q = X_d$.

5.28 Draw the steady-state, direct- and quadrature-axis phasor diagram for a salient-pole synchronous motor with reactances X_d and X_q and armature resistance R_a. From this phasor diagram, show that the torque angle δ between the generated voltage $\hat{E}_{af}$ (which lies along the quadrature axis) and the terminal voltage $\hat{V}_t$ is given by

$$\tan\delta = \frac{I_a X_q \cos\phi + I_a R_a \sin\phi}{V_t + I_a X_q \sin\phi - I_a R_a \cos\phi}$$

Here ϕ is the phase angle of the armature current $\hat{I}_a$ and V_t, considered to be negative when $\hat{I}_a$ lags $\hat{V}_t$.

5.29 Repeat Problem 5.28 for synchronous generator operation, in which case the equation for δ becomes

$$\tan\delta = \frac{I_a X_q \cos\phi + I_a R_a \sin\phi}{V_t - I_a X_q \sin\phi + I_a R_a \cos\phi}$$

5.30 What maximum percentage of its rated output power will a salient-pole motor deliver without loss of synchronism when operating at its rated terminal voltage with zero field excitation ($E_{af} = 0$) if $X_d = 0.90$ per unit and $X_q = 0.65$ per unit? Compute the per-unit armature current and reactive power for this operating condition.

5.31 If the synchronous motor of Problem 5.30 is now operated as a synchronous generator connected to an infinite bus of rated voltage, find the minimum per-unit field excitation (where 1.0 per unit is the field current required to achieve rated open-circuit voltage) for which the generator will remain synchronized at (*a*) half load and (*b*) full load.

5.32 A salient-pole synchronous generator with saturated synchronous reactances $X_d = 1.57$ per unit and $X_q = 1.34$ per unit is connected to an infinite bus of rated voltage through an external impedance $X_{bus} = 0.11$ per unit. The generator is supplying its rated MVA at 0.95 power factor lagging, as measured at the generator terminals.

a. Draw a phasor diagram, indicating the infinite-bus voltage, the armature current, the generator terminal voltage, the excitation voltage, and the rotor angle (measured with respect to the infinite bus).

b. Calculate the per-unit terminal and excitation voltages, and the rotor angle in degrees.

5.33 A salient-pole synchronous generator with saturated synchronous reactances $X_d = 0.78$ per unit and $X_q = 0.63$ per unit is connected to a rated-voltage infinite bus through an external impedance $X_{bus} = 0.09$ per unit.

a. Assuming the generator to be supplying only reactive power
   (i) Find minimum and maximum per-unit field excitations (where 1.0 per unit is the field current required to achieve rated open-circuit voltage) such that the generator does not exceed its rated terminal current.
   (ii) Using MATLAB, plot the armature current as a function of field excitation as the field excitation is varied between the limits determined in part (i).

b. Now assuming the generator to be supplying 0.25 per unit rated real power, on the same axes add a plot of the per-unit armature current as a function of field excitation as the field current is varied in the range for which the per-unit armature current is less than 1.0 per unit.

c. Repeat part (b) for generator output powers of 0.50 and 0.75 per unit. The final result will be a plot of V-curves for this generator in this configuration.

5.34 A two-phase permanent-magnet ac motor has a rated speed of 3000 r/min and a six-pole rotor. Calculate the frequency (in Hz) of the armature voltage required to operate at this speed.

5.35 A 5-kW, three-phase, permanent-magnet synchronous generator produces an open-circuit voltage of 208 V line-to-line, 60-Hz, when driven at a speed of 1800 r/min. When operating at rated speed and supplying a resistive load, its terminal voltage is observed to be 192 V line-to-line for a power output of 4.5 kW.

a. Calculate the generator phase current under this operating condition.

b. Assuming the generator armature resistance to be negligible, calculate the generator 60-Hz synchronous reactance.

c. Calculate the generator terminal voltage which will result if the motor generator load is increased to 5 kW (again purely resistive) while the speed is maintained at 1800 r/min.

5.36 Small single-phase permanent-magnet ac generators are frequently used to generate the power for lights on bicycles. For this application, these generators are typically designed with a significant amount of leakage inductance in their armature winding. A simple model for these generators is an ac voltage source $e_a(t) = \omega K_a \cos \omega t$ in series with the armature leakage inductance L_a and the armature resistance R_a. Here ω is the electrical frequency of the generated voltage which is determined by the speed of the generator as it rubs against the bicycle wheel.

Assuming that the generator is running a light bulb which can be modeled as a resistance R_b, write an expression for the minimum frequency ω_{min} which must be achieved in order to insure that the light operates at constant brightness, independent of the speed of the bicycle.