Multilevel PWM Converter Voltage Quality Asymptotic Time Domain Evaluation

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Abstract

An on-going extensive research on multi-level and matrix converters requires physically meaningful easy-to-calculate criteria to compare numerous suggested converter topologies and voltage modulation strategies. Frequency domain voltage Total Harmonic Distortion (THD) criterion approach is too complicated and non-transparent as it involves multiple Fourier integral and results in multiple series with Bessel function coefficients. Our approach is to evaluate converter voltage quality in time domain using Normalized Ripple Voltage Mean Square (NRVMS) criterion that is about squared voltage THD. This is an asymptotic approach in the frequency domain sense meaning that PWM carrier to fundamental frequency ratio is supposed to be infinitely large C/F->\infty. Or, alternatively, it is a quasi-static approach in the time domain sense meaning that the fundamental frequency signal is assumed constant on a single switching period. We present analytical NRVMS (THD) expressions that use elementary functions for an arbitrary number of levels N. NRVMS for multi-level DC/AC converter is obtained by double time integration – on PWM period and on output fundamental period (for matrix converter by triple integration – on PWM period, on input fundamental period and on output fundamental period). NRVMS criterion has a physical meaning of normalized PWM induced choke or electrical machine core loss that, under some realistic assumptions, is a major PWM loss mechanism dominating over PWM copper loss and PWM hysteresis core loss.

1. Voltage Quality Evaluation Methodology

We consider carrier based voltage modulation strategies. In general, these can be subdivided into Phase Shifted (PS) and Level Shifted (LS) multi-carrier strategies. LS strategies are, in turn, split into Phase Disposition (PD), Phase Opposite Disposition (POD) etc. A relationship with Space Vector PWM will be shown later.

Our approach is first to consider a DC type PWM modulation for a single-phase load fed by two inverter legs. One of the legs is controlled by a DC voltage command \( V_c \), the other – by the opposite voltage command \(-V_c\) . Normalized Ripple Voltage Mean Square (NRVMS) criterion is determined as a function of PWM duty ratio \( D \).

The next step is to consider a single-phase AC PWM modulation with sinusoidal voltage command variation. At this stage, NRVMS obtained for DC modulation is averaged on a fundamental period and becomes a function of modulation index \( M \).

The final step is to consider a three(multi)-phase converter. For a multi-phase converter, at best, a single-phase NRVMS holds or single-phase NRVMS is compromised dependent on carrier based voltage modulation strategy.
2. Single-Phase DC PWM

By a single-phase converter, engineers often mean a single converter leg with a split DC supply middle point access. As we find it useless for a three(multi)-phase analysis, we use the term “single-phase” for a single-phase fed by H-bridge (two inverter legs) that can formally be viewed as two-phase case.

For conventional two-level inverter, PWM voltage and ripple voltage (zero on average) are demonstrated on Fig.1.

![Fig.1. 2-Level PWM voltage (a) and ripple voltage (b) on a PWM period](image)

Here we assume a unipolar PWM voltage subject to Polarity Consistency Rule meaning that a positive (negative) load voltage is generated using positive (negative) voltage pulses. Bipolar PWM that involves both voltage pulse polarities essentially compromises PWM voltage quality producing excessive ripple voltage and PWM loss.

According to Fig.1, b, NRVMS for two-level DC PWM is calculated as

\[
NRVMS = (1-D)^2 D + (-D)^2 (1-D) = D(1-D).
\]  

Expression (1) may be further normalized for maximum unity \( D_{\text{MAX}} = 0.5 \) that gives

\[
NRVMS = 4D(1-D).
\]  

Now consider a three-level converter (Fig.2).
For a multi-level converter, according to the Extended Polarity Consistency Rule, a DC load voltage $V_L$ must be produced by switching between the two nearest voltage levels available.

The examples for $0 < V_L < 0.5V_{dc}$ and $0.5V_{dc} < V_L < V_{dc}$ involving PS modulation are shown in Fig.3 and Fig.4 respectively. For $V_L = 0.5V_{dc}$, the load voltage is theoretically DC-like.

![Fig.3. Three-level load voltage for $0 < V_L < 0.5V_{dc}$ by PS modulation](image)

![Fig.4. Three-level load voltage for $0.5V_{dc} < V_L < V_{dc}$ by PS modulation](image)

NRVMS calculation for multi-level PWM is piece-wise analytical similar to (1). Accounting for normalization (2), NRVMS criterion for multi-level DC PWM is shown in Fig.5, 6.

3. Single-Phase AC PWM

The assumption here is that a fundamental voltage may be considered constant on a PWM period that is a quasi-static time domain approach. Equivalently, it can be considered a frequency domain asymptotic approach in the sense that PWM carrier to fundamental frequency ratio is supposed infinitely large C/F->oo.
Given sinusoidal duty ratio variation (with a magnitude equal to a modulation index $M$; instead of negative duty ratios, it is better to consider negative output voltage polarity), we have to average NRVMS obtained for DC PWM (Fig.5, 6) on a fundamental period along sinusoidal duty ratio trajectories (Fig.5, 6 graphs must be extended for negative output voltages using even symmetry).

The results are given in Fig.7, 8 (NRVMS for two-level is normalized for maximum unity).
For conventional 2-level converter (Fig.7), NRVMS analytical expression is

\[ NRVMS_2 = \pi M - \frac{\pi^2}{4} M^2 \]

For a 3-level converter (Fig.7, 8)

\[
\begin{align*}
0 \leq M < 0.5 : & \quad NRVMS_3 = \frac{\pi}{2} M - \frac{\pi^2}{4} M^2; \\
0.5 \leq M \leq 1 : & \quad NRVMS_3 = \frac{\pi}{2} M - \frac{\pi^2}{4} M^2 - \frac{\pi^2}{4} \text{ARCSIN} \left( \frac{0.5}{M} \right) + \pi \sqrt{M^2 - 0.25}.
\end{align*}
\]

A general expression for an arbitrary N-level converter

\[
NRVMS_N(M) = \begin{cases} 
0 \leq M < \frac{1}{N-1} : & \frac{\pi}{N-1} M - \frac{\pi^2}{4} M^2; \\
\frac{K}{N-1} \leq M \leq \frac{K+1}{N-1} : & \frac{\pi}{N-1} M - \frac{\pi^2}{4} M^2 - \frac{K(K+1)}{2(N-1)^2} \pi^2 + \\
& + \frac{2\pi}{(N-1)^2} \sum_{i=1}^{K} i \text{ARCSIN} \left( \frac{i}{(N-1)M} \right) + \frac{2\pi}{(N-1)} \sum_{i=1}^{K} \sqrt{M^2 - \frac{i^2}{(N-1)^2}}.
\end{cases}
\]

may be proven using mathematical induction. Conventional voltage THD approximation closed-form expression is (remember NRVMS normalization for being unity for two-level)

\[ THD = \frac{\sqrt{2 \text{NRVMS}}}{\pi}. \]
4. Three-Phase AC PWM

The above NRVMS expressions were obtained while evaluating voltage quality of a single-phase multilevel DC/AC converter. The question is whether they hold for a three-phase converter as well?

The answer is:
- for two-level converter, yes;
- for multi-level converter, it depends on whether selected carrier based strategy is equivalent or not to the so-called Nearest-Three Virtual-Space-Vector voltage modulation strategy for a three-phase case.

There is an intuitive criterion that allows checking carrier based voltage modulation strategy equivalence to the Nearest-Three Virtual-Space-Vector one:

- consider single-phase DC voltage modulation process by selected strategy and verify that the Extended Polarity Consistency Rule holds (output voltage is produced by the two nearest DC levels) for an arbitrary voltage commands and shifts (zero sequence inserted voltages).

Two-level PWM voltage graphs without and with voltage commands shift are shown in Fig.9.

![Two-level PWM voltage graphs](image)

Fig.9. Two-level PWM: without (a) and with (b) voltage command shift

It is clearly seen that the effect of voltage commands shift (zero sequence voltage insertion) is a different load voltage pulses time distribution (placement) that does not affect NRVMS criterion.
Now observe a voltage shift impact on a three-level carrier based PS PWM (Fig.10). If a carriers cross point falls in-between shifted voltage commands (Fig.10, b), then the load voltage shape changes dramatically being far from just voltage pulses time redistribution and thus compromising NRVMS.

Therefore, a conclusion is that for a multi-level multi-phase carrier based Phase Shifted PWM the NRVMS criterion is compromised compared to a single-phase case.

Consider now a voltage shift effect on a three-level carrier based LS PD PWM (Fig. 11). No matter what is a shift, its impact is just a load voltage pulses time redistribution that, by no means, affects the NRVMS just as in the two-level PWM case above.

Therefore, the conclusion here is that a multi-level multi-phase carrier based Level Shifted Phase Disposition PWM is optimal in the sense of NRVMS for line-line voltage (equal to that of a single-phase case) and equivalent in the NRVMS sense to the Nearest-Three Virtual-Space-Vector voltage modulation strategy.

It is worth mentioning that conductivity and switching losses distributions between the converter power stage semiconductor devices are different for Phase Shifted and Level Shifted carrier based voltage modulation strategies because switching patterns essentially differ.

Fig.10. Three-level PS PWM: without (a) and with (b) voltage command shift
5. Multi-Phase AC PWM

So far, for a three-(multi-)phase system we dealt with a line-line PWM voltage quality. In this section, we consider a phase PWM voltage quality assuming a PWM strategy that meets the Extended Polarity Consistency Rule (single-phase double-leg DC PWM for arbitrary DC voltage commands and shifts) and symmetrical multi-phase system with isolated neutral.

For a three-phase, phase voltage criterion

\[
N_{RVM}^{P(M)} = \frac{N_{RVM}^{LL}(M)}{3},
\]

where \( N_{RVM}^{LL}(M) \) is that obtained for single phase double-leg case.

For a five-phase,

\[
N_{RVM}^{P(M)} = \frac{N_{RVM}^{LL}(M) + N_{RVM}^{LL}(M/1.618)}{5},
\]

for a seven-phase,

\[
N_{RVM}^{P(M)} = \frac{N_{RVM}^{LL}(M) + N_{RVM}^{LL}(M/1.247) + N_{RVM}^{LL}(M/2.247)}{7}.
\]

The calculations according to the above formulae show phase voltage quality degradation with a phase count increase that may look somewhat surprising because, according to the engineers’ “folklore”, a multi-phase system is always better than a three-phase one.