

Chapter 4

Inverters

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Content

- Topologies
 - Single-phase inverters
 - Three-phase inverters
- Gating patterns
 - Square wave operation
 - Pulse width modulation (PWM)
 - Selected harmonic elimination (SHE)
 - Delta modulation
- Microprocessor control of inverters

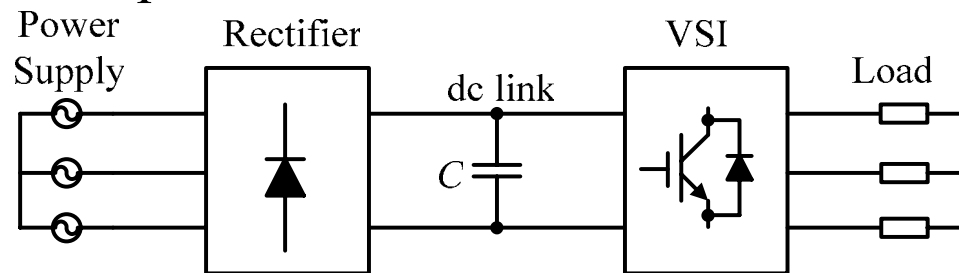
4.1 Introduction

- Inverters:
 - Input: dc voltage, fixed magnitude
 - Output: ac voltage, adjustable frequency and adjustable magnitude
- Main feature:
 - dc-to-ac conversion
 - Solid state device, no rotating machines.

4.1 Introduction

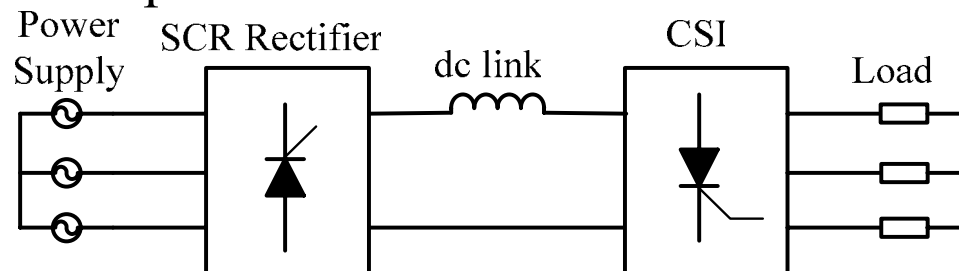
- Types

- Voltage source inverter (VSI): voltage source dc power supply. Output voltage is defined, based on the gating pattern. Output current is dependent on the load



Widely used
in industry

- Current source inverter (CSI): current source dc power supply. Output current is defined, based on the gating pattern. Output voltage is dependent on the load



4.2 Square wave operation

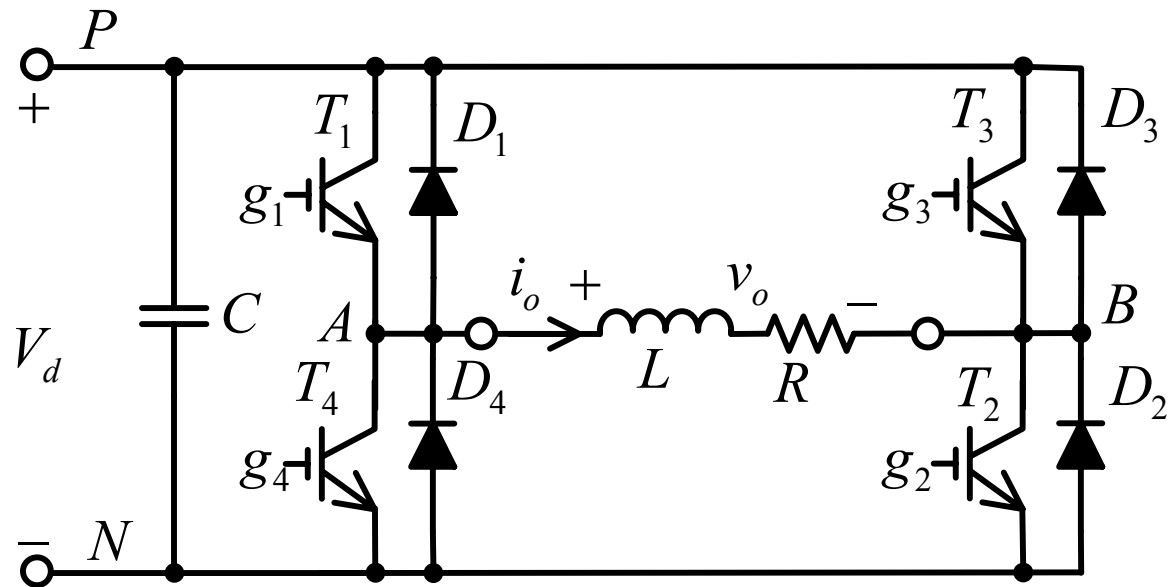
- Single-phase inverter
 - Circuit diagram

$T_1 \sim T_4$ – IGBTs,
switching devices

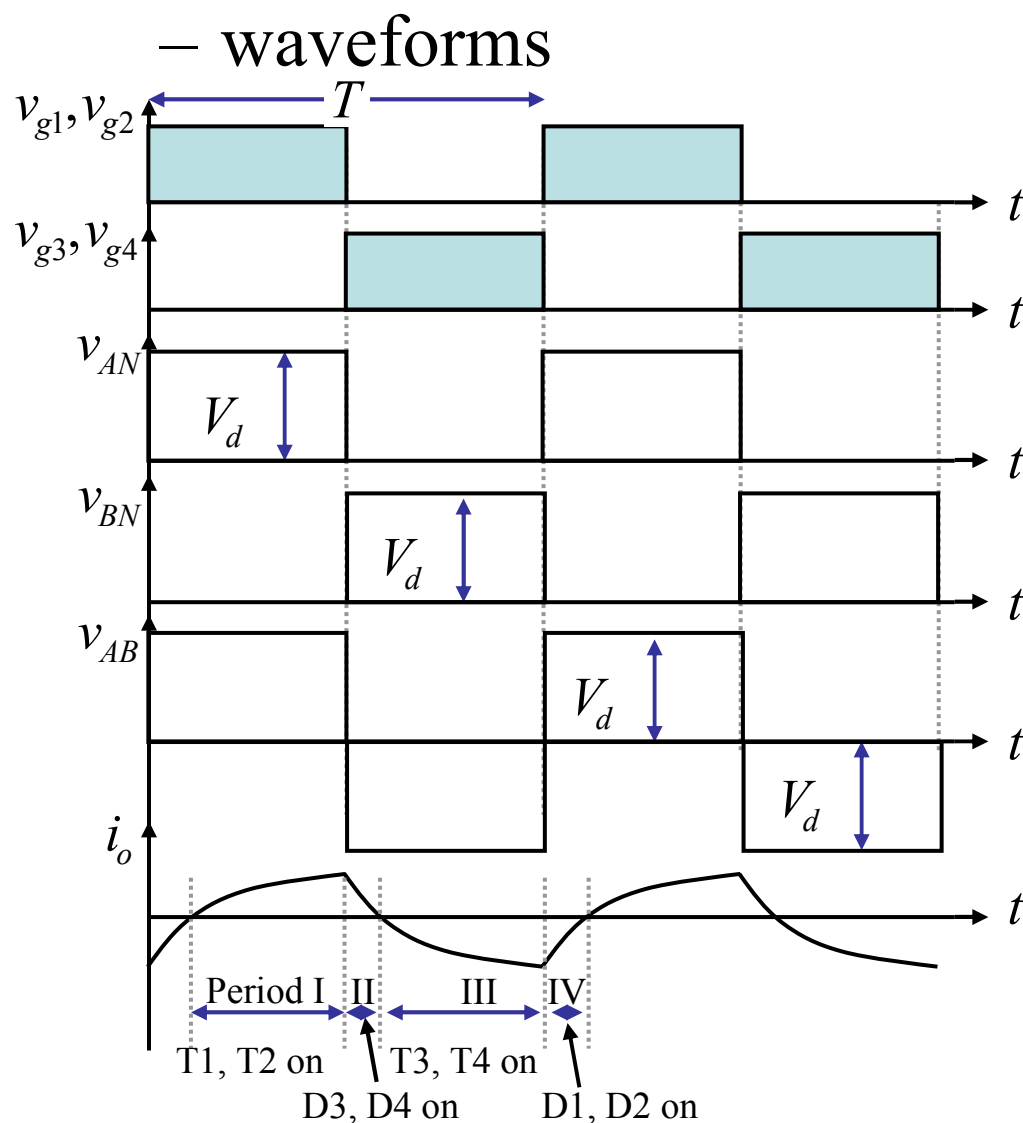
$D_1 \sim D_4$ – Freewheeling
diodes

C – dc filter capacitor
(VSI)

V_d – dc link voltage



4.2 Square wave operation



Period I:

$v_{g1}=v_{g2}>0 \rightarrow T_1, T_2$ on, current path:
 $V_d^+ \rightarrow T_1 \rightarrow \text{Load} \rightarrow T_2 \rightarrow V_d^-$

$v_o = V_d$

Period II:

$v_{g3}=v_{g4}>0 \rightarrow$ But $i_o > 0 \rightarrow D_3, D_4$ on,
 energy stored in L is releasing to V_d ,

current path:

$V_d^- \rightarrow D_4 \rightarrow \text{Load} \rightarrow D_3 \rightarrow V_d^+$

Period III:

$v_{g3}=v_{g4}>0 \rightarrow$ But $i_o < 0 \rightarrow T_3, T_4$ on,
 current path:

$V_d^+ \rightarrow T_3 \rightarrow \text{Load} \rightarrow T_4 \rightarrow V_d^-$

Period IV:

$v_{g1}=v_{g2}>0 \rightarrow$ But $i_o < 0 \rightarrow D_1, D_2$ on,
 energy stored in L is releasing to V_d ,

current path:

$V_d^- \rightarrow D_2 \rightarrow \text{Load} \rightarrow D_1 \rightarrow V_d^+$

4.2 Square wave operation

– Analysis

- rms value of output voltage

$$V_{o,rms} = \sqrt{\frac{1}{T} \int_0^T v_o^2 dt} = V_d$$

- Fourier series output voltage

$$v_o(t) = \sum_{n=1,3,5,\dots}^{\infty} \frac{4V_d}{n\pi} \sin n\omega t = \frac{4V_d}{\pi} \left(\sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t + \dots \right)$$

- rms value of fundamental component

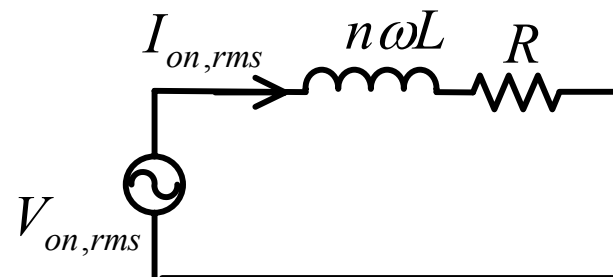
$$V_{o1,rms} = \frac{4V_d}{\pi} / \sqrt{2} = 0.9V_d$$

- Fourier series of output current

$$i_o(t) = \sum_{n=1,3,5,\dots}^{\infty} \sqrt{2} I_{on,rms} \sin(n\omega t - \theta_n) = \sum_{n=1,3,5,\dots}^{\infty} \frac{4V_d}{n\pi \sqrt{R^2 + (n\omega L)^2}} \sin(n\omega t - \theta_n)$$

use harmonic equivalent circuit

$$I_{on,rms} = \frac{V_{on,rms}}{\sqrt{R^2 + (n\omega L)^2}} \quad \text{and} \quad \theta_n = \tan^{-1} \frac{n\omega L}{R}$$



Harmonic
Equivalent circuit

$$\omega = 2\pi f = \frac{2\pi}{T}$$

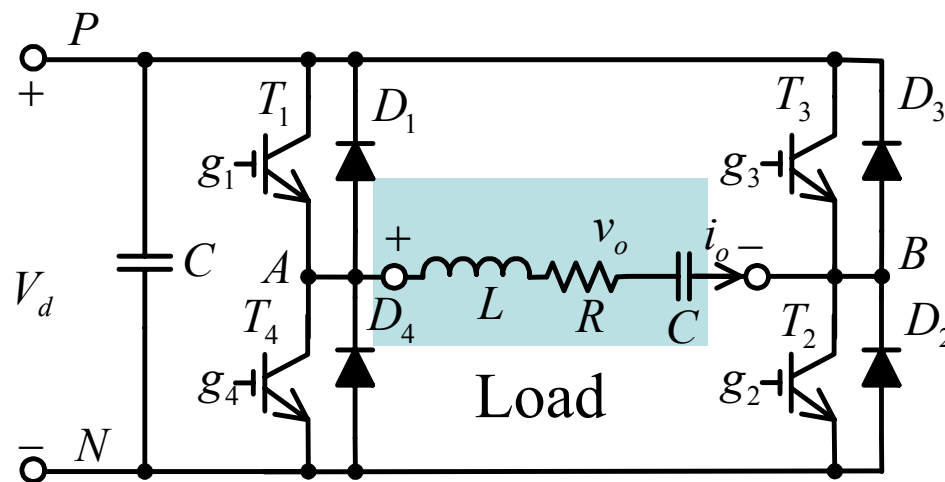
4.2 Square wave operation

– Post analysis

- Question 1: How to change output frequency
- Answer: Adjust the period T of the gate signal
- Question 2: How to change $V_{o,rms}$?
- Answer: Adjust dc link voltage V_d
- Question 3: Is i_o pure AC? no dc component in practical?
- Answer: No. usually a capacitor is used to isolate the dc component.

4.2 Example

– RLC load



Example

$R = 10\Omega$, $L = 31.5\text{mH}$, $C = 112\mu\text{F}$, $V_d = 220\text{V}$, $f_o = 60\text{Hz}$

Find : a) Fourier series of i_o (up to 9th order harmonic)

b) $I_{o1,rms}$, c) THD, d) P_{load} , and e) I_d

4.2 Example

a) Use harmonic equivalent circuit

$$X_L = jn\omega L = jn2\pi fL = jn \times 2\pi \times 60 \times 31.5 \times 10^{-3} = j11.87n\Omega$$

$$X_C = \frac{1}{jn\omega C} = -j \frac{1}{n \times 2\pi f C} = -j \frac{1}{n} \times \frac{1}{2\pi \times 60 \times 112 \times 10^{-6}} = -j \frac{23.68}{n} \Omega$$

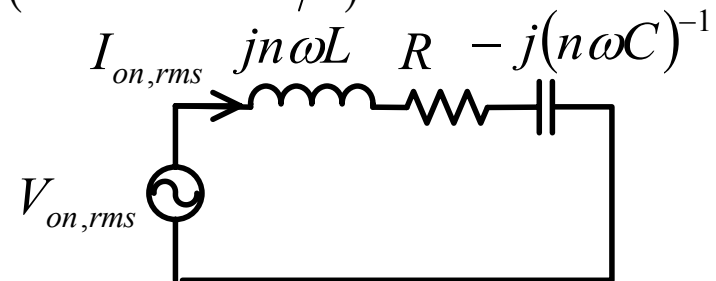
$$Z_n = \sqrt{R^2 + (X_L + X_C)^2} = \sqrt{100 + \left(11.87n - \frac{23.68}{n}\right)^2}$$

$$\theta_n = \tan^{-1} \frac{X_L + X_C}{R} = \tan^{-1} \frac{11.87n - 23.68/n}{10} = \tan^{-1}(1.187n - 2.368/n)$$

$$\begin{aligned} v_o &= \sum_{n=1,3,5,\dots}^{\infty} \frac{4V_d}{n\pi} \sin n\omega t \\ &= 280.1 \sin \omega t + 93.4 \sin 3\omega t + 56.02 \sin 5\omega t \\ &\quad + 40.02 \sin 7\omega t + 31.12 \sin 9\omega t + \dots \end{aligned}$$

$$I_{on,peak} = \frac{4V_d}{n\pi Z_n}$$

$$\begin{aligned} i_o &= 18.1 \sin(\omega t + 40.7^\circ) + 3.17 \sin(3\omega t - 70.2^\circ) + 1.0 \sin(5\omega t - 79.6^\circ) \\ &\quad + 0.5 \sin(7\omega t - 82.9^\circ) + 0.3 \sin(9\omega t - 84.5^\circ) + \dots \end{aligned}$$



Harmonic
Equivalent circuit

4.2 Example

$$\text{b) } I_{o1,rms} = \frac{18.1}{\sqrt{2}} = 12.8\text{A}$$

$$\begin{aligned}\text{c) } I_{o,rms} &\approx \sqrt{I_{o1,rms}^2 + I_{o3,rms}^2 + I_{o5,rms}^2 + I_{o7,rms}^2 + I_{o9,rms}^2} \\ &= \sqrt{\left(\frac{18.1}{\sqrt{2}}\right)^2 + \left(\frac{3.17}{\sqrt{2}}\right)^2 + \left(\frac{1.0}{\sqrt{2}}\right)^2 + \left(\frac{0.5}{\sqrt{2}}\right)^2 + \left(\frac{0.3}{\sqrt{2}}\right)^2} \\ &= 13.02\text{A}\end{aligned}$$

$$\text{THD} = \frac{\sqrt{I_{o,rms}^2 - I_{o1,rms}^2}}{I_{o1,rms}} = \frac{\sqrt{13.02^2 - 12.8^2}}{12.8} = 18.6\%$$

$$\text{d) } P_{load} = I_{o,rms}^2 R = 13.02^2 \times 10 = 1.695\text{kW}$$

$$\text{or : } P_{load} = \sum_{n=1,3,5,\dots}^{\infty} V_{on,rms} I_{on,rms} \cos \theta_n \approx 1.69\text{kW}$$

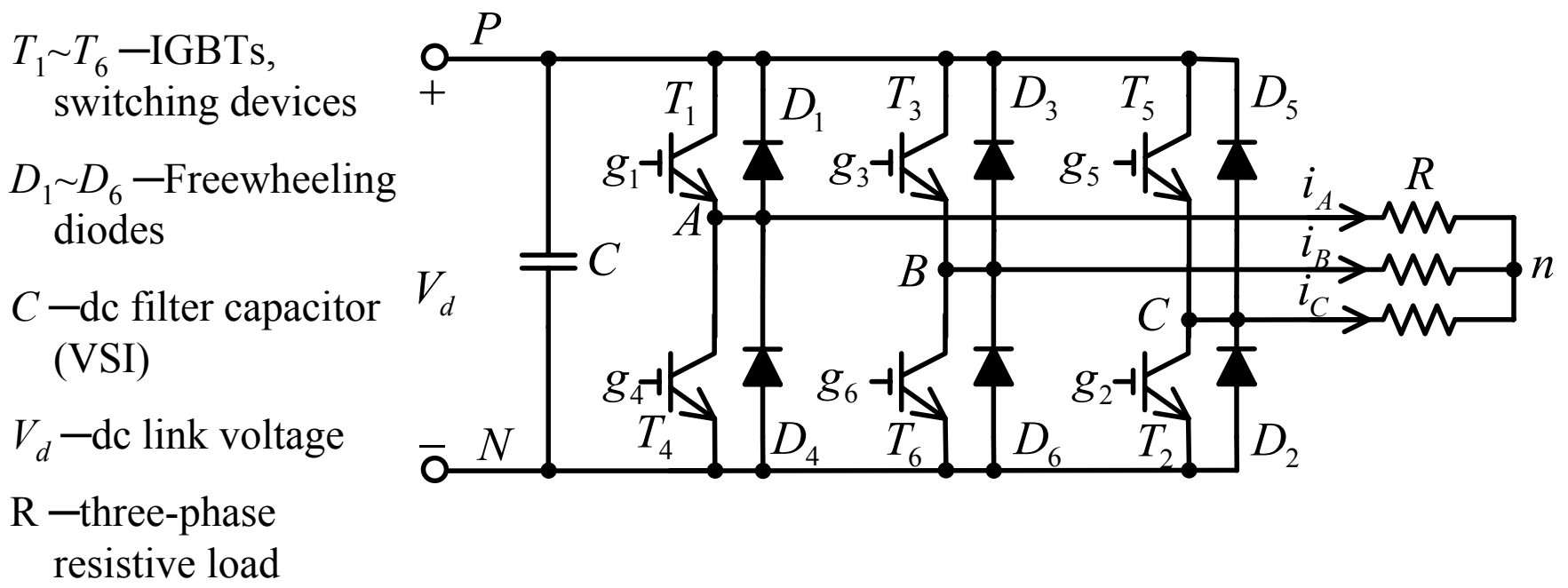
e) Lossless devices (ideal device)

$$P_{in} = P_{load} = 1.695\text{kW}$$

$$P_{in} = V_d I_d \Rightarrow I_d = \frac{P_{load}}{V_d} = \frac{1695}{220} = 7.7\text{A}$$

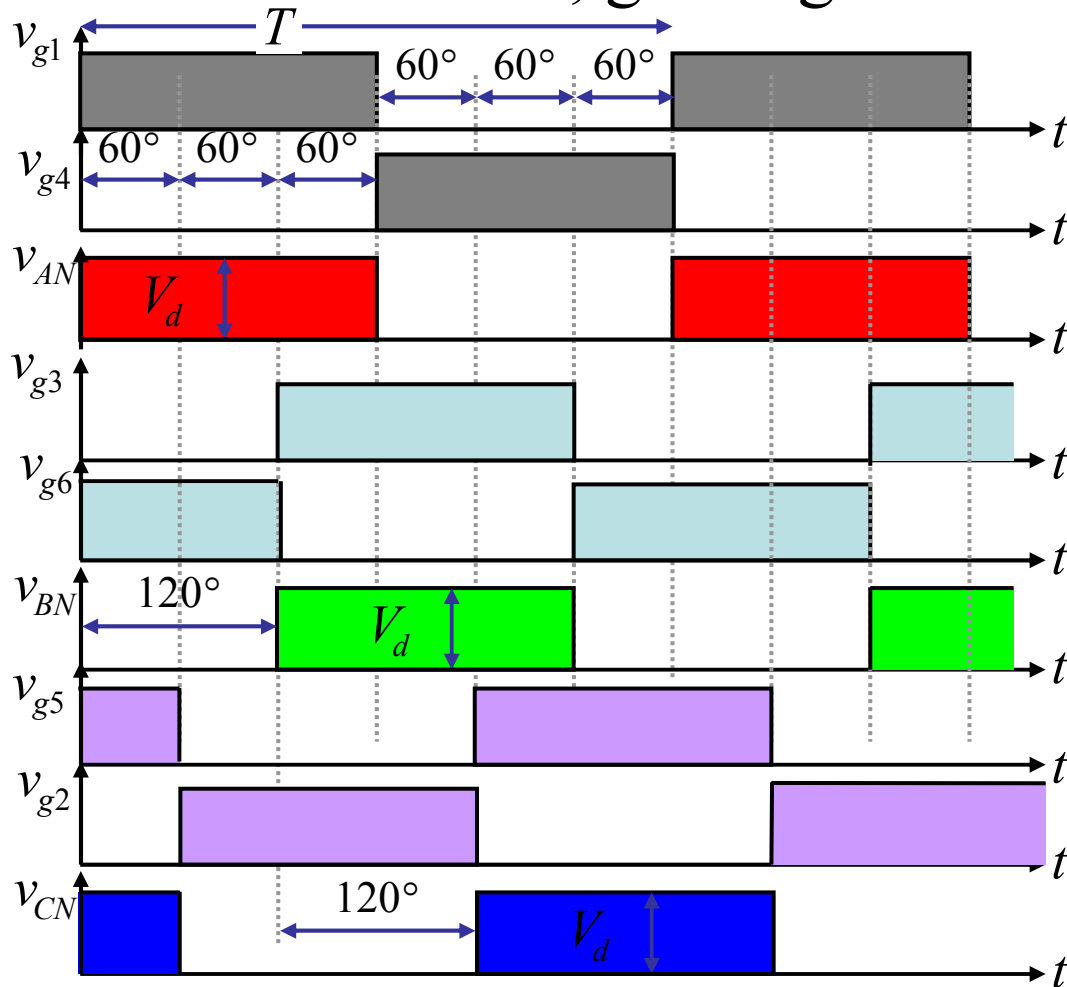
4.2 Square wave operation

- Three-phase inverter
 - Circuit diagram



4.2 Square wave operation

– Waveforms, gate signals



Note:

(1) v_{g1} and v_{g4} are complimentary

v_{g3} and v_{g6} are complimentary

v_{g5} and v_{g2} are complimentary

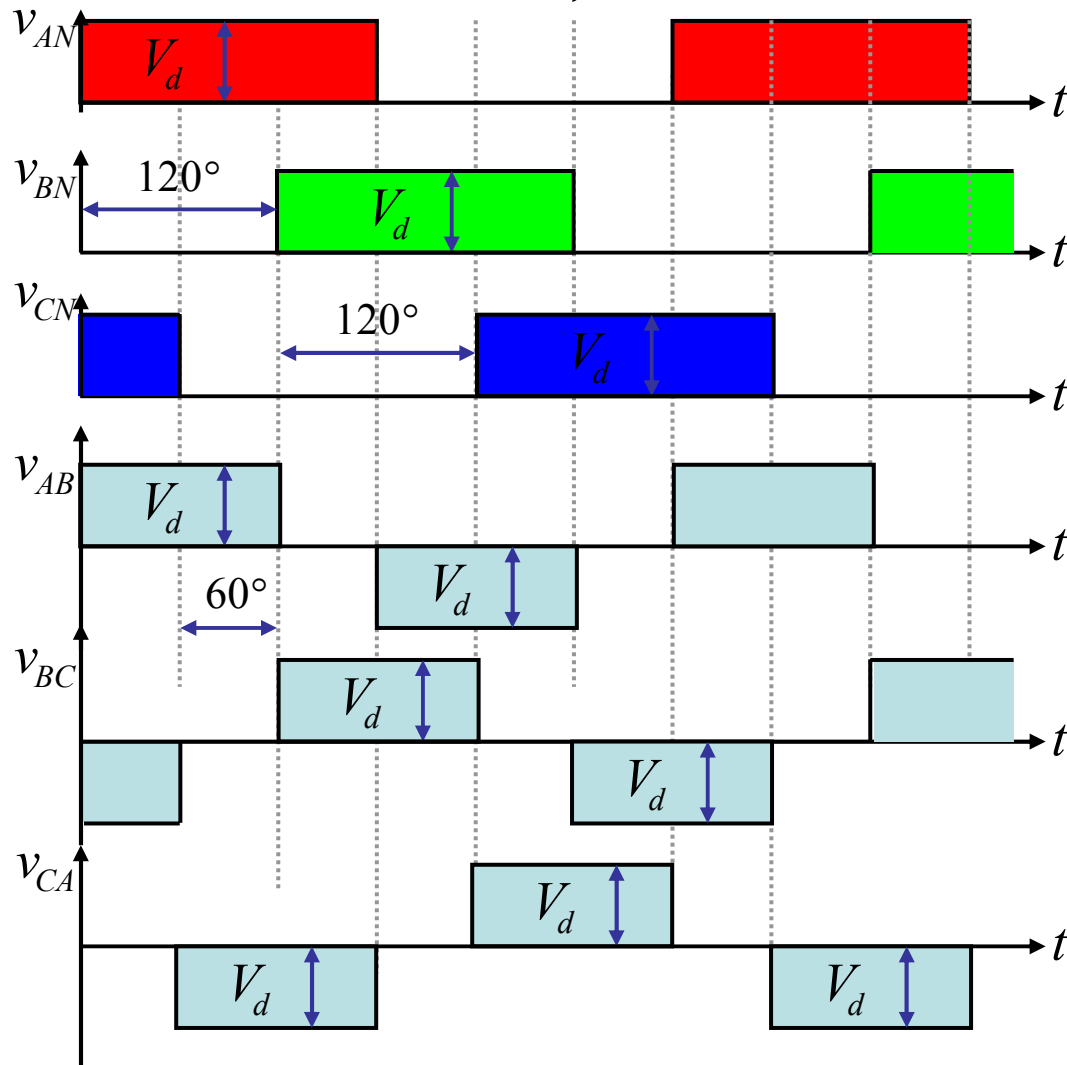
(2) V_{AN} is controlled by v_{g1} . They have the same wave shape

V_{BN} is controlled by v_{g3} . They have the same wave shape

V_{CN} is controlled by v_{g5} . They have the same wave shape

4.2 Square wave operation

– Waveforms, line-to-line voltages

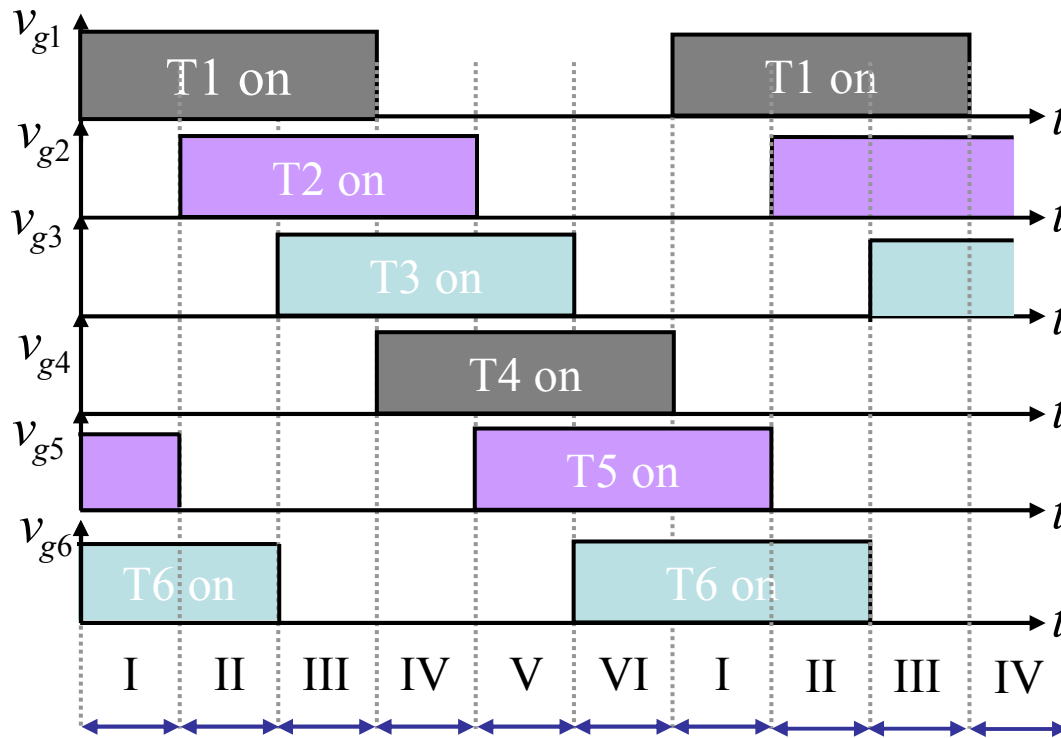


Note:

- (1) V_{AN} leads V_{BN} 120°
- V_{BN} leads V_{CN} 120°
- V_{CN} leads V_{AN} 120°
- (2) V_{AB} leads V_{BC} 120°
- V_{BC} leads V_{CA} 120°
- V_{CA} leads V_{AB} 120°
- (3) V_{AN} lags V_{AB} 30°
- V_{BN} lags V_{BC} 30°
- V_{CN} lags V_{CA} 30°

4.2 Square wave operation

– Switching sequence



Note:

Period I: T5, T6 and T1 on

Period II: T6, T1 and T2 on

Period III: T1, T2 and T3 on

Period IV: T2, T3 and T4 on

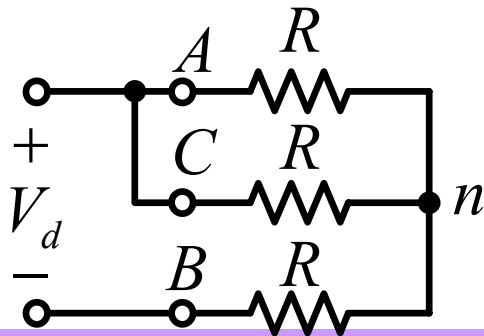
Period V: T3, T4 and T5 on

Period VI: T4, T5 and T6 on

4.2 Square wave operation

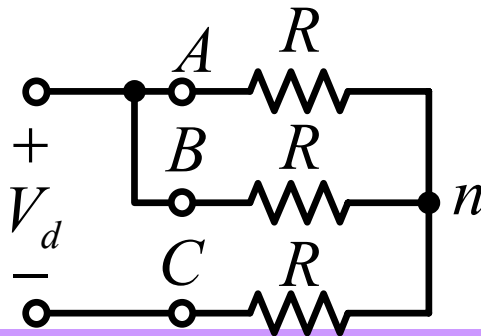
– Waveforms, phase voltage on load

Period I: T5, T6 and T1 on



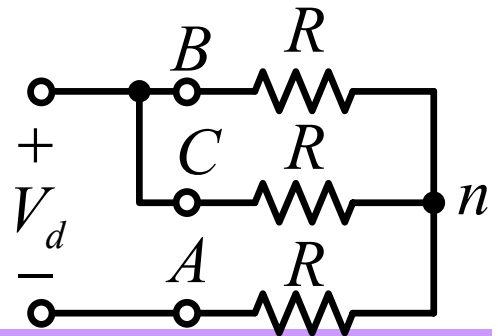
$$v_{An} = v_{Cn} = \frac{1}{3}V_d, \quad v_{Bn} = -\frac{2}{3}V_d$$

Period III: T1, T2 and T3 on



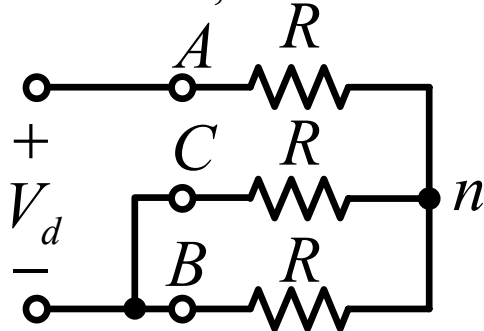
$$v_{An} = v_{Bn} = \frac{1}{3}V_d, \quad v_{Cn} = -\frac{2}{3}V_d$$

Period V: T3, T4 and T5 on



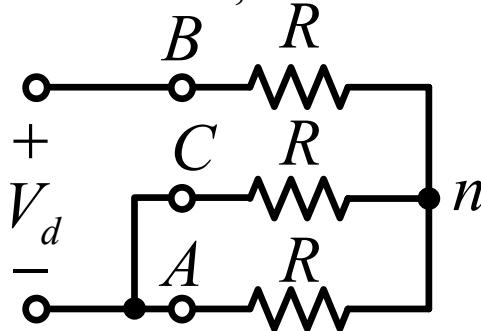
$$v_{Bn} = v_{Cn} = \frac{1}{3}V_d, \quad v_{An} = -\frac{2}{3}V_d$$

Period II: T6, T1 and T2 on



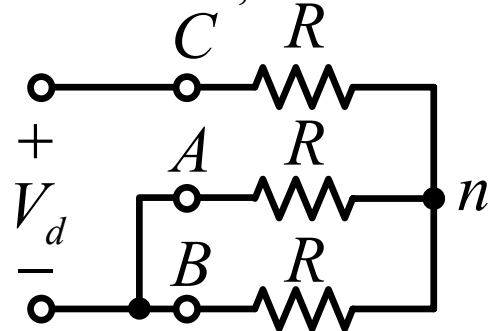
$$v_{An} = \frac{2}{3}V_d, \quad v_{Bn} = v_{Cn} = -\frac{1}{3}V_d$$

Period IV: T2, T3 and T4 on



$$v_{Bn} = \frac{2}{3}V_d, \quad v_{An} = v_{Cn} = -\frac{1}{3}V_d$$

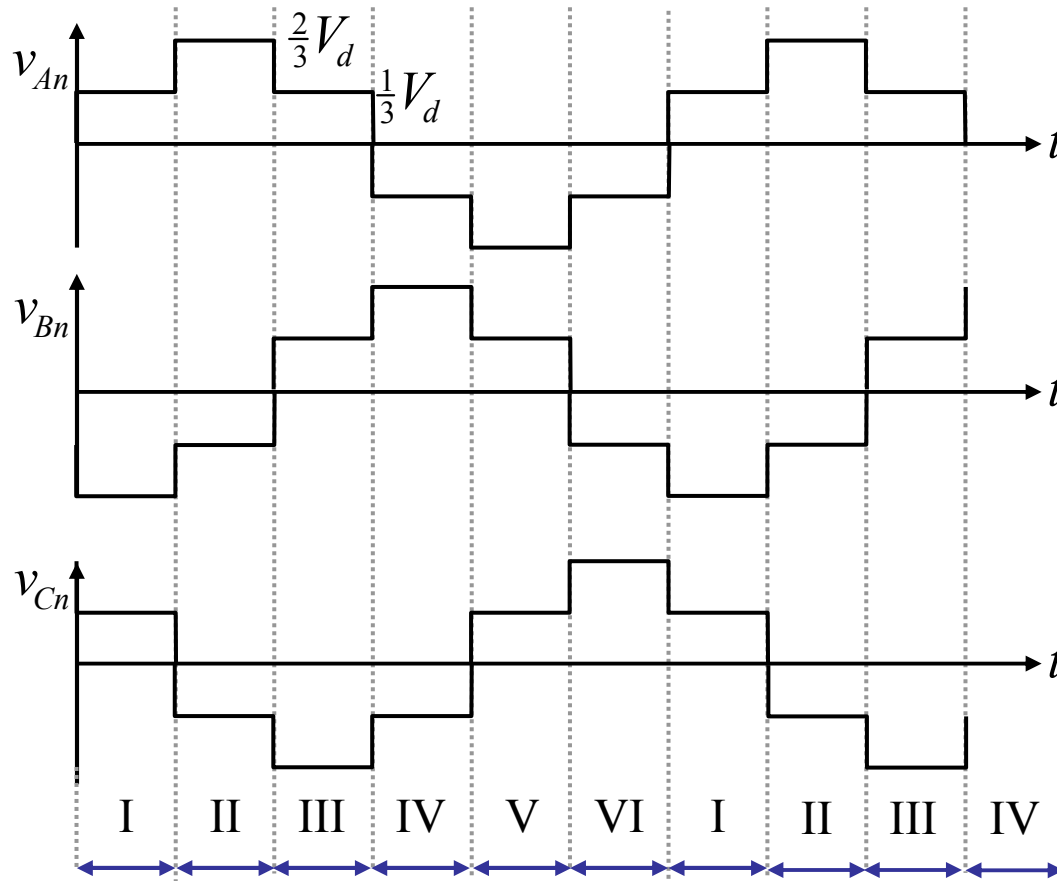
Period VI: T4, T5 and T6 on



$$v_{Cn} = \frac{2}{3}V_d, \quad v_{An} = v_{Bn} = -\frac{1}{3}V_d$$

4.2 Square wave operation

– Waveforms, phase voltage on load



Note:

$$V_{An} \text{ lags } V_{AB} 30^\circ$$

$$V_{Bn} \text{ lags } V_{BC} 30^\circ$$

$$V_{Cn} \text{ lags } V_{CA} 30^\circ$$

4.2 Square wave operation

– Analysis, line-to-line voltage

- rms value

$$V_{LL,rms} = \sqrt{\frac{1}{T} \int_0^T v_o^2 dt} = \sqrt{\frac{2}{3}} V_d = 0.816V_d$$

- Fourier series

$$\begin{aligned} v_{AB}(t) &= \sum_{n=1,3,5,\dots}^{\infty} \frac{4V_d}{n\pi} \sin \frac{n\pi}{2} \sin \frac{n\pi}{3} \sin \left(n\omega t + \frac{n\pi}{6} \right) \\ &= \frac{2\sqrt{3}V_d}{\pi} \left[\sin \left(\omega t + \frac{\pi}{6} \right) - \frac{1}{5} \sin \left(5\omega t + \frac{5\pi}{6} \right) - \frac{1}{7} \sin \left(7\omega t + \frac{7\pi}{6} \right) + \dots \right] \end{aligned}$$

- rms value of fundamental component

$$V_{AB1,rms} = \frac{2\sqrt{3}V_d}{\pi} / \sqrt{2} = \frac{\sqrt{6}}{\pi} V_d = 0.78V_d$$

- triple - order harmonics

$$V_{AB3,rms} = \frac{1}{\sqrt{2}} \frac{4V_d}{3\pi} \sin \frac{3\pi}{3} = 0, \quad V_{AB9,rms} = \frac{1}{\sqrt{2}} \frac{4V_d}{9\pi} \sin \frac{9\pi}{3} = 0$$

- Other harmonics

$$V_{AB5,rms} = \frac{1}{\sqrt{2}} \frac{4V_d}{5\pi} \sin \frac{5\pi}{3} = 0.156V_d$$

4.2 Square wave operation

– Analysis, phase voltage

- rms value

$$V_{p,rms} = \sqrt{\frac{1}{T} \int_0^T v_o^2 dt} = \sqrt{\frac{1}{\pi} \left[\left(\frac{V_d}{3} \right)^2 \frac{\pi}{3} + \left(\frac{2V_d}{3} \right)^2 \frac{\pi}{3} + \left(\frac{V_d}{3} \right)^2 \frac{\pi}{3} \right]} = \sqrt{\frac{2}{9}} V_d = 0.471 V_d = \frac{V_{LL,rms}}{\sqrt{3}}$$

- Fourier series

$$v_{An}(t) = \sum_{n=1,3,5,\dots}^{\infty} \frac{8V_d}{3n\pi} \sin \frac{n\pi}{2} \sin \frac{n\pi}{3} \cos \frac{n\pi}{6} \sin n\omega t = \frac{2V_d}{\pi} \left(\sin \omega t + \frac{1}{5} \sin 5\omega t + \frac{1}{7} \sin 7\omega t + \dots \right)$$

- rms value of fundamental component

$$V_{An1,rms} = \frac{2V_d}{\pi} / \sqrt{2} = \frac{\sqrt{2}}{\pi} V_d = 0.45 V_d$$

- triple - order harmonics

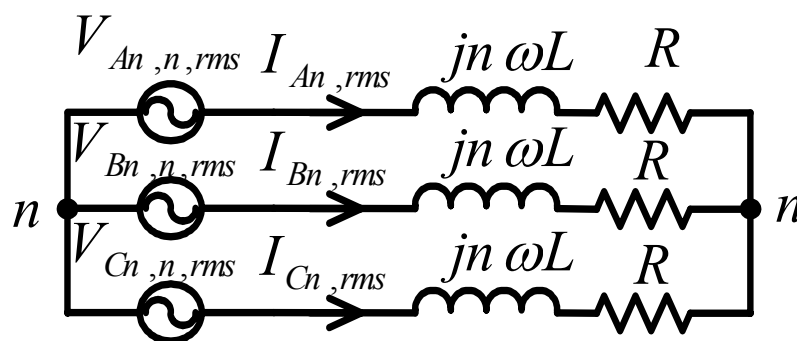
$$V_{An3,rms} = 0, \quad V_{An9,rms} = 0$$

- Other harmonics

$$V_{An5,rms} = \frac{1}{\sqrt{2}} \frac{2V_d}{5\pi} = 0.09 V_d$$

4.2 Square wave operation

- Analysis, harmonic equivalent circuit



Harmonic Equivalent circuit

- Fourier series of i_A

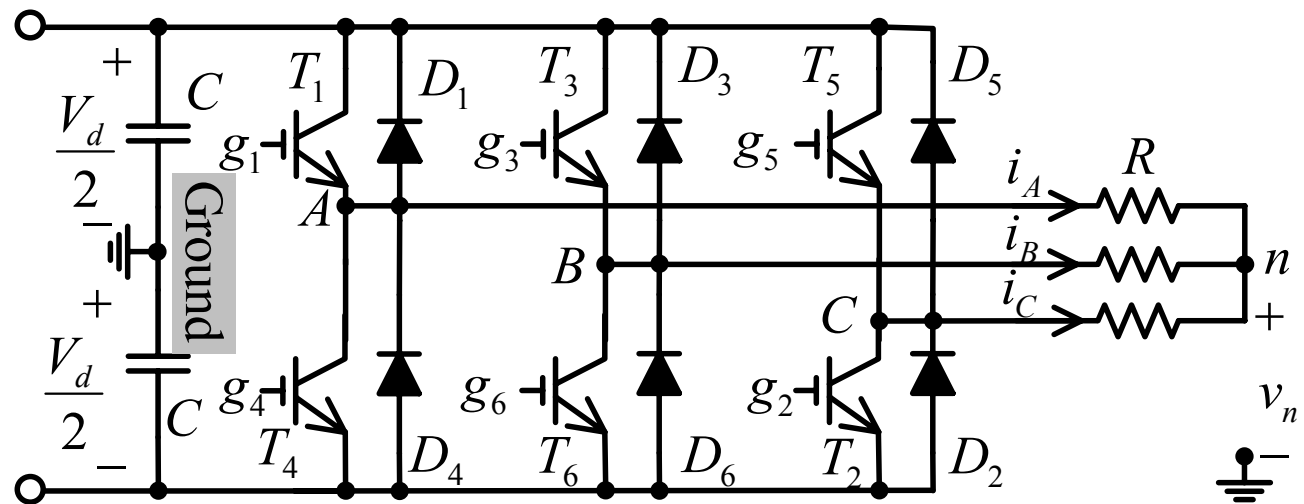
$$i_A = \sum_{n=1,3,5,7,\dots}^{\infty} \frac{4V_d}{\sqrt{3n\pi} \sqrt{R^2 + (n\omega L)^2}} \sin \frac{n\pi}{2} \sin \frac{n\pi}{3} \sin(n\omega t - \theta_n)$$

where $\theta_n = \tan^{-1} \frac{n\omega L}{R}$

4.2 Square wave operation

– Load neutral voltage

Note: In practical design, the neutral of capacitors is not grounded because of the grounded three-phase power supply.



Neutral point grounded three-phase inverter

v_n : voltage of load neutral respect to ground

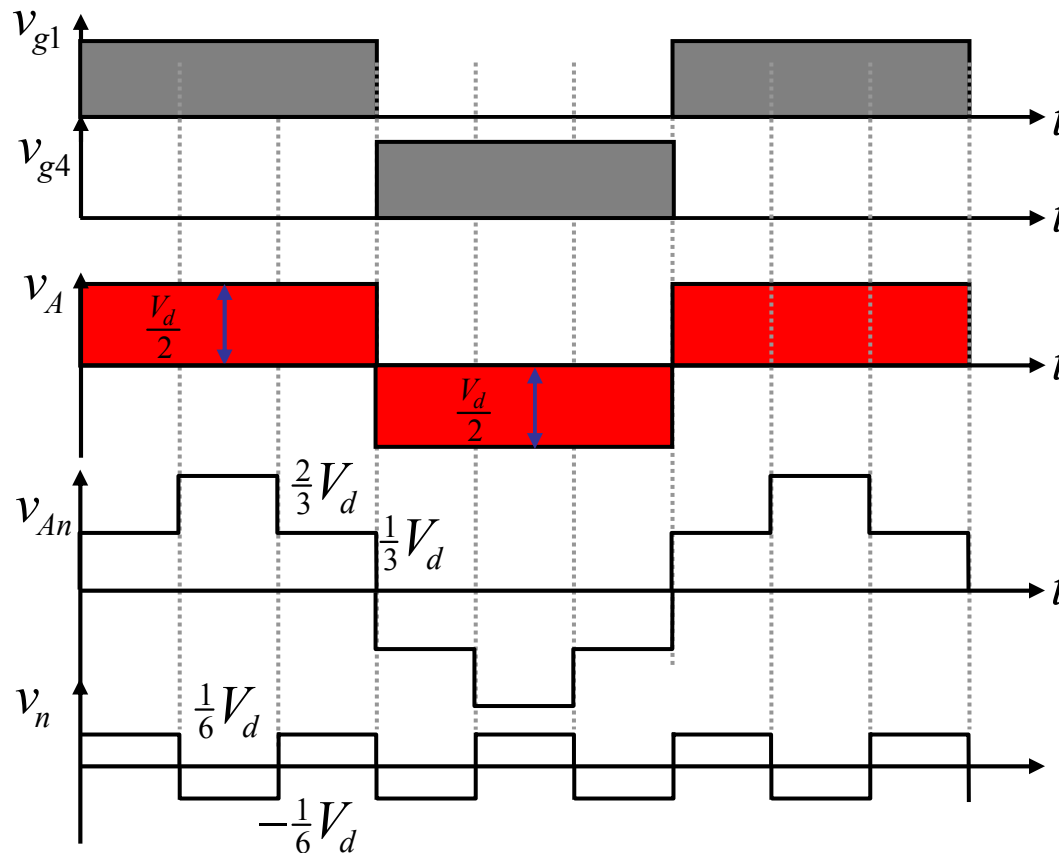
v_A : voltage of node A respect to ground

v_B : voltage of node B respect to ground

v_C : voltage of node C respect to ground

4.2 Square wave operation

– Load neutral voltage



- frequency : $3f_1$, where f_1 is fundamental frequency
- magnitude
peak - to - peak value : $\pm \frac{1}{6}V_d$
- harmonic component :
Triplen - order harmonics of v_A

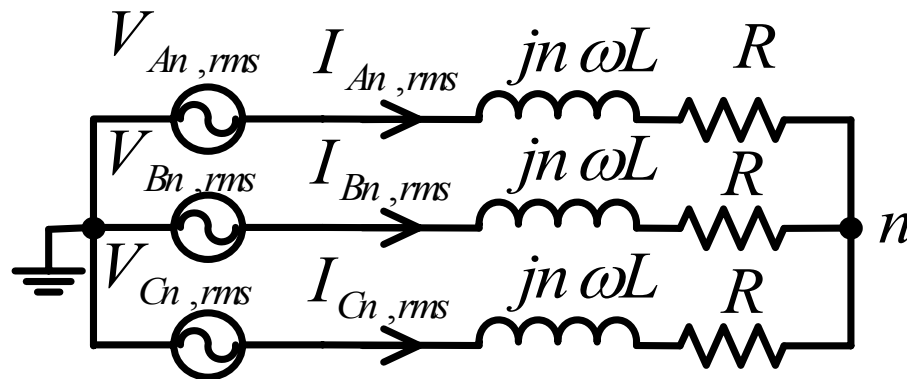
$$v_A = \frac{2V_d}{\pi} \left(\sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t + \frac{1}{7} \sin 7\omega t + \frac{1}{9} \sin 9\omega t + \dots \right)$$

$$v_{An} = \frac{2V_d}{\pi} \left(\sin \omega t + \frac{1}{5} \sin 5\omega t + \frac{1}{7} \sin 7\omega t + \dots \right)$$

$$v_n = \frac{2V_d}{\pi} \left(\frac{1}{3} \sin 3\omega t + \frac{1}{9} \sin 9\omega t + \frac{1}{15} \sin 15\omega t + \dots \right)$$

4.2 Square wave operation

– Load neutral voltage



Harmonic Equivalent circuit

- Triple-order harmonic:

voltages are in phase. But no current path between node n and ground. Then the voltage will be applied to the load neutral

- Other harmonics:

voltages are not in phase (120° lagging or leading). The three harmonic voltages are three-phase balanced voltages, then load neutral voltage is zero.

4.2 Square wave operation

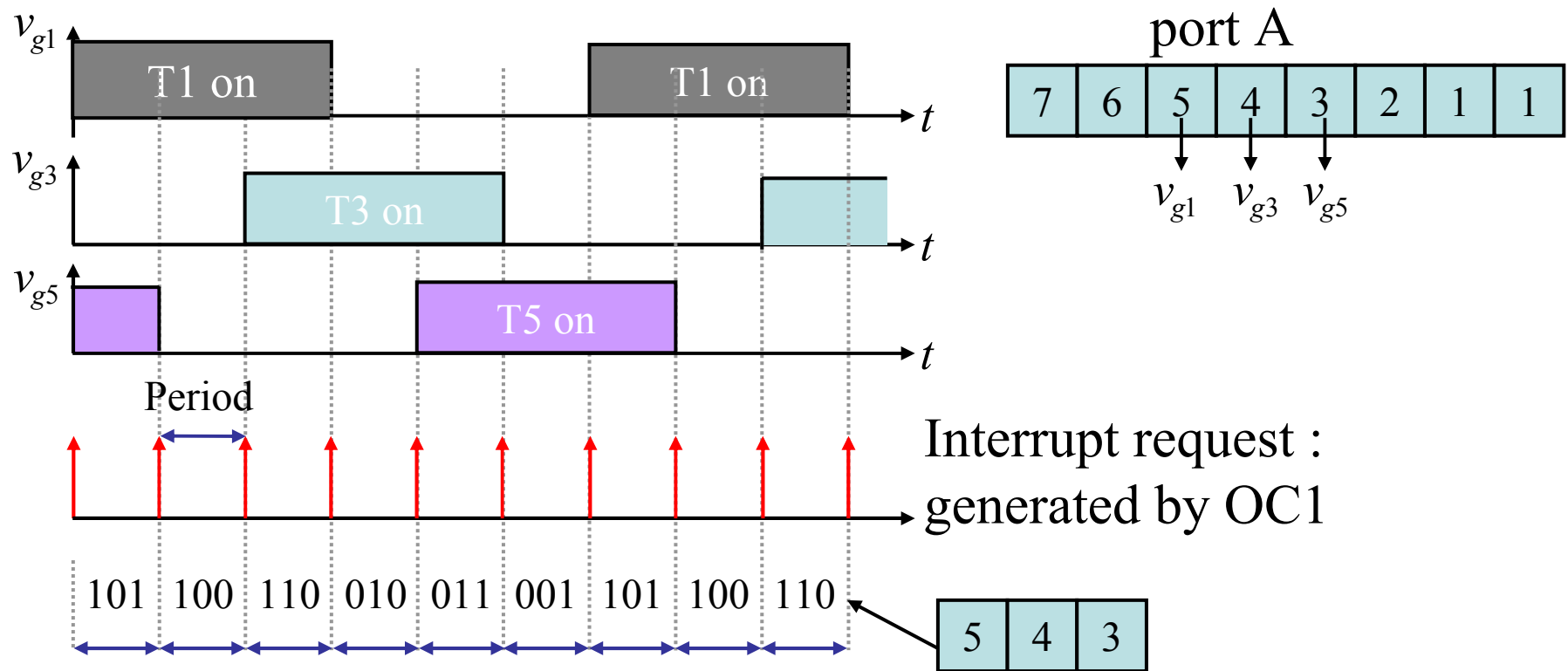
- Post analysis
 - Question 1: How to change output frequency
 - Answer: Adjust the period T of the gate signal
 - Question 2: How to change $V_{LL,rms}$?
 - Answer: Adjust dc link voltage V_d
- Feature:
 - Has low order harmonic, 5th, 7th, 13th, 15th
 - Voltage THD is high
 - How to reduce the voltage THD? Use low pass filter (LPF)
 - Suitable for high power application due to very low switching frequency
 - Usually, the multiple converters are connected with phase shift transformer to reduce the harmonics.

4.3 Project 3

- Objective: use microprocessor to generate IGBT gate signals for three-phase inverter
 - Variable frequency: 10-70Hz
 - Square wave operation (with a fixed dc voltage \rightarrow ac output voltage is fixed)
- Gate signals: three gate signals are to be generated. Other three signals are obtained by logic inverters.
 - v_{g1} and v_{g4} are complimentary
 - v_{g3} and v_{g6} are complimentary
 - v_{g5} and v_{g2} are complimentary

4.2 Square wave operation

- Gate signal generation
use port A to generate the gate signals



4.2 Square wave operation

- Interrupt Service Routine

Task

- Determine with gating pattern (1,2,3,...6) should be used
- Send the gate signals (101, 100, ...) to port A
- Determine the time interval (“period”) for the next interrupt
- Clear flag and return

4.2 Square wave operation

- How to determine the “period”

$$\text{Period} = \frac{1}{6f_1} \text{ sec} = \frac{1}{6f_1} f_{E\text{-cycle}} [\# \text{ of E - clocck}] = \frac{10^6}{6f_1} [\# \text{ of E - clocck}]$$

For example :

$$f_1 = 20\text{Hz} \rightarrow \text{Period} = \frac{10^6}{6f_1} = \frac{10^6}{6 \times 20} = 8333 [\# \text{ of E - clocck}]$$

where f_1 is fundamental frequency of output

4.2 Square wave operation

- How to change frequency
 - Change period according to frequency
 - Due to the slow uP, use look-up table to calculate division

Table	FDB	16667 (10Hz)
	FDB	15152 (11Hz)
	FDB	13889 (12Hz)
	⋮	⋮
	FDB	8333 (20Hz)

- You are required to develop the full table until 70Hz

4.3 Pulse Width Modulation

- Purpose of using PWM
 - To make inverter output voltage adjustable
 - To make inverter output frequency adjustable
 - To eliminate low order harmonics
- Method
 - Change the pulse width according to the modulating waveform (sinusoidal, trapezoidal, et al)
 - Carrier waveform: triangular wave

4.3 Pulse Width Modulation

- Principle
 - Half bridge inverter

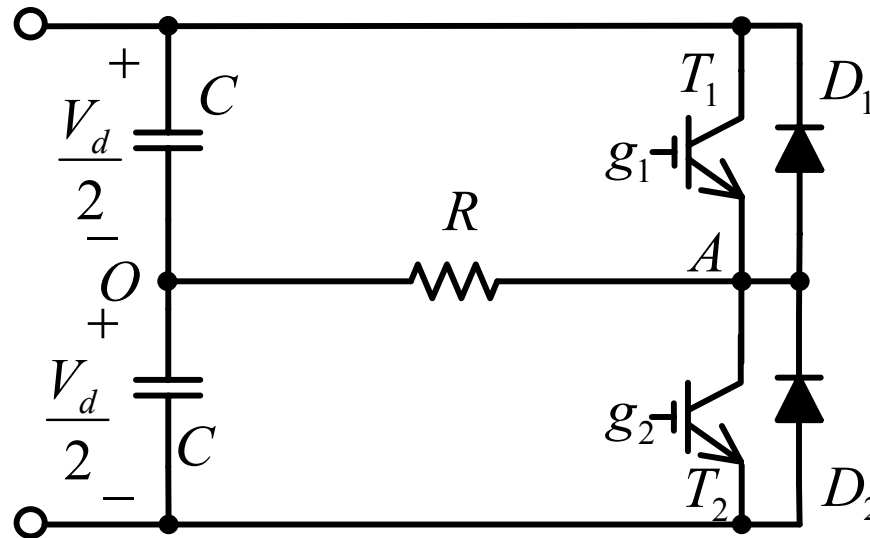
T_1, T_2 – IGBTs,
switching devices

D_1, D_2 – Freewheeling
diodes

C – dc filter capacitors
(VSI)

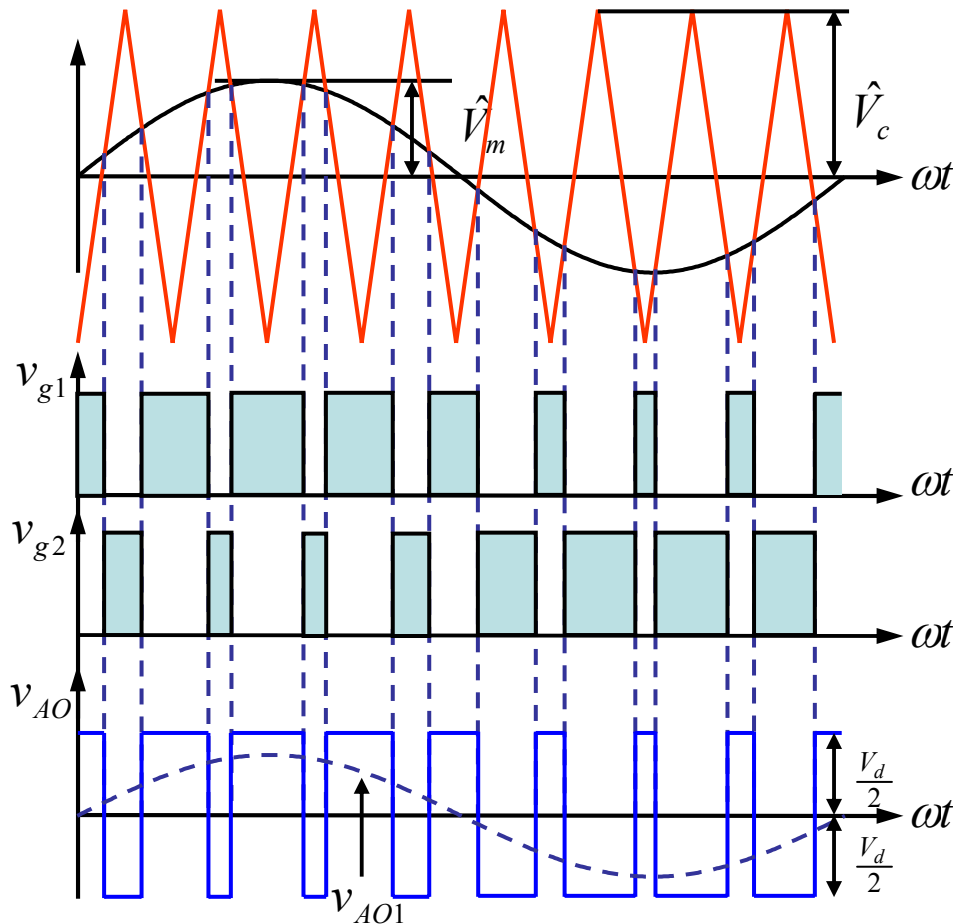
V_d – dc link voltage

R – load



4.3 Pulse Width Modulation

– Waveforms, gate signals



Note:

- (1) v_{g1} and v_{g2} are complimentary
- (2) v_m is modulating wave (sine)
 v_c is carrier wave (triangular)
- (3) When $v_m \geq v_c \rightarrow v_{g1} = \text{logic "1"}$
 $v_m < v_c \rightarrow v_{g1} = \text{logic "0"}$
- (4) When $v_{g1} = \text{logic "1"} \rightarrow T1$
 $\text{on} \rightarrow v_{AO} = +V_d/2$
When $v_{g1} = \text{logic "0"} \rightarrow T2$
 $\text{on} \rightarrow v_{AO} = -V_d/2$

4.3 Pulse Width Modulation

– Terminology

- Amplitude modulation index (ratio)

$$m_a = \frac{\hat{V}_m}{\hat{V}_c}$$

where \hat{V}_m is peak value of v_m

\hat{V}_c is peak value of v_c

usually $0 < m_a \leq 1$

- Frequency modulation index (ratio)

$$m_f = \frac{f_c}{f_m}$$

where f_m is frequency of v_m

f_c is frequency of v_c

usually $m_f > 9$

- How to find fundamental component?

use Fourier analysis

$$a_n = \frac{1}{\pi} \int_0^{2\pi} v_{AO} \cos n\omega t d\omega t \quad \text{and} \quad b_n = \frac{1}{\pi} \int_0^{2\pi} v_{AO} \sin n\omega t d\omega t$$

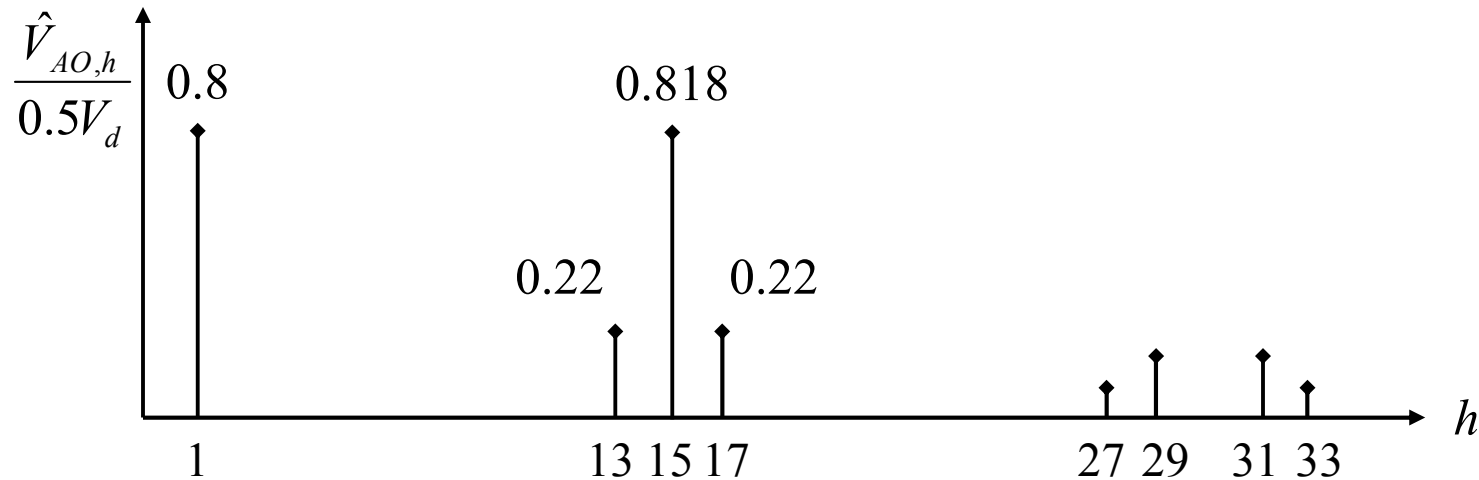
- Fundamental frequency?

Fundamental frequency = f_m

4.3 Pulse Width Modulation

– Spectrum • Given : $m_a = 0.8$, $m_f = 15$, $f_m = 60\text{Hz}$

Find spectrum of v_{AO}



Assume $V_d = 100\text{V}$, $V_{AO,1} = ?$

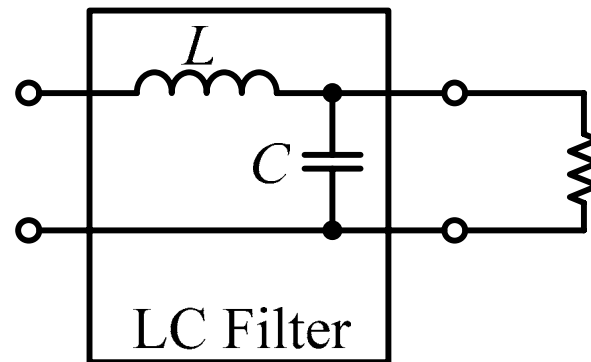
$$\frac{\hat{V}_{AO,1}}{0.5V_d} = 0.8 \rightarrow \hat{V}_{AO,1} = 0.8 \times 0.5V_d = 0.8 \times 0.5 \times 100 = 40\text{V}$$

$$V_{AO,1} = \frac{\hat{V}_{AO,1}}{\sqrt{2}} = \frac{40}{\sqrt{2}} = 28.28\text{V}$$

4.3 Pulse Width Modulation

– Features

- No low order harmonics: in the example, no 3rd, 5th, 7th, 9th, 11th order harmonics
- High order harmonics can be easily filtered out



$f \uparrow \rightarrow X_L \uparrow \rightarrow$ Blocking harmonic current
 $\rightarrow X_C \downarrow \rightarrow$ Bypassing harmonic current

4.3 Pulse Width Modulation

- How to determine harmonic components
 - Use table 8-1 on text page 207
 - Note:
 - $m_f \geq 9$, if $m_f < 9$, table 8-1 may not be used
 - Based on half bridge inverter

$h \backslash m_a$	0.2	0.4	0.6	0.8	1.0
1 st	0.2	0.4	0.6	0.8	1.0
m_f	1.242	1.15	1.006	0.818	0.601
$m_f \pm 2$	0.016	0.061	0.131	0.22	0.318
$m_f \pm 4$
$2m_f \pm 1$
$2m_f \pm 3$
$2m_f \pm 5$

}

$\hat{V}_{AO,h}$
 $0.5V_d$

4.3 Pulse Width Modulation

– Conclusions

- The fundamental component is proportional to amplitude modulation index (m_a)
- The fundamental frequency is determined by the frequency of modulating waveform (v_m)

– Questions

- Q: How to change inverter output frequency?
- A: Change the frequency of v_m (modulating frequency)
- Q: How to change inverter output voltage?
- A: Change the amplitude modulation index.

4.3 Pulse Width Modulation

– Example

• Given : Half bridge inverter, $V_d = 300\text{V}$, $m_a = 0.8$, $m_f = 39$, $f_m = 47\text{Hz}$

Find : (1) $V_{AO1,rms} = ?$

(2) Dominant harmonics, $m_f - 2$, m_f , $m_f + 2$ (rms value)

Solution :

Check $m_f = 39 > 9 \rightarrow$ Table 8 - 1 can be used

(1) fundamental component at $m_a = 0.8$

$$\frac{\hat{V}_{AO,1}}{0.5V_d} = 0.8 \rightarrow V_{AO,1} = \frac{0.8 \times 0.5 \times 300}{\sqrt{2}} = 84.85\text{V}$$

4.3 Pulse Width Modulation

– Example

- Given : Half bridge inverter, $V_d = 300\text{V}$, $m_a = 0.8$, $m_f = 39$, $f_m = 47\text{Hz}$

Find : (1) $V_{AO1,rms} = ?$

(2) Dominant harmonics, $m_f - 2$, m_f , $m_f + 2$ (rms value)

$$(2) m_f - 2 = 37^{\text{th}}, \frac{\hat{V}_{AO,37}}{0.5V_d} = 0.22 \rightarrow V_{AO,37} = \frac{0.22 \times 0.5 \times 300}{\sqrt{2}} = 23.3$$

$$m_f = 39^{\text{th}}, \frac{\hat{V}_{AO,39}}{0.5V_d} = 0.818 \rightarrow V_{AO,39} = \frac{0.818 \times 0.5 \times 300}{\sqrt{2}} = 86.7$$

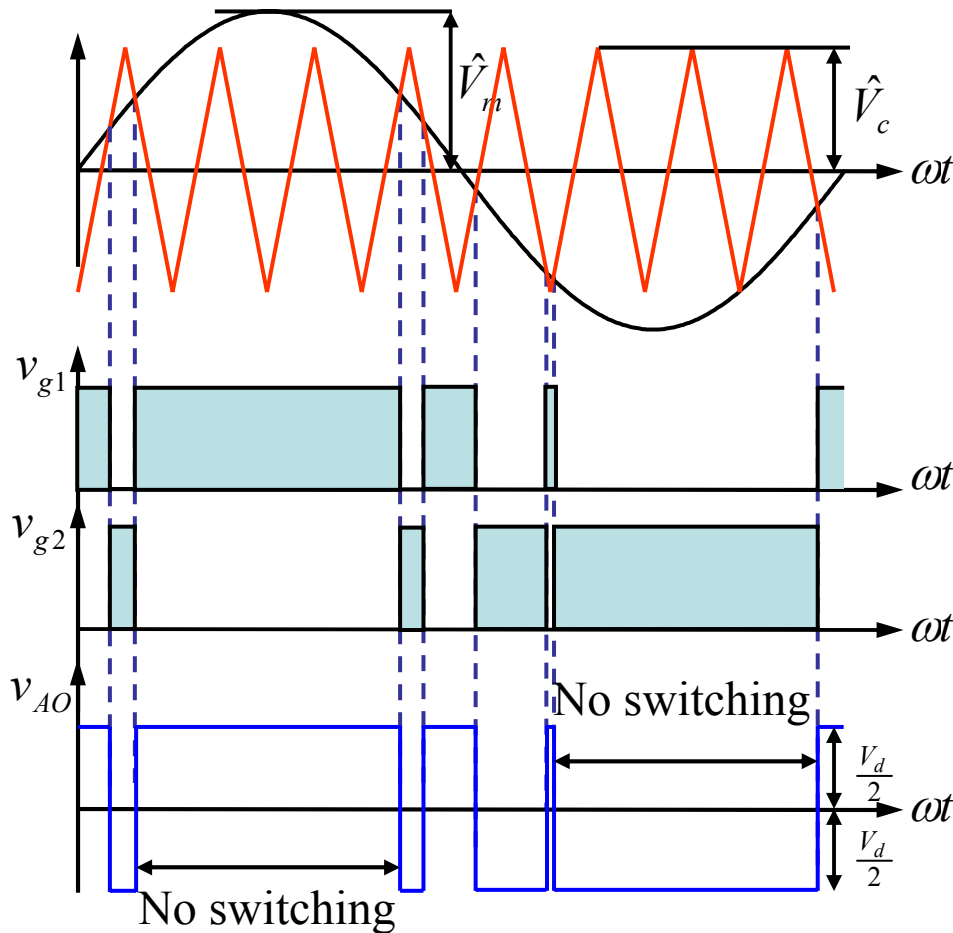
$$m_f + 2 = 41^{\text{st}}, \frac{\hat{V}_{AO,41}}{0.5V_d} = 0.22 \rightarrow V_{AO,41} = \frac{0.22 \times 0.5 \times 300}{\sqrt{2}} = 23.3$$

$$2m_f - 1 = 77^{\text{th}}, V_{AO,77} = 33.3$$

$$2m_f + 1 = 79^{\text{th}}, V_{AO,79} = 33.3$$

4.3 Pulse Width Modulation

– Over modulation

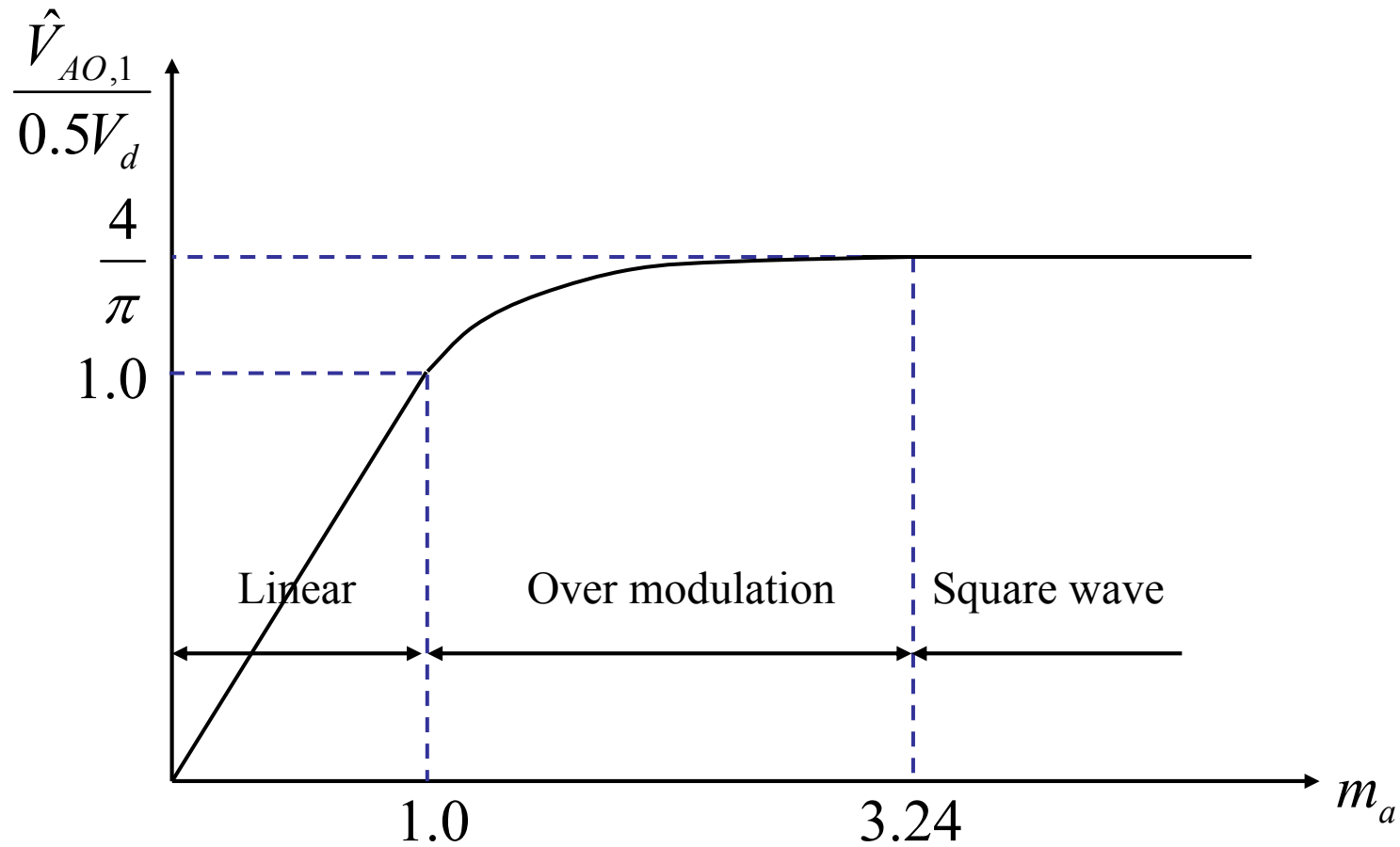


- Define : if $m_a = \frac{\hat{V}_m}{\hat{V}_c} > 1$
 → over modulation

- Waveform :
 if $m_a \gg 1$,
 v_{AO} become a squarewave

4.3 Pulse Width Modulation

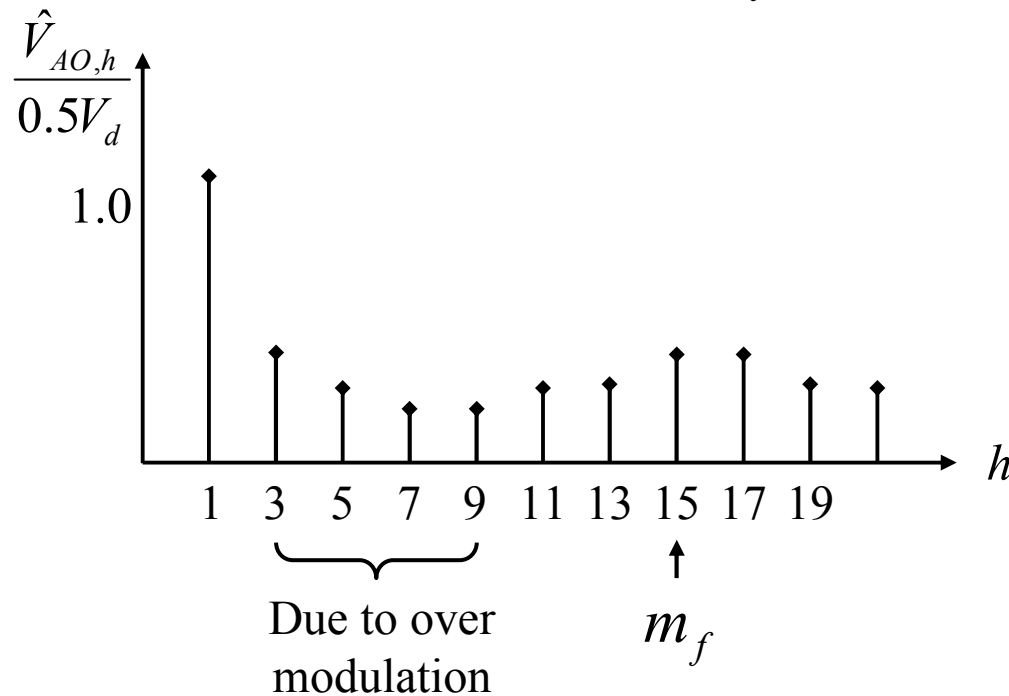
– Fundamental versus m_a



4.3 Pulse Width Modulation

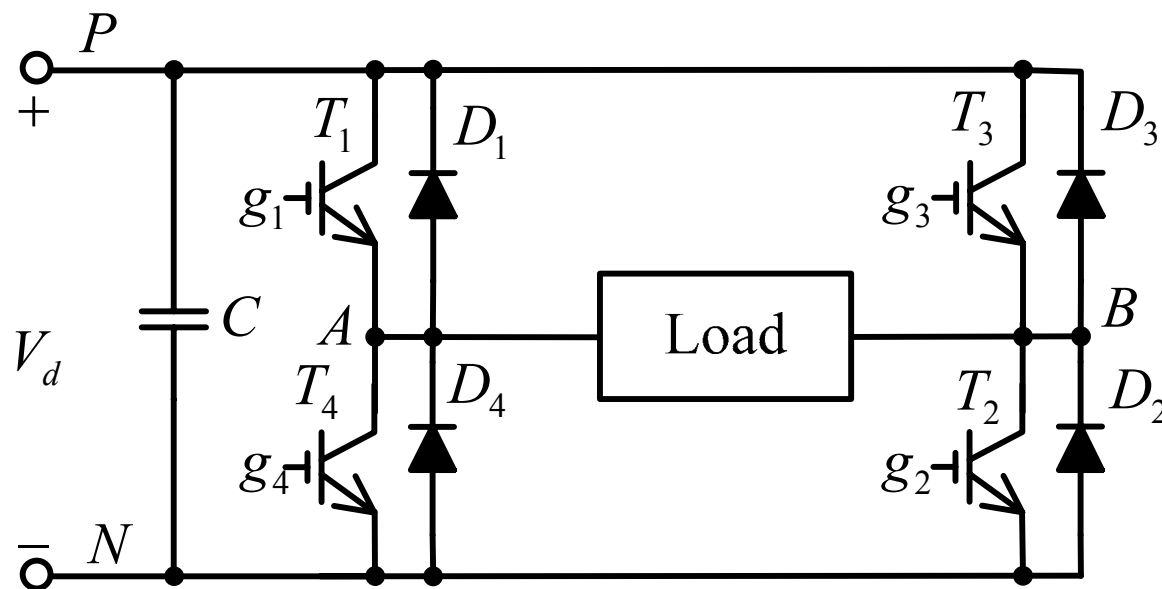
- Drawback of over modulation
 - Low order harmonic components start to appear

Example : $m_a = 2.5, m_f = 15$



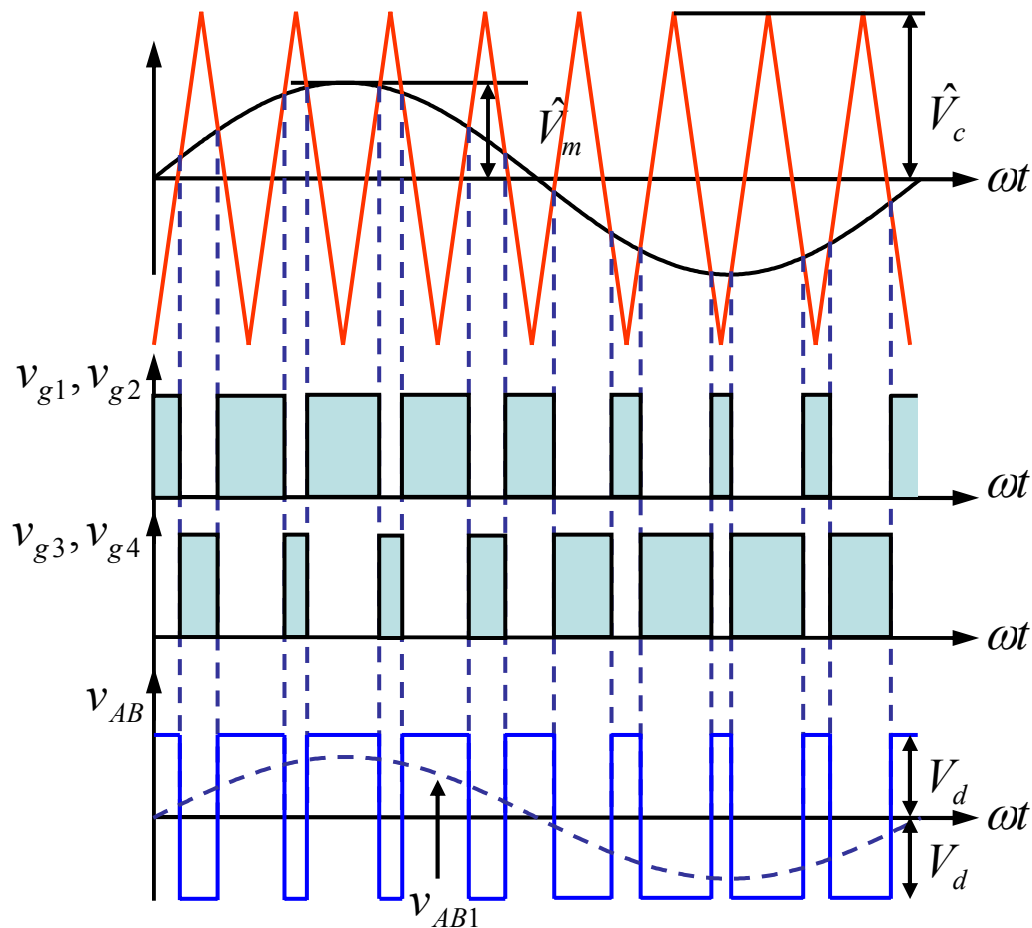
4.4 Single-phase full bridge inverter with PWM operation

- Circuit diagram



4.4 Single-phase full bridge inverter with PWM operation

- Waveforms, bipolar PWM



Note:

(1) v_{g1} and v_{g4} are complimentary signals

v_{g2} and v_{g3} are complimentary signals

(2) $v_m \geq v_c \rightarrow v_{g1} = v_{g2} = \text{logic "1"}$

$v_m < v_c \rightarrow v_{g1} = v_{g2} = \text{logic "0"}$

(3) When $v_{g1} = v_{g2} = \text{logic "1"}$ \rightarrow T1 and T2 on $\rightarrow v_{AB} = V_d$

When $v_{g1} = v_{g2} = \text{logic "0"}$ \rightarrow T3 and T4 on $\rightarrow v_{AB} = -V_d$

4.4 Single-phase full bridge inverter with PWM operation

- How to determine harmonic components
 - Use table 8-1 on text page 207

– Note:

Table 8-1 is designed for half bridge inverter

Half bridge inverter: $V_{AO,p-p} = V_d$

Full bridge inverter: $V_{AB,p-p} = 2V_d$

(Coefficients in Table 8-1) $\times 2$ for full bridge inverter

- Given : Full bridge inverter, bipolar PWM, $V_d = 300\text{V}$, $m_a = 0.8$,

$$m_f = 39, f_m = 47\text{Hz}$$

Find : (1) $V_{AB1,rms} = ?$

(2) Dominant harmonics, $m_f - 2, m_f, m_f + 2$ (rms value)

4.4 Single-phase full bridge inverter with PWM operation

- Example, bipolar PWM

Solution:

(1) Check $m_f = 39 > 9 \rightarrow$ Table 8 - 1 can be used

fundamental component at $m_a = 0.8$

$$\frac{\hat{V}_{AB,1}}{0.5V_d} = 0.8 \times 2 \rightarrow V_{AB,1} = \frac{0.8 \times 0.5 \times 300 \times 2}{\sqrt{2}} = 169.7V$$

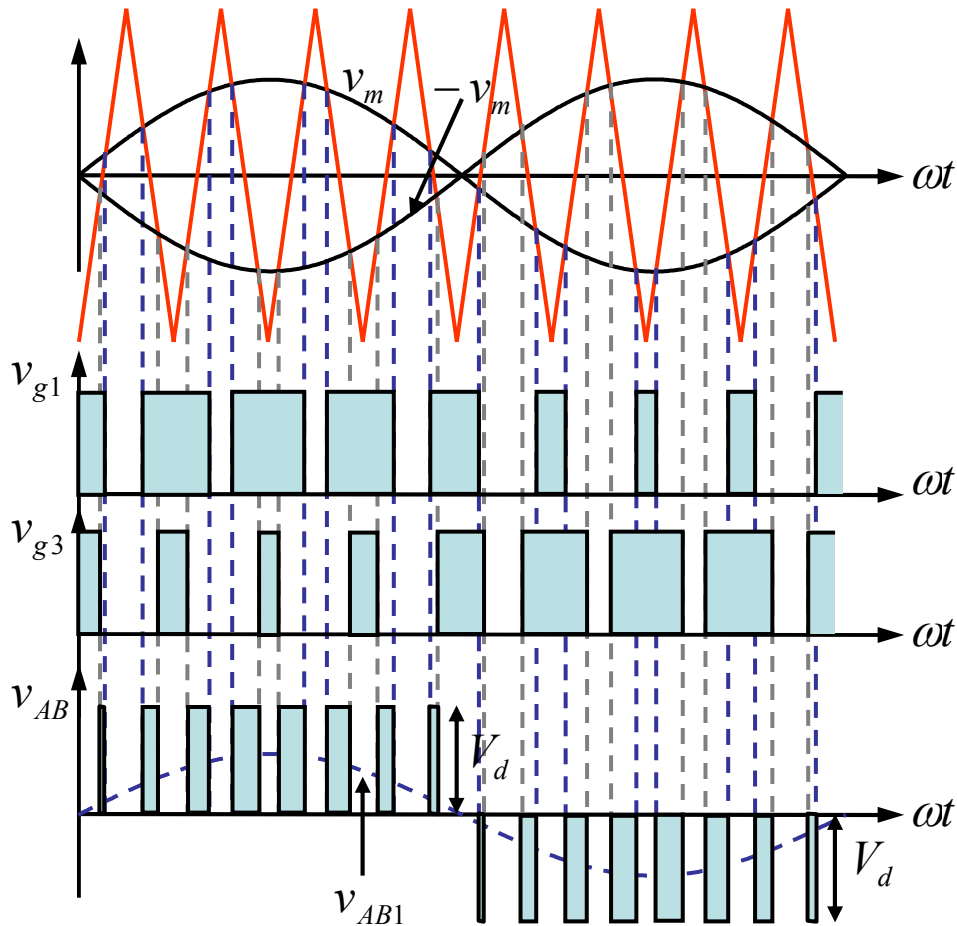
$$(2) m_f - 2 = 37^{\text{th}}, \frac{\hat{V}_{AB,37}}{0.5V_d} = 0.22 \times 2 \rightarrow V_{AB,37} = \frac{0.22 \times 0.5 \times 300 \times 2}{\sqrt{2}} = 46.7$$

$$m_f = 39^{\text{th}}, \frac{\hat{V}_{AB,39}}{0.5V_d} = 0.818 \times 2 \rightarrow V_{AB,39} = \frac{0.818 \times 0.5 \times 300 \times 2}{\sqrt{2}} = 173.5$$

$$m_f + 2 = 41^{\text{st}}, \frac{\hat{V}_{AB,41}}{0.5V_d} = 0.22 \times 2 \rightarrow V_{AB,41} = \frac{0.22 \times 0.5 \times 300 \times 2}{\sqrt{2}} = 46.7$$

4.4 Single-phase full bridge inverter with PWM operation

- Waveforms, unipolar PWM



Note:

(1) $v_m \geq v_c \rightarrow v_{g1} = \text{logic "1"}, v_{g4} = \text{logic "0"}$

$v_m < v_c \rightarrow v_{g1} = \text{logic "0"}, v_{g4} = \text{logic "1"}$

$-v_m \geq v_c \rightarrow v_{g3} = \text{logic "1"}, v_{g2} = \text{logic "0"}$

$-v_m < v_c \rightarrow v_{g3} = \text{logic "0"}, v_{g2} = \text{logic "1"}$

(2) $v_{g1} = v_{g3} = \text{logic "1"} \rightarrow T1 \text{ and } T3$
 $\text{on} \rightarrow v_{AB} = 0$

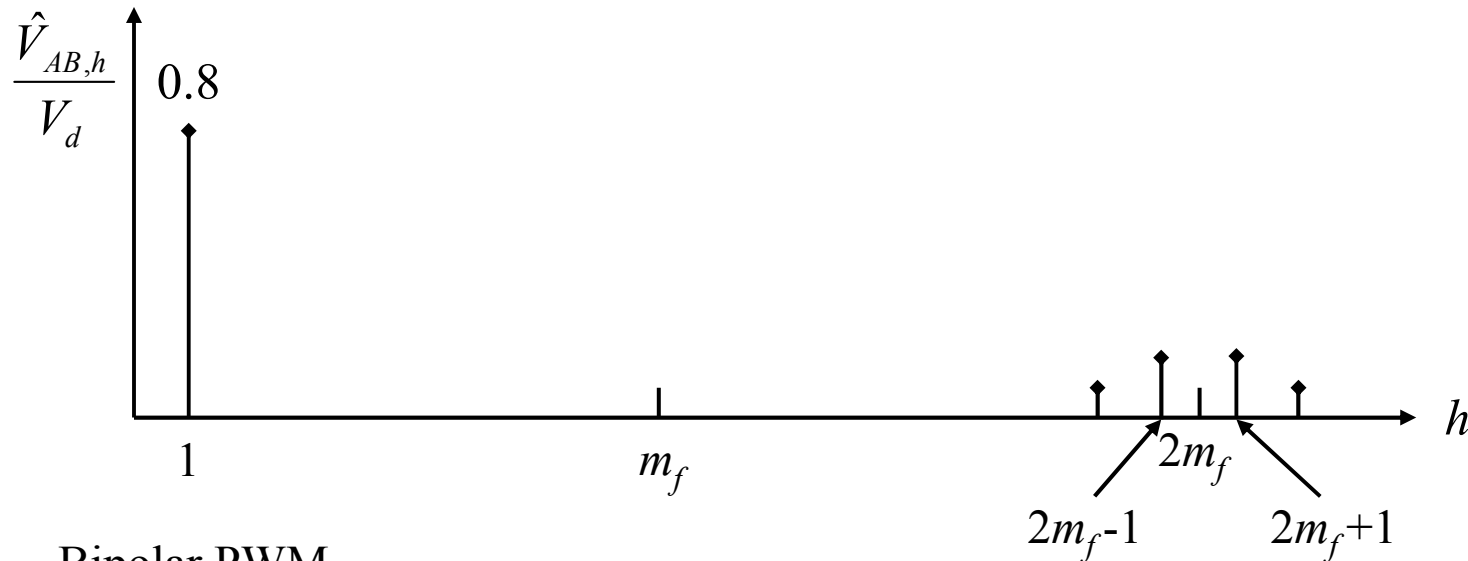
$v_{g1} = v_{g3} = \text{logic "0"} \rightarrow T4 \text{ and } T2$
 $\text{on} \rightarrow v_{AB} = 0$

$v_{g1} = \text{logic "1"}$ and $v_{g3} = \text{logic "0"}$
 $\rightarrow T1 \text{ and } T2 \text{ on} \rightarrow v_{AB} = V_d$

$v_{g1} = \text{logic "0"}$ and $v_{g3} = \text{logic "1"}$
 $\rightarrow T4 \text{ and } T3 \text{ on} \rightarrow v_{AB} = -V_d$

4.4 Single-phase full bridge inverter with PWM operation

- Spectrum for unipolar PWM



Bipolar PWM

Dominant harmonics: $m_f, m_f \pm 2, m_f \pm 4$

Unipolar PWM

Dominant harmonics: $2m_f \pm 1, 2m_f \pm 3$

4.4 Single-phase full bridge inverter with PWM operation

- Example, unipolar PWM

- Given : Full bridge inverter, unipolar PWM, $V_d = 300\text{V}$, $m_a = 0.8$,

$$m_f = 38, f_m = 47\text{Hz}$$

Find : (1) $V_{AB1,rms} = ?$

(2) Dominant harmonics, $2m_f \pm 1, 2m_f \pm 3$ (rms value)

(3) THD (consider 4 dominant harmonics only)

Solution :

(1) Check $m_f = 38 > 9 \rightarrow$ Table 8 - 1 can be used

fundamental component at $m_a = 0.8$

$$\frac{\hat{V}_{AB,1}}{0.5V_d} = 0.8 \times 2 \rightarrow V_{AB,1} = \frac{0.8 \times 0.5 \times 300 \times 2}{\sqrt{2}} = 169.7\text{V}$$

4.4 Single-phase full bridge inverter with PWM operation

- Example, unipolar PWM

(2) Four dominant harmonics, $2m_f - 3 = 73^{\text{rd}}$ (3431Hz), $2m_f - 1 = 75^{\text{th}}$ (3525Hz),
 $2m_f + 1 = 77^{\text{th}}$ (3619Hz), $2m_f + 3 = 79^{\text{th}}$ (3713Hz)

$$2m_f - 3 = 73^{\text{rd}}, \frac{\hat{V}_{AB,73}}{0.5V_d} = 0.139 \times 2 \rightarrow V_{AB,73} = \frac{0.139 \times 0.5 \times 300 \times 2}{\sqrt{2}} = 29.5$$

$$2m_f - 1 = 75^{\text{th}}, \frac{\hat{V}_{AB,75}}{0.5V_d} = 0.314 \times 2 \rightarrow V_{AB,75} = \frac{0.314 \times 0.5 \times 300 \times 2}{\sqrt{2}} = 66.6$$

$$2m_f + 1 = 77^{\text{th}}, \frac{\hat{V}_{AB,77}}{0.5V_d} = 0.314 \times 2 \rightarrow V_{AB,77} = \frac{0.314 \times 0.5 \times 300 \times 2}{\sqrt{2}} = 66.6$$

$$2m_f + 3 = 79^{\text{rd}}, \frac{\hat{V}_{AB,79}}{0.5V_d} = 0.139 \times 2 \rightarrow V_{AB,79} = \frac{0.139 \times 0.5 \times 300 \times 2}{\sqrt{2}} = 29.5$$

$$(3) V_{AB,rms} = \sqrt{V_{AB,1}^2 + V_{AB,73}^2 + V_{AB,75}^2 + V_{AB,77}^2 + V_{AB,79}^2} = \sqrt{169.7^2 + 29.5^2 + 66.6^2 + 66.6^2 + 29.5^2} = 198.5 \text{ V}$$

$$\text{THD} = \frac{\sqrt{V_{AB,rms}^2 - V_{AB,1}^2}}{V_{AB,1}} = \frac{\sqrt{198.5^2 - 169.7^2}}{169.7} = 0.61$$

4.4 Single-phase full bridge inverter with PWM operation

- Summary

- Table 8-1 can be used for

- (1) Single-phase half bridge inverter

- (2) Single-phase full bridge inverter with bipolar PWM

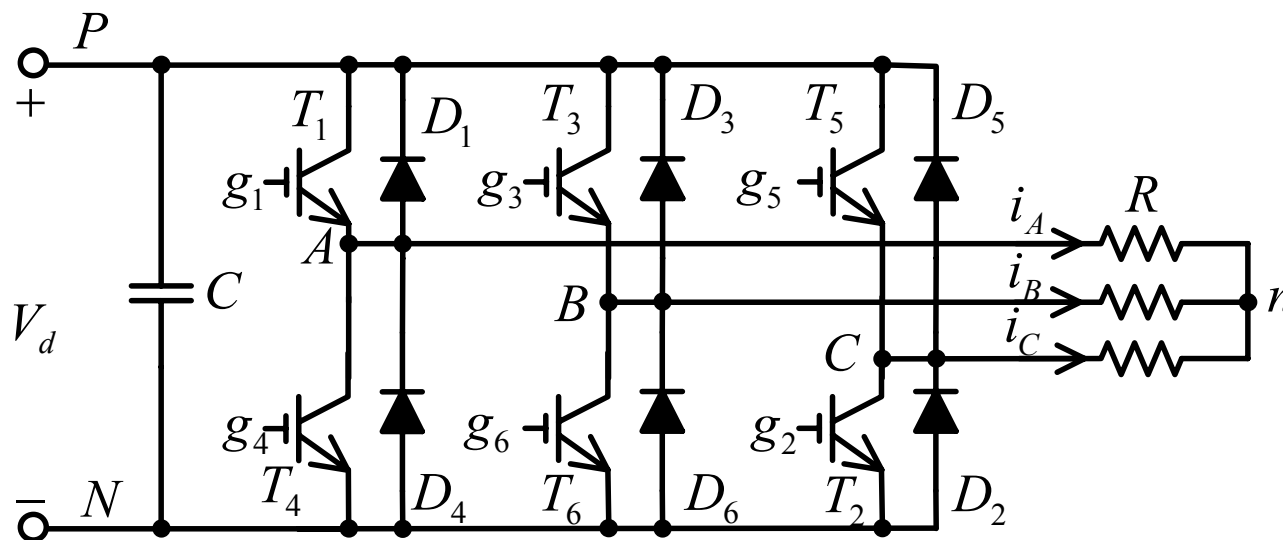
- Note: coefficient $\times 2$, dominant harmonics: $m_f, m_f \pm 2, m_f \pm 4$

- (3) Single-phase full bridge inverter with unipolar PWM

- Note: coefficient $\times 2$, dominant harmonics: $2m_f \pm 1, 2m_f \pm 3$

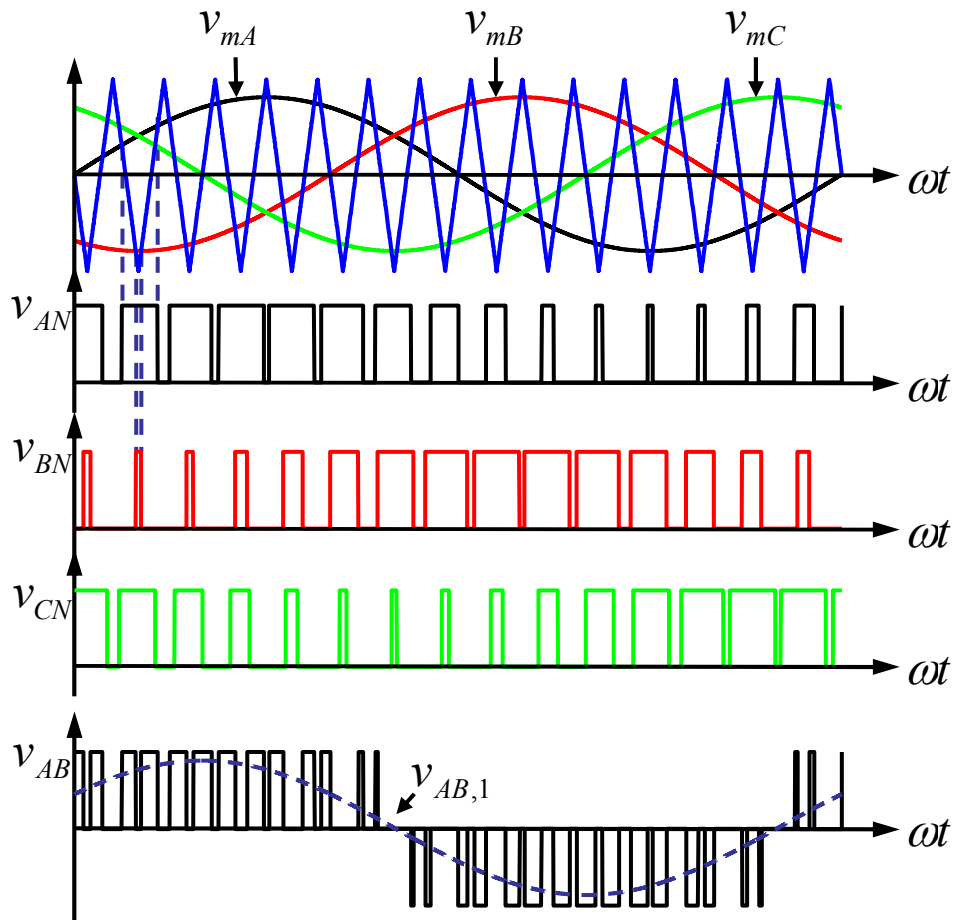
4.5 Three-phase inverter with PWM operation

- Circuit diagram



4.5 Three-phase inverter with PWM operation

- Waveforms

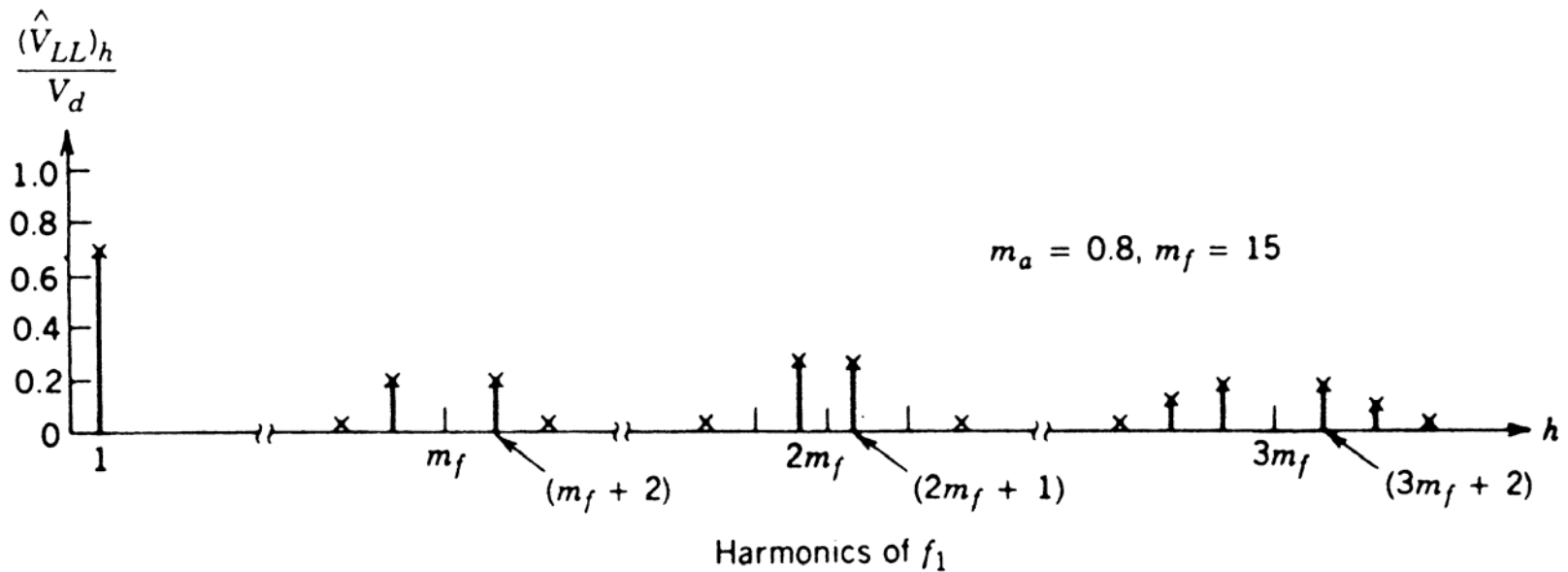


Note:

- (1) Modulating waves: three-phase sine waves (v_{mA} , v_{mB} , v_{mC}) with adjustable amplitude and frequency
- (2) Carrier wave: triangular wave, fixed amplitude, frequency may be adjusted, depends on applications
- (3) $v_{mA} \geq v_c \rightarrow T1$ on $\rightarrow v_{AN} = V_d$
 $v_{mA} < v_c \rightarrow T4$ on $\rightarrow v_{AN} = 0$
 v_{g1} and v_{g4} are complementary
- (4) $v_{mB} \geq v_c \rightarrow T3$ on $\rightarrow v_{BN} = V_d$
 $v_{mB} < v_c \rightarrow T6$ on $\rightarrow v_{BN} = 0$
 v_{g3} and v_{g6} are complementary
- (5) $v_{mC} \geq v_c \rightarrow T5$ on $\rightarrow v_{CN} = V_d$
 $v_{mC} < v_c \rightarrow T2$ on $\rightarrow v_{CN} = 0$
 v_{g5} and v_{g2} are complementary

4.5 Three-phase inverter with PWM operation

- Spectrum



No low order harmonic

4.5 Three-phase inverter with PWM operation

- How to determine harmonic components
 - Use table 8-2 on text page 228

$h \backslash m_a$	0.2	0.4	0.6	0.8	1.0
1 st	0.122	0.245	0.367	0.49	0.612
$m_f \pm 2$	0.010	0.037	0.080	0.135	0.195
$m_f \pm 4$	–	–	–	0.005	0.011
$2m_f \pm 1$	0.116	0.2	0.227	0.192	0.111
$2m_f \pm 5$	–	–	–	0.008	0.020
$3m_f \pm 2$
$3m_f \pm 4$

rms value!

$$\frac{V_{LL,h}}{V_d}$$

4.5 Three-phase inverter with PWM operation

- Example

- Given : Three - phase PWM inverter, $V_d = 240\text{V}$, $m_a = 1$,

$$m_f = 21, f_m = 100\text{Hz}$$

Find : (1) rms value of the fundamental line - to - line voltage

(2) rms value of 4100Hz and 4300Hz harmonic voltage

Solution :

- (1) Use table 8 - 2

fundamental component at $m_a = 1$

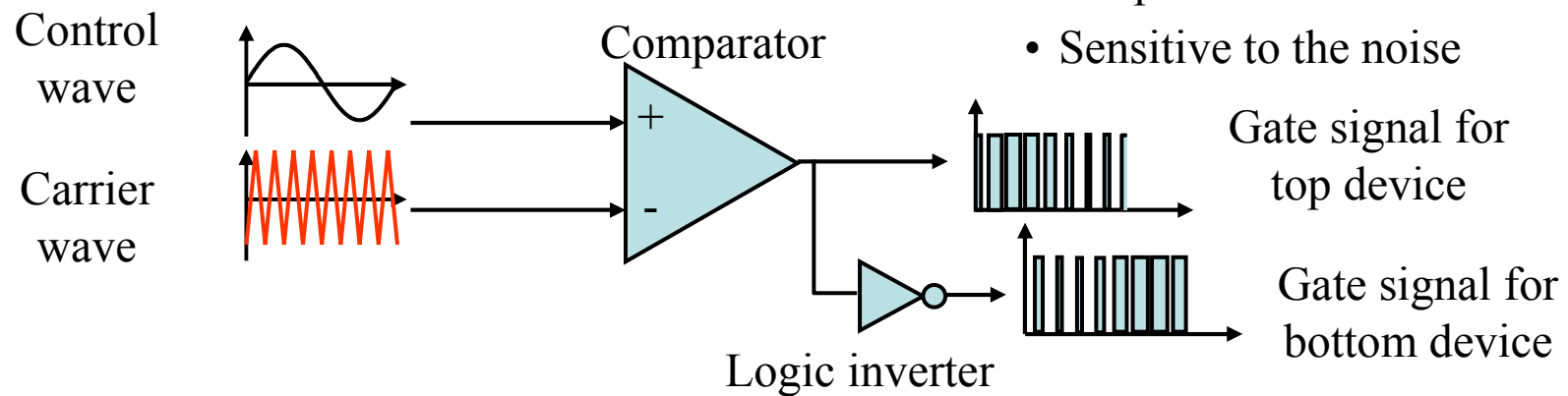
$$\frac{V_{LL,1}}{V_d} = 0.612 \rightarrow V_{LL,1} = 0.612 \times 240 = 146.9\text{V}$$

(2) 4100Hz : $h = \frac{4100}{100} = 41 = 2m_f - 1 \rightarrow \frac{V_{LL,41}}{V_d} = 0.111 \rightarrow V_{LL,41} = 0.111 \times 240 = 26.4\text{V}$

4300Hz : $h = \frac{4300}{100} = 43 = 2m_f + 1 \rightarrow \frac{V_{LL,43}}{V_d} = 0.111 \rightarrow V_{LL,43} = 0.111 \times 240 = 26.4\text{V}$

4.6 PWM Signal Generation

- Analog circuit



Features:

- Simple circuit and low cost
- Sensitive to the noise

Synchronous PWM:

The triangular waveform is synchronized with the sine wave

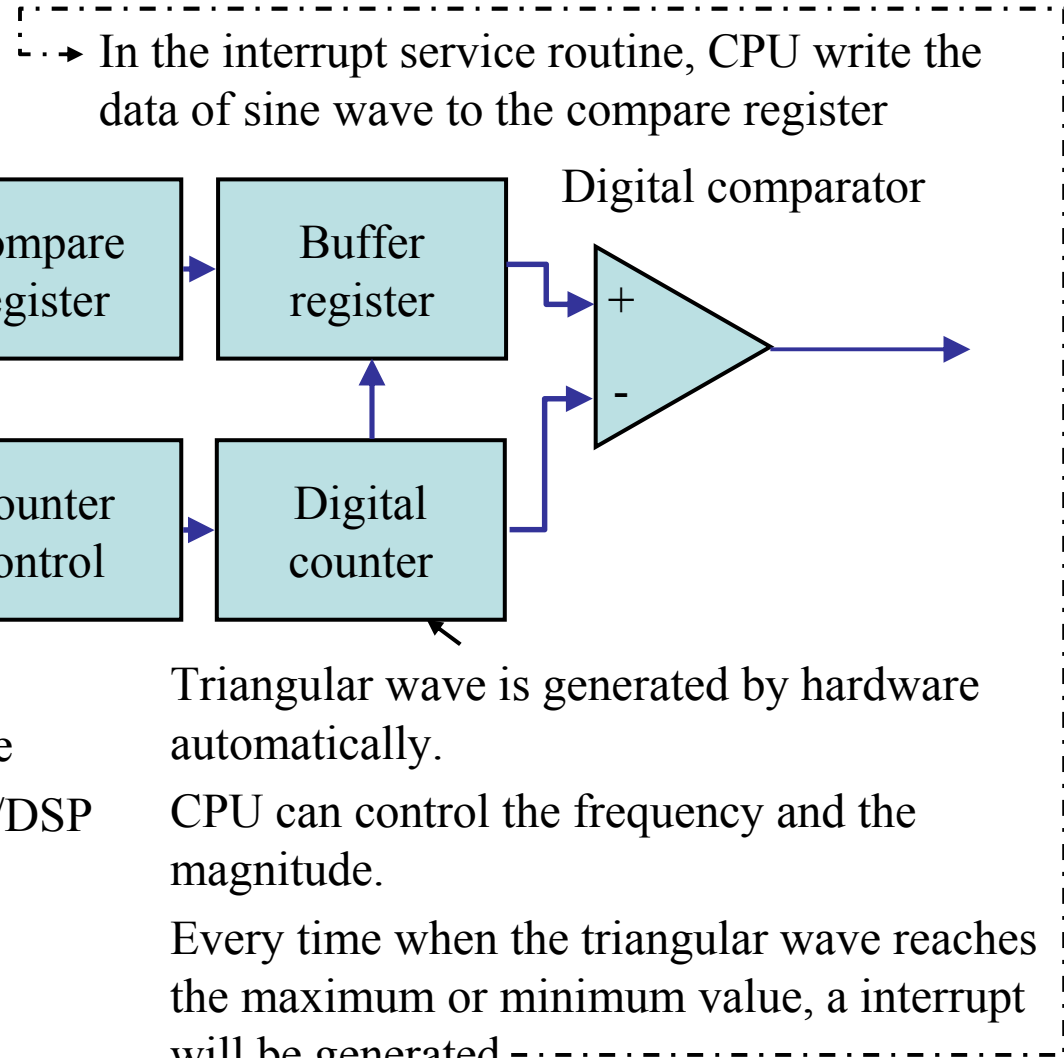
Asynchronous PWM:

The triangular waveform is NOT synchronized with the sine wave

Producing “sub-harmonics”: the frequency of the harmonic is not integer times of fundamental frequency.

4.6 PWM Signal Generation

- Digital circuit



Advantages:

- Stable, very robust to the noise
- Easy to be controlled by CPU/DSP
- Reduce the load of CPU/DSP

Disadvantages:

- Expensive hardware
- High frequency counter

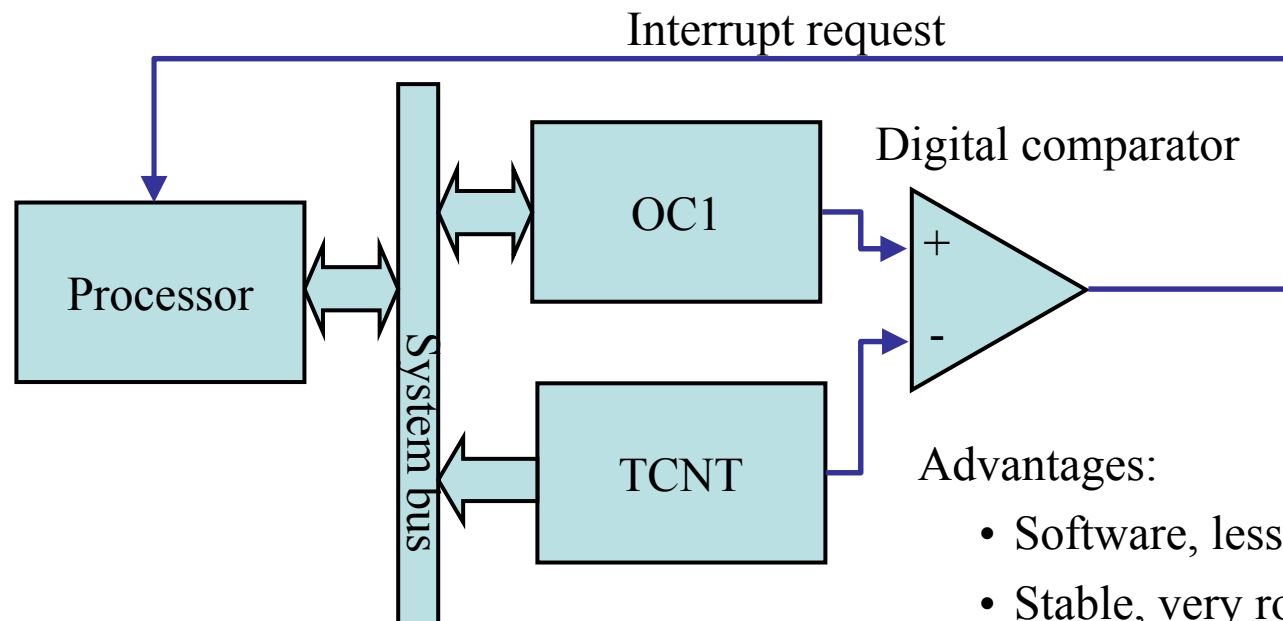
Triangular wave is generated by hardware automatically.

CPU can control the frequency and the magnitude.

Every time when the triangular wave reaches the maximum or minimum value, a interrupt will be generated.

4.6 PWM Signal Generation

- Software



Advantages:

- Software, less hardware
- Stable, very robust to the noise
- Easy to be controlled by CPU/DSP

Disadvantages:

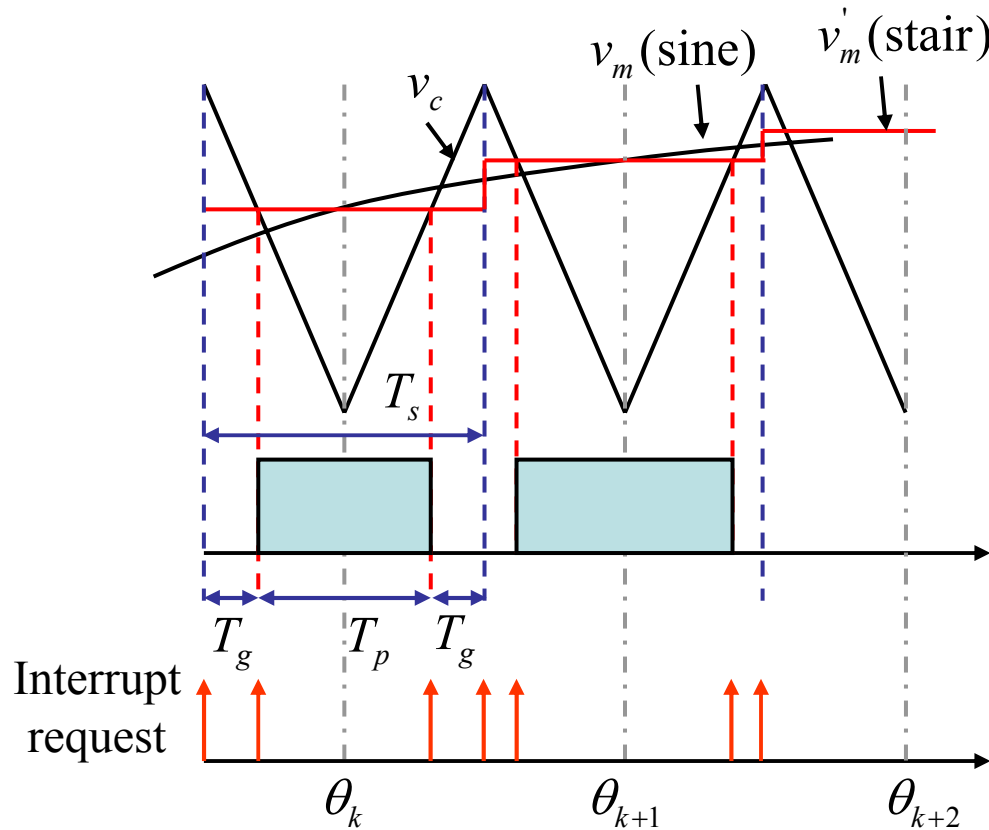
- Increase the calculation load of CPU/DSP

TCNT: Free running counter

OC1: output compare register #1

4.6 PWM Signal Generation

- Software



- use v'_m to approximate v_m for simplicity

- T_s – sample period

- $T_p = \frac{T_s}{2} [1 + m_a \sin \theta_k]$

$$T_g = \frac{T_s - T_p}{2}$$

- $\theta_{k+1} = \theta_k + \Delta\theta$

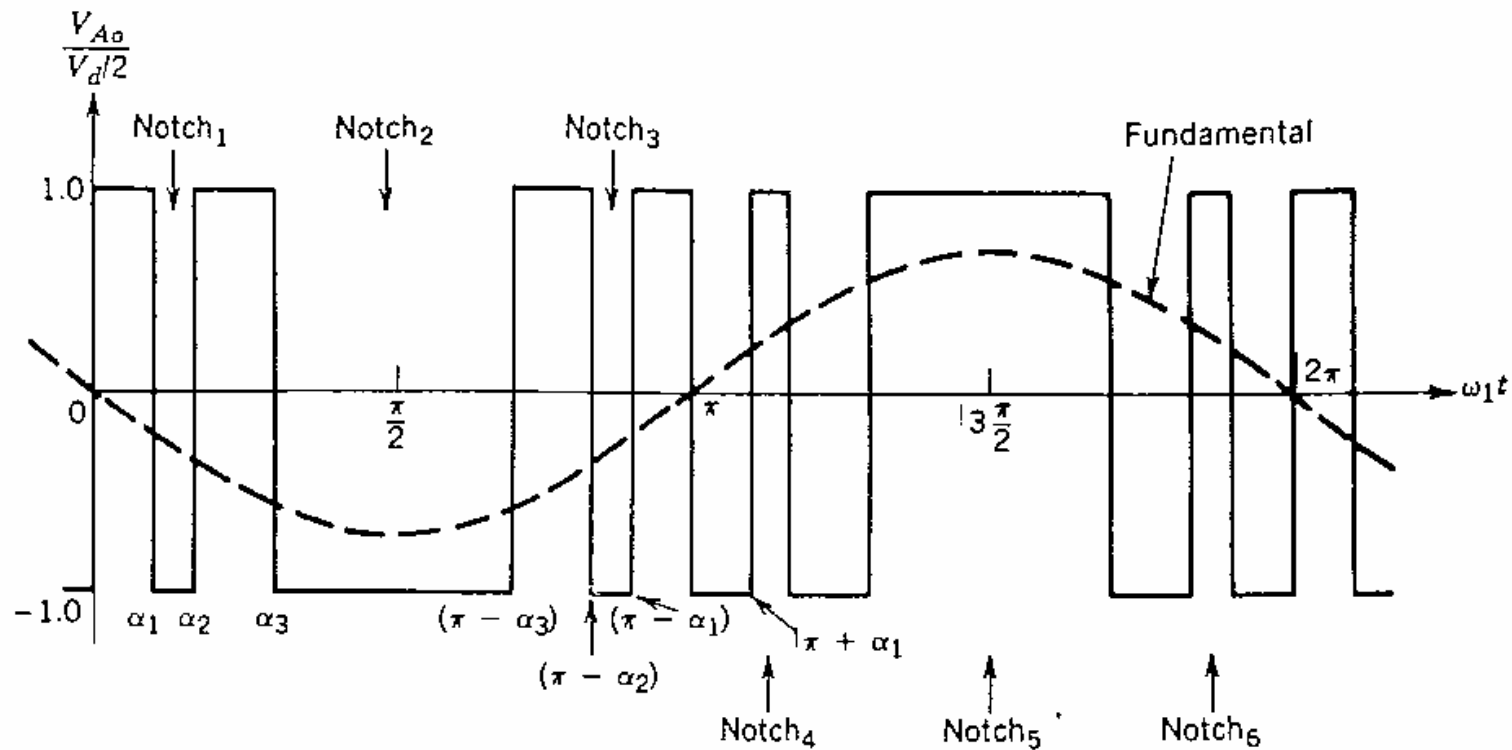
$$\Delta\theta = \omega \cdot \Delta t$$

4.6 PWM Signal Generation

- Software programming
 - At t_1 , the number in free running counter (TCNT) matches the number of the output compare register (OC1), an interrupt request will be made.
 - Interrupt service routine
 - Determine the type of interval (gap, or pulse)
 - Set the output pin to logic “0” for gap interval or logic “1” for pulse interval.
 - Calculate the time interval for following T period
 - Clear the interrupt flag and return
 - If the calculation in the interrupt service routine can not be completed during T_g , the program will crash. What shall we do?

4.7 Selected Harmonic Elimination Techniques (SHE)

- Purpose:
 - To eliminate a number of unwanted low order harmonics
 - To control the fundamental output voltage
- Waveforms



4.7 Selected Harmonic Elimination Techniques (SHE)

- Waveforms

- Three independent switching angles: α_1 , α_2 , and α_3
 - Given α_1 , α_2 , and α_3 , the other switching angles are determined due to symmetrical
- α_1 , α_2 , and α_3 are used to
 - Eliminate two harmonics (usually 5th and 7th), and
 - To control the fundamental output voltage
- α_1 , α_2 , and α_3 should be pre-calculated (offline calculation)
 - Use numerical iteration methods to solve a set of nonlinear equations.

4.7 Selected Harmonic Elimination Techniques (SHE)

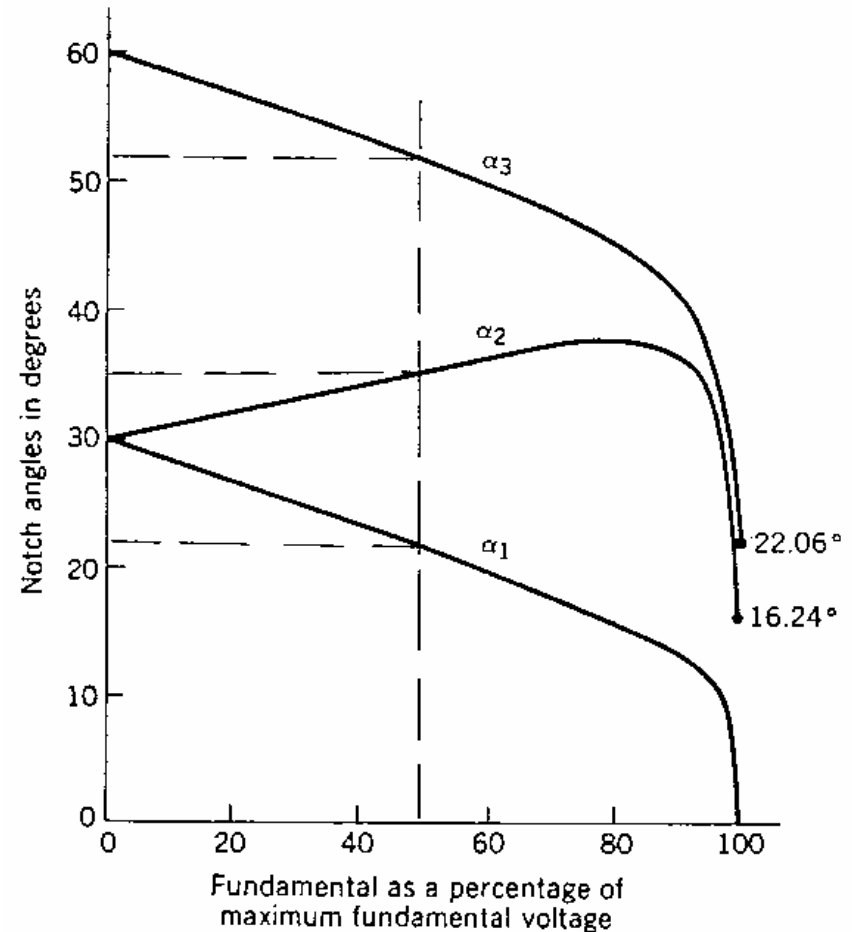
- Calculations results

Example:

At 50% of maximum fundamental voltage, $\alpha_1=22^\circ$, $\alpha_2=36^\circ$ and $\alpha_3=52^\circ$.

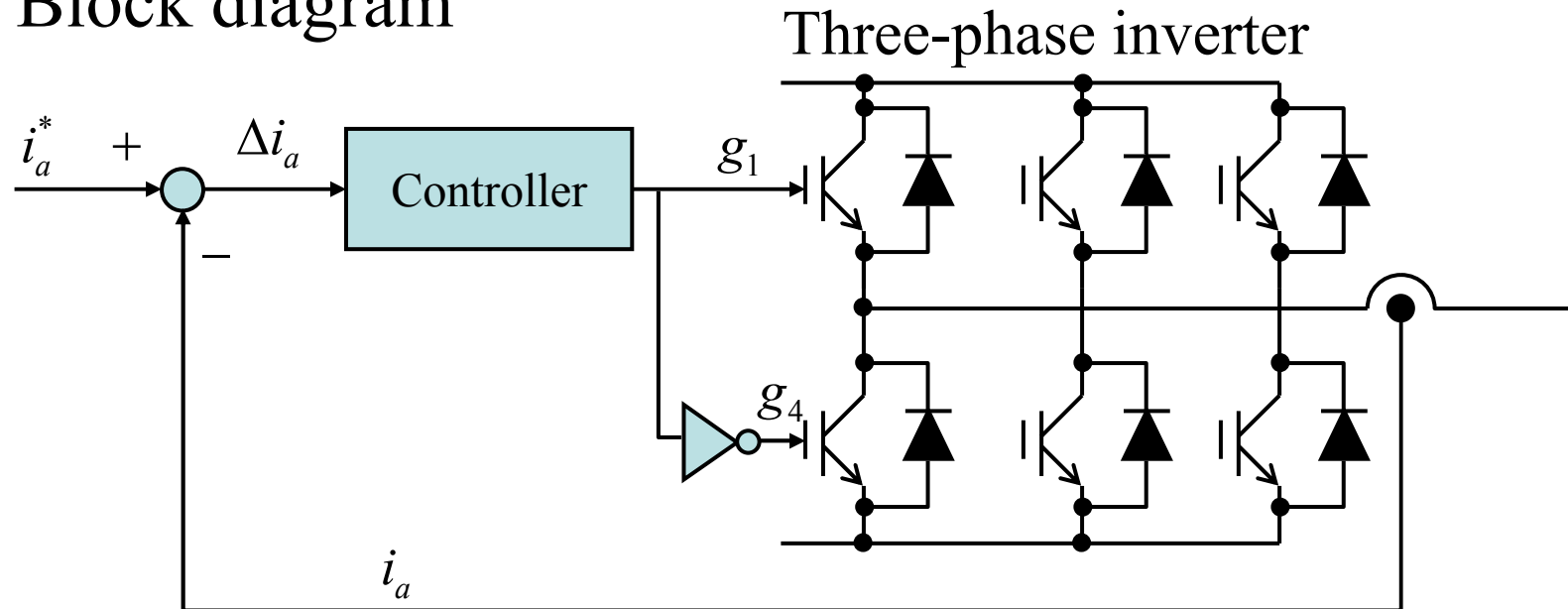
- Microprocessor programming
 - Use look-up table

fundamental	α_1	α_2	α_3
1%	29.2°	30.3°	59.5°
2%	29.6°	30.8°	59.1°
3%
4%
...



4.8 Delta Modulation

- Block diagram



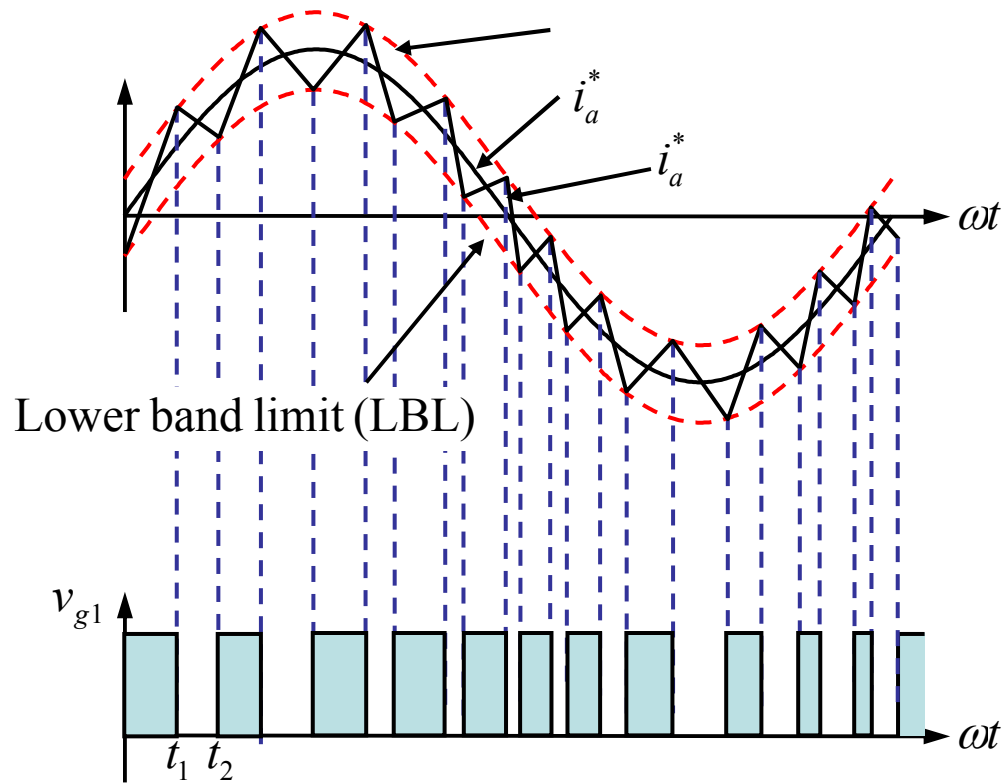
i_a^* , reference current (phase A)

i_a , actual inverter output current (phase A)

Δi_a , error signal (phase A)

4.8 Delta Modulation

- Waveforms Upper band limit (UBL)



Note:

- (1) Assume $v_{g1} = "1"$ $\rightarrow T_1$
on $\rightarrow i_a \uparrow$ until t_1
- (2) At t_1 , i_a reaches the UBL
 $\rightarrow v_{g1} = "0"$ $\rightarrow v_{g4} = "1"$ $\rightarrow T_4$
on $\rightarrow i_a \downarrow$ until t_2
- (3) At t_2 , i_a reaches the LBL
 $\rightarrow v_{g1} = "1"$ $\rightarrow v_{g4} = "0"$ $\rightarrow T_1$
on $\rightarrow i_a \uparrow$

As a result, the actual current i_a will be kept within the upper and lower band limits

4.8 Delta Modulation

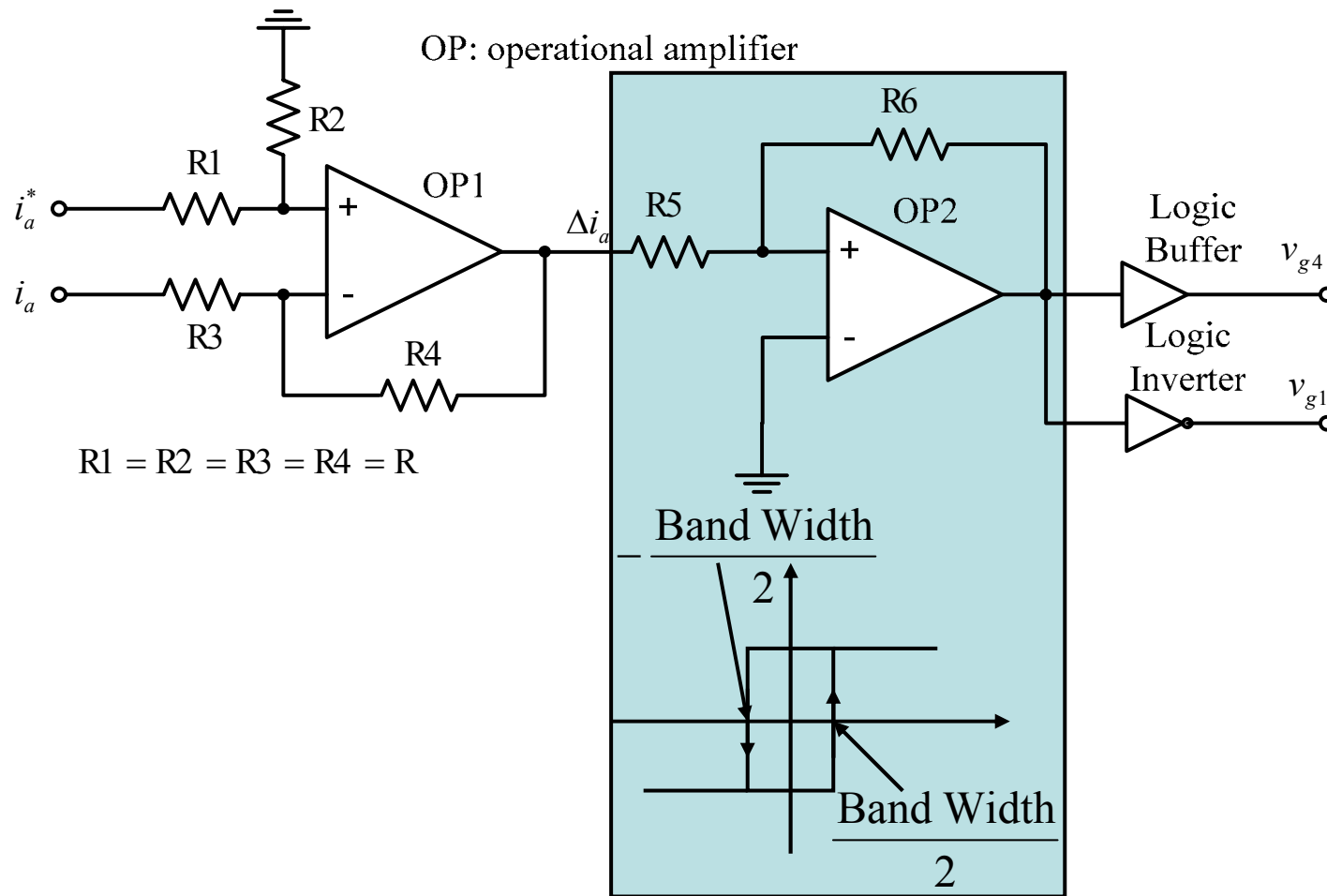
- Q &A
 - Q: How to adjust the frequency of i_a ?
 - A: Adjust the frequency of reference current
 - Q: How to adjust the magnitude of i_a ?
 - A: Adjust the magnitude of reference current
 - Q: What if the band width is reduces?
 - A: i_a will follow reference current more closely. But switching frequency↑

4.8 Delta Modulation

- Summary
 - If reference current is sine wave, actual current is also sine wave on which some high order harmonics are superimposed. High order harmonics can be filtered out easily.
 - No low order harmonics.
 - Inverter output current can be accurately controlled.

4.8 Delta Modulation

- Implementation



Summary

- Voltage source inverter, Topologies
 - Single-phase inverters
 - Half-bridge inverter
 - Full-bridge (H-Bridge) inverter
 - Three-phase inverters
- Gating patterns
 - Square wave operation
 - Pulse width modulation (PWM)
 - Selected harmonic elimination (SHE)
 - Delta modulation
- Harmonics
 - Fourier series
 - Harmonic components, table 8-1 and 8-2
 - Dominant harmonic component
- Waveforms
 - Gate signals
 - Voltage waveforms
- Signal generation
 - Analog circuit
 - Digital circuit
 - Microprocessor