RYERSON UNIVERSITY

# Chapter 1
# DC-DC Switch-mode Converters

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## Content

- Introduction
- Buck converters
- One-quadrant chopper
- Two-quadrant chopper
- Review of 68HC11 MPP board
- Microprocessor control of dc-dc converters

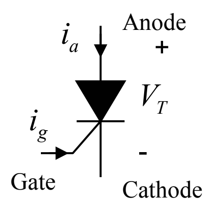
# 1.1 Introduction

- Switching devices
  - The devices are operated like a switch.
  - Popular switching devices
    - SCR – silicon controlled rectifier (thyristor)
    - GTO – gate turn off thyristor
    - IGBT – insulated gate bipolar transistors
    - POWER MOSFET – mental oxide semiconductor field effect transistor



# 1.1 Introduction

- SCR – Silicon controlled rectifier (thyristor)




$i_a$ – Anode current

$i_g$ – Gate current

To turn it on, $i_g>0$

To turn it off, $V_T<0$ : (apply a reverse voltage)→ $i_a=0$→the SCR is turned off.

Note: a "-ve" $i_g$ will not turn the device off.

Device rating: $i_a$=10A~6000A

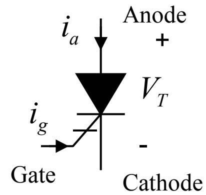
$V_T$=50V~6500V

Operating frequency: up to 500Hz

Main applications: ac-dc converters/rectifiers

## 1.1 Introduction

- GTO – Gate turn off thyristor




$i_a$ – Anode current

$i_g$ – Gate current

To turn it on, $i_g$>0

To turn it off, $i_g$<0

Current-controlled device

Device rating: $i_a$=10A~6000A

$V_T$=50V~6000V

Operating frequency: up to 1KHz

Main applications: high power dc-ac converters (inverters)

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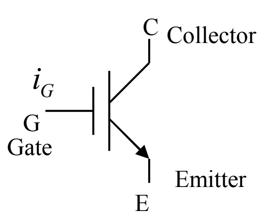
## 1.1 Introduction

- IGBT – Insulated gate bipolar transitors




To turn it on, $V_{GE}$>0

To turn it off, $V_{GE}$<0

Voltage-controlled device ($i_G$=0 at steady state)

Device rating: up to 1200A/4500V

Operating frequency: up to 20KHz

Main applications: dc-ac converters (inverters)

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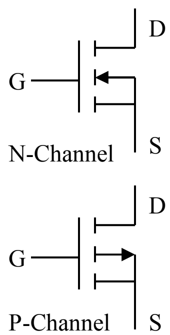
# 1.1 Introduction

- POWER MOSFET – mental oxide semiconductor field effect transistor




•For N-Channel devices:

To turn it on, $V_{GS}>0$

To turn it off, $V_{GS}<0$

•For P-Channel devices:

To turn it on, $V_{GS}<0$

To turn it off, $V_{GS}>0$

Device rating: up to 150A/1000V

Operating frequency: up to 200KHz

Main applications: dc-dc converters (switch mode power supplies)



# 1.1 Introduction

- Device rating

| Switching devices | Single device rating (max.) | Operating frequency (max.) | Gate control |
|---|---|---|---|
| SCR | 4500A/6500V | 500Hz | Current |
| GTO | 4500A/6000V | 1KHz | Current |
| IGBT | 1200A/4500V | 20KHz | Voltage |
| POWER MOSFET | 150A/1000V | 200KHz | Voltage |

# 1.1 Introduction

- Summary
  - SCR and GTO are current-controlled devices. The device turn-on is controlled by gate current. SCR can NOT be turned off by applying a "-ve" gate current.
  - IGBT and POWER MOSFET are voltage-controlled devices. The device turn-on and turn-off are controlled by the gate voltage. In power converters, these devices will be either turned on (shorted) or turned off (open circuit). They NEVER operate in active mode.
  - All these devices are unidirectional.
  - In power electronics, all these devices are simplified as ideal switches:
    - when turn on, saturation voltage is zero;
    - when turn off, leakage current is zero.
  - Increase switching frequency → decrease device rating

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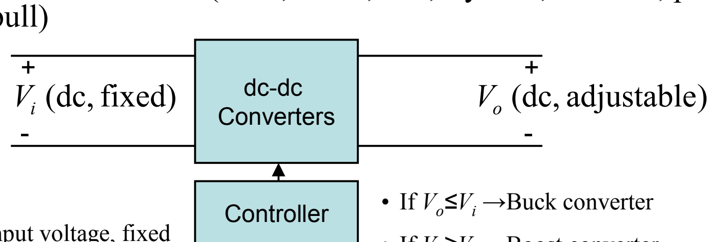
# 1.1 Introduction

- Power converters
  - dc-dc converters (buck, boost, cuk, flyback, forward, push-pull)




$V_i$ – dc input voltage, fixed

$V_o$ – dc output voltage, adjustable

- If $V_o \le V_i$ →Buck converter
- If $V_o \ge V_i$ →Boost converter
- $V_o \le V_i$ or $V_o \ge V_i$ → Buck-boost converter, Cuk converter

Main functions:
- Adjustable output voltage (dc).
- A good output voltage regulation.

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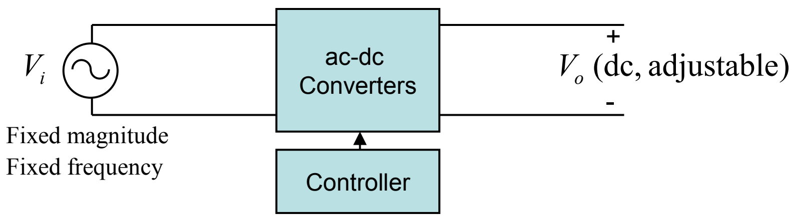
## 1.1 Introduction

– ac-dc converters (rectifier)




$V_i$ – ac input voltage, fixed magnitude and fixed frequency

$V_o$ – dc output voltage, adjustable

Main functions:

- Ac-dc conversion
- Adjustable dc output

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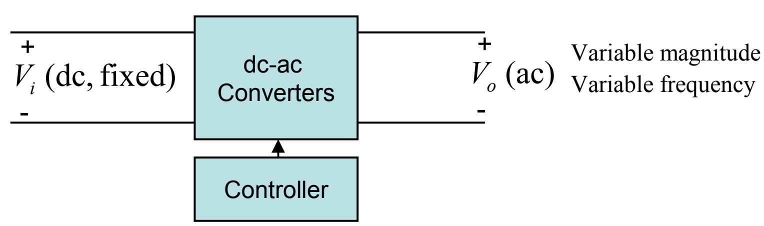
## 1.1 Introduction

– dc-ac converters (inverters)




$V_i$ – dc input voltage, fixed

$V_o$ – ac output voltage, adjustable magnitude and adjustable frequency

Main functions:

- dc-ac conversion
- Adjustable magnitude and frequency output

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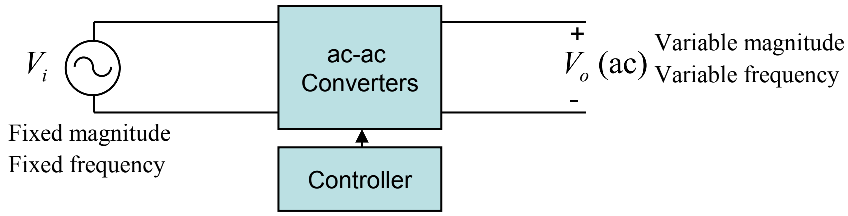
# 1.1 Introduction

– ac-ac converters (cycloconverter, matrix converter)




$V_i$ – ac input voltage, fixed magnitude and fixed frequency

$V_o$ – ac output voltage, adjustable magnitude and adjustable frequency

Main functions:

- Ac-ac direct conversion
- Adjustable magnitude and frequency output

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# 1.1 Introduction

– Conclusions
  - The input voltage/frequency of the converters is usually fixed
  - The output voltage/frequency of the converters is usually adjustable and regulated.

– The converters to be studied in ELE754, which are widely used in industry.
  - dc-dc converters
  - ac-dc converters
  - dc-ac converters

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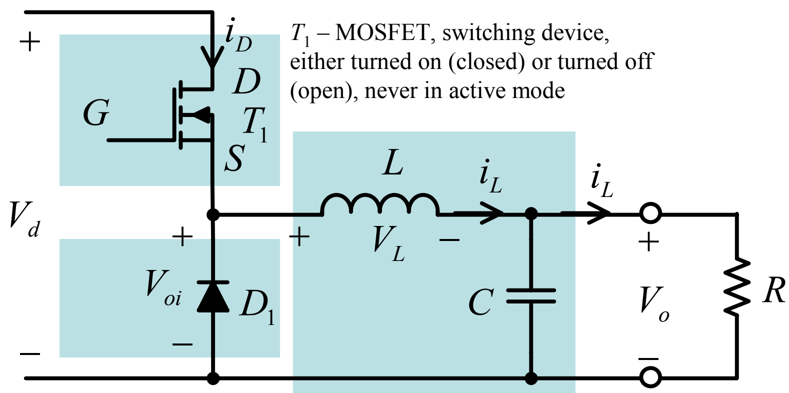
# 1.2 Buck Converters

- Topology




$D_1$ – free wheeling diode, provides current path for the current of inductor $L$ when $T_1$ is off.

$L, C$ – Low pass filter



# 1.2 Buck Converters

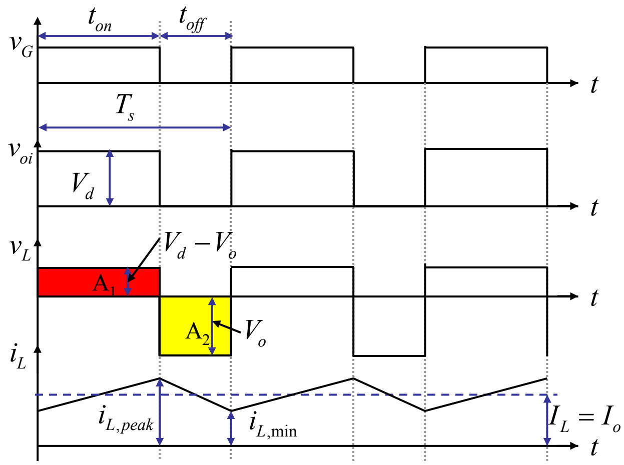
- Features
  - High efficiency
    - Switching device loss

      $T_1$ is on → $V_{DS}$≈0V→$P_{LOSS}=V_{DS}\times i_d$≈0W

      $T_1$ is off → $i_d$≈0A→$P_{LOSS}=V_{DS}\times i_d$≈0W
    - $L$, $C$ and diode loss → negligible
  - Small size
    - High switching frequency → small size of $L$ and $C$.
- Assumption:
  - Ideal switches: $V_{DS,\text{on}}$=0V (MOSFET $T_1$) and $V_{D1,\text{on}}$=0V (Diode $D_1$)
  - $C=\infty \rightarrow V_o$=constant, no voltage ripple

# 1.2 Buck Converters

- Waveforms (continues current mode)




$T_s$ – switching period

$D$ – duty cycle (ratio),

$$D = \frac{t_{on}}{T_s}$$

$I_L$ – average choke current.

$I_o$ – output current.

$V_d$ – input dc voltage.

$V_o$ – output dc voltage.

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# 1.2 Buck Converters

– Summary
  - $T_{on}$ period: $T_1$ on → $V_{oi}=V_d$ → $i_L$↑, energy stored in $L$, $v_L=V_d-V_o$ due to $C=\infty$ and $V_o$=constant.
  - $T_{off}$ period: $T_1$ off → energy in $L$ released through $C$, $R$ and $D_1$→ $D_1$ is on, $v_{oi}=0$.
  - Average output current $I_o$=?

    No dc current increase →$I_o=I_L$
  - Output voltage $V_o$=?

    Ideal inductor → No dc voltage drop on the inductor →

    Area $A_1$=Area $A_2$.

    →$(V_d-V_o)t_{on}=V_o t_{off}=V_o(T_s-t_{on})$

    →$V_o=V_d D$

    $0\le D\le 1$→$V_o\le V_d$→buck converter, or step-down converter

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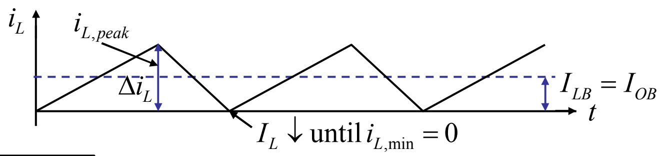
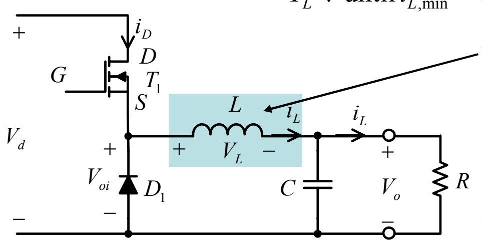
## 1.2 Buck Converters

- Boundary between continuous and discontinuous current mode

$i_L$ continuous →continuous current mode
$i_L$ discontinuous →discontinuous current mode




$v_L = L\frac{di_L}{dt} \rightarrow v_L = L\frac{\Delta i_L}{\Delta t} \rightarrow \Delta i_L = \frac{v_L}{L}\Delta t$

During $t_{on}$, $v_L = V_d - V_o$

$I_{LB} = \frac{1}{2} i_{L,peak} = \frac{1}{2}\frac{v_L}{L}\Delta t = \frac{1}{2}\frac{V_d - V_o}{L} t_{on}$

$= \frac{1}{2L}(V_d - V_o) D T_s$

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## 1.2 Buck Converters

- Identify the operation mode of buck converter

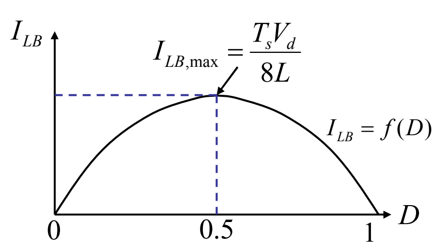
$$I_{LB} = \frac{1}{2L}(V_d - V_o) D T_s$$

If $I_L > I_{LB}$ → continuous current mode

If $I_L < I_{LB}$ → discontinuous current mode

$\because V_o = D V_d$

$\therefore I_{LB} = \frac{T_s V_d}{2L} D(1-D) = f(D)$

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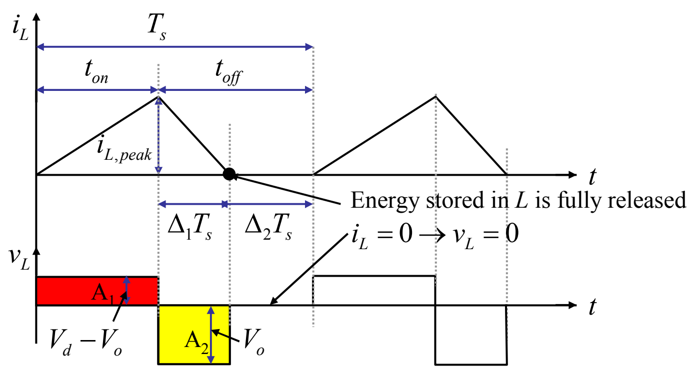
# 1.2 Buck Converters

- Discontinues current mode
  - Waveforms






# 1.2 Buck Converters

– Output voltage $V_o$=?

Area $A_1$ = Area $A_2$

$$\rightarrow (V_d - V_o) D T_s = V_o \Delta_1 T_s$$

$$\rightarrow V_o = \frac{D}{D + \Delta_1} V_d \qquad (1)$$

$$\Delta i_L = \frac{V_L}{L} \Delta t \rightarrow i_{L,peak} = \frac{V_o}{L} \Delta_1 T_s$$

$$\rightarrow \Delta_1 = \frac{L i_{L,peak}}{V_o T_s} = \frac{I_o}{4 I_{LB,\max} D} \qquad (2)$$

$(2) \rightarrow (1)$

$$V_o = \frac{D^2}{D^2 + \frac{1}{4}\left(I_o / I_{LB,\max}\right)} V_d$$ for discontinuous current mode

# 1.2 Buck Converters

- Summary
  - Assumption
    - Ideal switches, switching device and diode
    - Constant output voltage
  - Key equations

$$\Delta i_L = \frac{V}{L}\Delta t$$

$$V_o = DV_d \quad \text{for continuous current mode}$$

$$V_o = \frac{D^2}{D^2 + \frac{1}{4}\left(I_o / I_{LB,\max}\right)} V_d \quad \text{for discontinuous current mode}$$

$$I_{LB} = \frac{T_s V_d}{2L} D(1-D) \quad I_L > I_{LB} \rightarrow \text{continuous current mode}$$

$$I_L < I_{LB} \rightarrow \text{discontinuous current mode}$$

$$I_{LB.\max} = \frac{T_s V_d}{8L}$$

  - Waveforms
    - $v_G$, $v_L$, $i_L$

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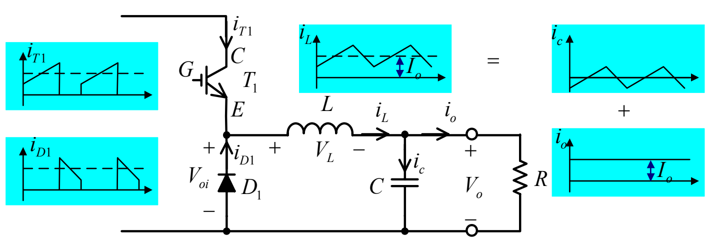
# 1.2 Buck Converters

- Output voltage ripple
  - Filter capacitance is not infinite, practical value
  - Ripple voltage $\Delta V_o / V_o$ usually is less than 1%
  - Assume
    - All the ripple component of inductor current flows through the capacitor.
    - The dc component of inductor current flows through the load resistor
  - Pick continuous current mode as example: current waveforms

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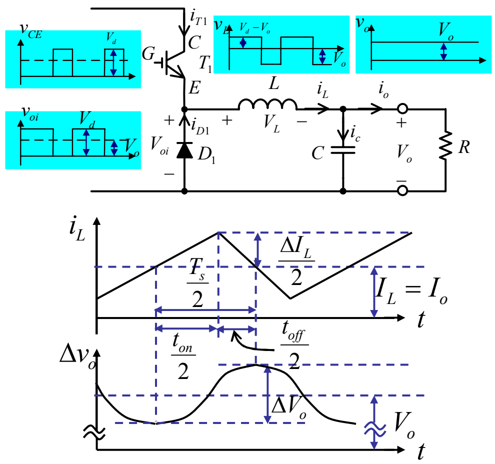
# 1.2 Buck Converters

– Voltage waveforms




$$\Delta V_o = \frac{1}{C}\int_0^{\frac{T_s}{2}} i_c dt = \frac{1}{C}\int_0^{\frac{T_s}{2}} (i_L - I_L)dt$$

$$= \frac{1}{C}\left(\frac{\Delta I_L}{2}\frac{t_{on}}{2}\frac{1}{2} + \frac{\Delta I_L}{2}\frac{t_{off}}{2}\frac{1}{2}\right) = \frac{\Delta I_L T_s}{8C} \quad (1)$$

$$\Delta I_L = \frac{V_o}{L}(1-D)T_s \quad (2)$$

$(2) \rightarrow (1)$

$$\Delta V_o = \frac{T_s}{8C}\frac{V_o}{L}(1-D)T_s$$

$$\frac{\Delta V_o}{V_o} = \frac{\pi^2}{2}(1-D)\left(\frac{f_c}{f_s}\right)^2 \quad (3)$$

where, $f_c = \frac{1}{2\pi\sqrt{LC}}$ is corner frequency of LC filter and $f_s = \frac{1}{T_s}$ is switching frequency.



# 1.2 Buck Converters

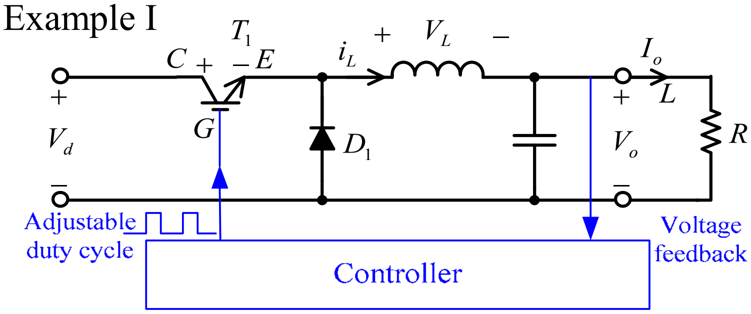
– Reduce voltage ripple

- Decrease the corner frequency → $L\uparrow$ or $C\uparrow$ → Larger size of the converter.
- Increase the switching frequency → switching loss ↑ → efficiency ↓ .
- How to reduce switching loss?
  - Use low switching loss device, such as MOSFET
  - Use soft switching topology, such as ZVS, ZCS)

– $f_s \approx (10 \sim 100) f_c \rightarrow \Delta V_o / V_o$ usually is less than 1%

# Example I

- Example I




Given : $V_d = 150\text{V}$, $f_{sw} = 20\text{KHz}$, $L = 1\text{mH}$, $C = 47.0\mu\text{F}$,

$V_o = 48\text{V}$ and load resistance $R = 10\Omega$, Assume the output voltage is kept constant.

Find : (1) operating mode, (2) ripple voltage $\dfrac{\Delta V_o}{V_o}$

(3) inductor's current ripple $\Delta I_L$, (4)duty cycle if $R = 100\Omega$



# Example I

Solution (1) :

step 1 $\Rightarrow I_o = \dfrac{V_o}{R} = \dfrac{48}{10} = 4.8\text{A}$

step 2 $\Rightarrow I_L = I_o = 4.8\text{A}$

step 3 $\Rightarrow$ Assume continuous current mode : $D = \dfrac{V_o}{V_d} = \dfrac{48}{150} = 0.32$

step 4 $\Rightarrow I_{LB} = \dfrac{T_s V_d}{2L} D(1-D) = \dfrac{50\times10^{-6}\times150}{2\times1\times10^{-3}}\times0.32\times(1-0.32) = 0.816\text{A}$

where $T_s = 1/f_{sw} = \dfrac{1}{20\times10^3} = 50\times10^{-6}\text{s}$

step 5 $\Rightarrow I_L > I_{LB} \rightarrow$ Continuous current mode

# Example I

Solution (2):

In continuous current mode:

$$\frac{\Delta V_o}{V_o} = \frac{\pi^2}{2}(1-D)\left(\frac{f_c}{f_s}\right)^2 = \frac{\pi^2}{2}(1-0.32)\left(\frac{734.13}{20\times10^3}\right)^2 = 4.5\text{mV}$$

$$\text{where } f_c = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1\times10^{-3}\times47\times10^{-6}}} = 734.13\text{Hz}$$

Solution (3):

Current ripple

$$\Delta I_L = \frac{V_d - V_o}{L}t_{on} = \frac{V_d - V_o}{L}DT_s = \frac{150-48}{1\times10^{-3}}\times0.32\times50\times10^{-6} = 1.632\text{A}$$

or

$$\Delta I_L = \frac{V_o}{L}t_{off} = \frac{V_o}{L}(1-D)T_s = \frac{48}{1\times10^{-3}}\times(1-0.32)\times50\times10^{-6} = 1.632\text{A}$$



# Example I

Solution (4):

step 1 ⇒ $I_o = \frac{V_o}{R} = \frac{48}{100} = 0.48\text{A}$

step 2 ⇒ $I_L = I_o = 0.48\text{A}$

step 3 ⇒ Assume continuous current mode: $D = \frac{V_o}{V_d} = 0.32$

step 4 ⇒ $I_{LB} = 0.816\text{A}$

step 5 ⇒ $I_L < I_{LB} \rightarrow$ disontinuous current mode

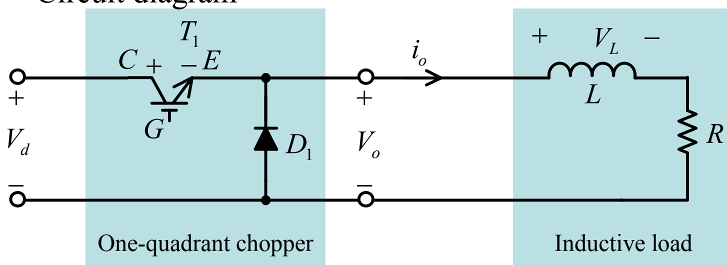
step 6 ⇒ use $V_o = \frac{D^2}{D^2 + \frac{1}{4}\left(I_o / I_{LB,\max}\right)}V_d \rightarrow$

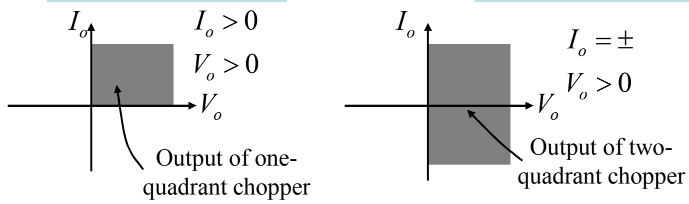
$$D = \sqrt{\frac{\frac{V_o}{4}\left(I_o / I_{LB,\max}\right)}{V_d - V_o}} = \sqrt{\frac{\frac{48}{4}(0.48/0.9375)}{150-48}} = 0.2454$$

$$\text{where } I_{LB,\max} = \frac{T_s V_d}{8L} = \frac{50\times10^{-6}\times150}{8\times1\times10^{-3}} = 0.9375$$

# 1.3 One Quadrant Chopper

- One quadrant chopper with RL load
  - Circuit diagram

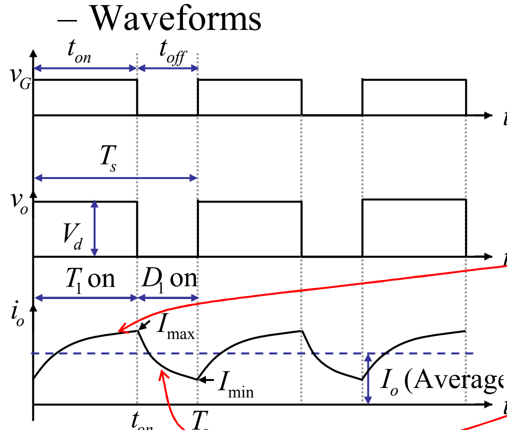





# 1.3 One Quadrant Chopper

- Waveforms




$i_o$ does not increase or decrease linearly!

when $T_1$ is on:

$$v_o = V_d = L\frac{di_o}{dt} + Ri_o \rightarrow$$

$$i_o(t) = \frac{V_d}{R}\left(1 - e^{-\frac{t}{\tau}}\right) + I_{min}e^{-\frac{t}{\tau}}$$

when $T_1$ is off ($D_1$ is on):

$$v_o = 0 = L\frac{di_o}{dt} + Ri_o \rightarrow$$

$$i_o(t) = I_{max}e^{-(t-t_{on})/\tau}$$

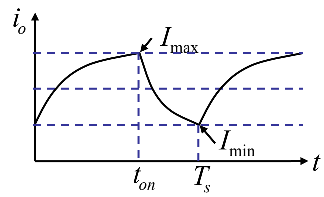
where $\tau = L/R$ is time constant of inductive load

Note: $\tau\frac{dx}{dt} = -x + b \rightarrow x(t) = x_0 e^{-t/\tau} + b\left(1 - e^{-t/\tau}\right)$,

where $x_0 = x(0)$ is initial value and $b = x(\infty)$ is final value.

# 1.3 One Quadrant Chopper

– $I_{\max}$ and $I_{\min}$




From current waveform

$$i_o(t) = \frac{V_d}{R}\left(1 - e^{-\frac{t}{\tau}}\right) + I_{\min} e^{-\frac{t}{\tau}}, 0 \le t < t_{on} \qquad (1)$$

$$i_o(t) = I_{\max} e^{-(t-t_{on})/\tau}, t_{on} \le t < T_s \qquad (2)$$

From Eq.(1) At $t = t_{on}, i_o(t_{on}) = I_{\max} \rightarrow I_{\max} = \frac{V_d}{R}\left(1 - e^{-\frac{t_{on}}{\tau}}\right) + I_{\min} e^{-\frac{t_{on}}{\tau}}$ (3)

From Eq.(2) At $t = T_s, i_o(T_s) = I_{\min} \rightarrow I_{\min} = I_{\max} e^{-\frac{T_s - t_{on}}{\tau}}$ (4)

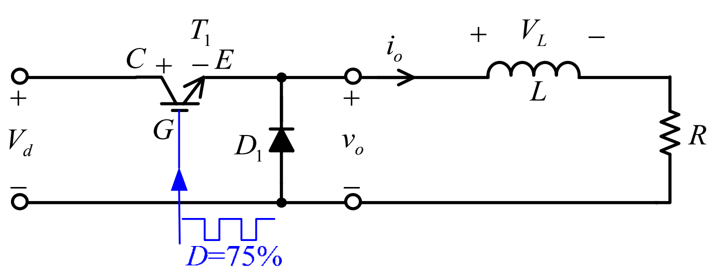
Solve Eq.(3) and Eq.(4)

$$I_{\max} = \frac{V_d}{R}\frac{1 - e^{-t_{on}/\tau}}{1 - e^{-T_s/\tau}} \text{ and } I_{\min} = \frac{V_d}{R}\frac{e^{t_{on}/\tau} - 1}{e^{T_s/\tau} - 1}$$



# Example II

- Example II




Given : one quadrant chopper with RL load (inductive load)

$V_d = 120\text{V}, f_{sw} = 1\text{KHz}, L = 4.5\text{mH}, R = 1.5\Omega, D = 0.75$

Find : (1) inductor's current ripple $\Delta I_L$, (2) average $I_o$ and $V_o$

## Example II

Solution (1) :

$$\text{step 1} \Rightarrow \tau = \frac{L}{R} = \frac{4.5\times10^{-3}}{1.5} = 3\text{ms}$$

$$\text{step 2} \Rightarrow T_s = \frac{1}{f_{sw}} = \frac{1}{1000} = 1\text{ms}$$

$$\text{step 3} \Rightarrow t_{on} = DT_s = 0.75\times1 = 0.75\text{ms}$$

$$\text{step 4} \Rightarrow I_{\max} = \frac{V_d}{R}\frac{1-e^{-t_{on}/\tau}}{1-e^{-T_s/\tau}} = \frac{120}{1.5}\frac{1-e^{-0.75/3}}{1-e^{-1/3}} = 62.4\text{A}$$

$$I_{\min} = \frac{V_d}{R}\frac{e^{t_{on}/\tau}-1}{e^{T_s/\tau}-1} = \frac{120}{1.5}\frac{e^{0.75/3}-1}{e^{1/3}-1} = 57.5\text{A}$$

$$\text{step 5} \Rightarrow \Delta I_L = I_{\max} - I_{\min} = 62.4 - 57.5 = 4.9\text{A}$$

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## Example II

Solution (2) :

$$V_{o,avg} = DV_d = 0.75\times120 = 90\text{V}$$

$$I_{o,avg} = \frac{V_{o,avg}}{R} = \frac{90}{1.5} = 60\text{A}, \text{(because of no dc voltage drop on the inductor)}$$

Also

$$I_{o,avg} \approx \frac{I_{\max}+I_{\min}}{2} = \frac{62.4+57.5}{2} = 60\text{A}$$

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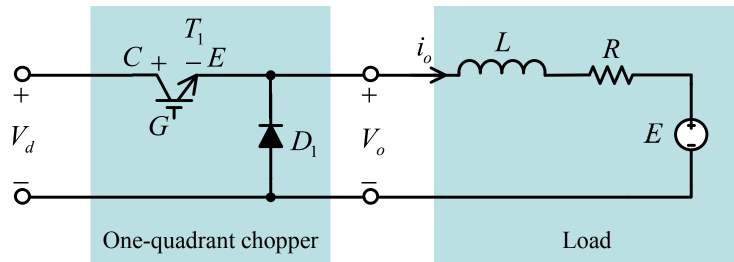
# 1.3 One Quadrant Chopper

- One quadrant chopper with RLE load
  - Circuit diagram

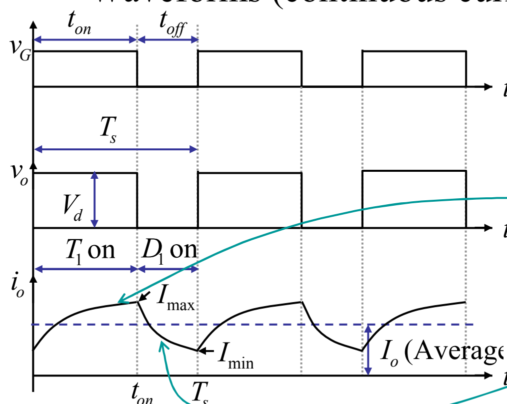



**Load: DC motor equivalent circuit**
**L: armature winding inductance**
**R: armature winding resistance**
**E: back emf**



# 1.3 One Quadrant Chopper

– Waveforms (continuous current mode)




when $T_1$ is on :

$$v_o = V_d = L\frac{di_o}{dt} + Ri_o + E \rightarrow$$

$$i_o(t) = \frac{V_d - E}{R}\left(1 - e^{-\frac{t}{\tau}}\right) + I_{min}e^{-\frac{t}{\tau}}$$

when $T_1$ is off ($D_1$ is on) :

$$v_o = 0 = L\frac{di_o}{dt} + Ri_o + E \rightarrow$$

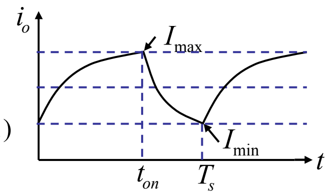
$$i_o(t) = -\frac{E}{R}\left(1 - e^{-(t-t_{on})/\tau}\right) + I_{max}e^{-(t-t_{on})/\tau}$$

where $\tau = L/R$ is time constant of the load

$$V_{o,avg} = \frac{\text{Area A1}}{T_s} = \frac{V_d t_{on}}{T_s} = V_d\frac{t_{on}}{T_s} = DV_d$$

# 1.3 One Quadrant Chopper

– $I_{max}$ and $I_{min}$




From current waveform

$$i_o(t) = \frac{V_d - E}{R}\left(1 - e^{-\frac{t}{\tau}}\right) + I_{min} e^{-\frac{t}{\tau}}, 0 \le t < t_{on} \qquad (1)$$

$$i_o(t) = -\frac{E}{R}\left(1 - e^{-(t-t_{on})/\tau}\right) + I_{max} e^{-(t-t_{on})/\tau}, t_{on} \le t < T_s \qquad (2)$$

From Eq.(1) At $t = t_{on}, i_o(t_{on}) = I_{max} \to I_{max} = \frac{V_d - E}{R}\left(1 - e^{-\frac{t_{on}}{\tau}}\right) + I_{min} e^{-\frac{t_{on}}{\tau}}$ (3)

From Eq.(2) At $t = T_s, i_o(T_s) = I_{min} \to I_{min} = -\frac{E}{R}\left(1 - e^{-(T_s - t_{on})/\tau}\right) + I_{max} e^{-(T_s-t_{on})/\tau}$ (4)

Solve Eq.(3) and Eq.(4)

$$I_{max} = \frac{V_d}{R}\frac{1 - e^{-t_{on}/\tau}}{1 - e^{-T_s/\tau}} - \frac{E}{R} \text{ and } I_{min} = \frac{V_d}{R}\frac{e^{t_{on}/\tau} - 1}{e^{T_s/\tau} - 1} - \frac{E}{R}$$

**Note: continuous current mode!**

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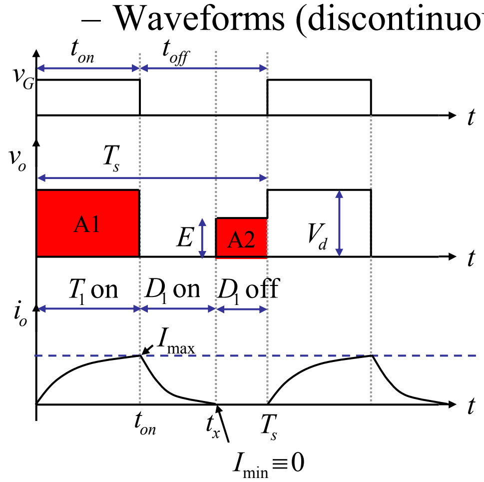
# 1.3 One Quadrant Chopper

– Waveforms (discontinuous current mode)




(1) $t_{on} \le t < t_x$,
$L$ is releasing the energy to the load
$\to D_1$ is on $\to V_o = 0$

(2) at $t_x$,
The energy in $L$ is fully released
$\to i_o = 0$

(3) $t_x \le t < T_s$,
$i_o = 0 \to D_1$ is off $\to V_o = E$

$$V_{o,avg} = \frac{\text{AreaA1} + \text{AreaA2}}{T_s} = \frac{V_d t_{on} + E(T_s - t_x)}{T_s} = V_d \frac{t_{on}}{T_s} + E\frac{T_s - t_x}{T_s}$$

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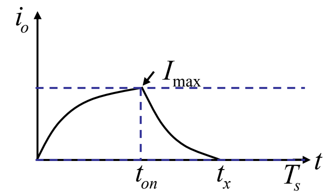
# 1.3 One Quadrant Chopper

– $I_{max}$ =? and $I_{min}$≡0




From current waveform

$$i_o(t) = \frac{V_d - E}{R}\left(1 - e^{-\frac{t}{\tau}}\right), 0 \le t < t_{on} \qquad (1)$$

$$i_o(t) = -\frac{E}{R}\left(1 - e^{-(t-t_{on})/\tau}\right) + I_{max} e^{-(t-t_{on})/\tau}, t_{on} \le t < t_x \qquad (2)$$

$$\text{From Eq.(1) At } t = t_{on}, i_o(t_{on}) = I_{max} \rightarrow I_{max} = \frac{V_d - E}{R}\left(1 - e^{-\frac{t_{on}}{\tau}}\right) \qquad (3)$$

$$\text{From Eq.(2) At } t = t_x, i_o(t_x) = 0 \rightarrow 0 = -\frac{E}{R}\left(1 - e^{-(t_x-t_{on})/\tau}\right) + I_{max} e^{-(t_x-t_{on})/\tau} \qquad (4)$$

Solve Eq.(4)

$$t_x = \tau \ln\left\{e^{t_{on}/\tau}\left[1 + \frac{V_d - E}{E}\left(1 - e^{-t_{on}/\tau}\right)\right]\right\}$$

**Note: discontinuous current mode!**



# 1.3 One Quadrant Chopper

– Summary

- Equations

Continuous current mode:

$$V_{o,avg} = \frac{t_{on}}{T_s} V_d = D V_d$$

$$I_{o,avg} = \frac{V_{o,avg} - E}{R}$$

$$I_{max} = \frac{V_d}{R}\frac{1 - e^{-t_{on}/\tau}}{1 - e^{-T_s/\tau}} - \frac{E}{R}$$

$$I_{min} = \frac{V_d}{R}\frac{e^{t_{on}/\tau} - 1}{e^{T_s/\tau} - 1} - \frac{E}{R} > 0$$

Discontinuous current mode:

$$V_{o,avg} = \frac{t_{on}}{T_s} V_d + \frac{T_s - t_x}{T_s} E \ne D V_d$$

$$I_{o,avg} = \frac{V_{o,avg} - E}{R}$$

$$I_{max} = \frac{V_d - E}{R}\left(1 - e^{-\frac{t_{on}}{\tau}}\right)$$

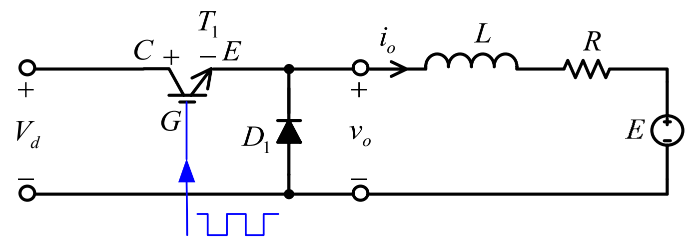
$$I_{min} \equiv 0$$

$$t_x = \tau \ln\left\{e^{t_{on}/\tau}\left[1 + \frac{V_d - E}{E}\left(1 - e^{-t_{on}/\tau}\right)\right]\right\}$$

- waveforms

## Example III

- Example III




Given : one quadrant chopper drives a dc motor

$V_d = 110\text{V}, f_{sw} = 400\text{Hz}, L = 0.2\text{mH}, R = 0.25\Omega, D = 0.5$ and $E = 40\text{V}$

Find : (1) operation mode (continuous or discontinuous) ,

(2) $I_{\max}$ and $I_{\min}$, (3) average $I_o$ and $V_o$



## Example III

Solution (1) :

step 1 ⇒ Assume continuous current mode

$$\text{step 2} \Rightarrow \tau = \frac{L}{R} = \frac{0.2 \times 10^{-3}}{0.25} = 0.8\text{ms}$$

$$\text{step 3} \Rightarrow T_s = \frac{1}{f_{sw}} = \frac{1}{400} = 2.5\text{ms}$$

$$\text{step 4} \Rightarrow t_{on} = DT_s = 0.5 \times 2.5 = 1.25\text{ms}$$

$$\text{step 5} \Rightarrow I_{\min} = \frac{V_d}{R} \frac{e^{t_{on}/\tau} - 1}{e^{T_s/\tau} - 1} - \frac{E}{R} = \frac{110}{0.25} \frac{e^{1.25/0.8} - 1}{e^{2.5/0.8} - 1} - \frac{40}{0.25} = -84\text{A}$$

step 6 ⇒ $I_{\min} < 0 \rightarrow$ discontinuous current mode

## Example III

Solution (2):

$$I_{\max} = \frac{V_d - E}{R}\left(1 - e^{-\frac{t_{on}}{\tau}}\right) = \frac{110-40}{0.25}\left(1 - e^{-1.25/0.8}\right) = 221.2\text{A}$$

$$I_{\min} = 0\text{A}$$

Solution (3):

$$\text{step1} \Rightarrow t_x = \tau \ln\left\{e^{t_{on}/\tau}\left[1 + \frac{V_d - E}{E}\left(1 - e^{-t_{on}/\tau}\right)\right]\right\}$$

$$= 0.8 \times 10^{-3}\left\{e^{1.25/0.8}\left[1 + \frac{110-40}{40}\left(1 - e^{-1.25/0.8}\right)\right]\right\} = 1.94\text{ms}$$

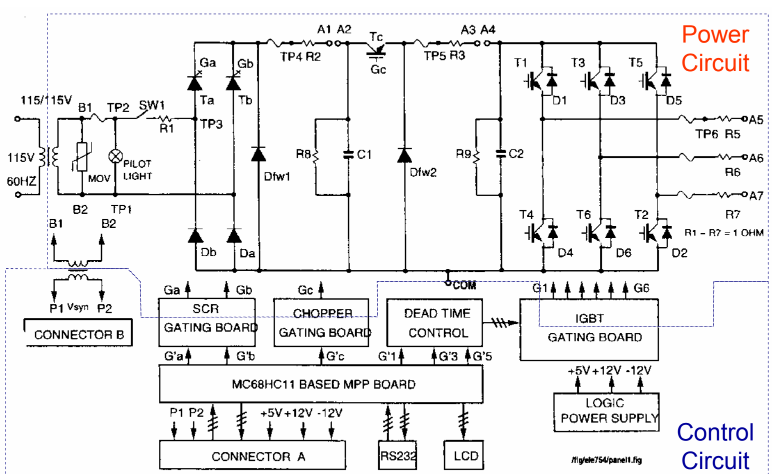
$$\text{step2} \Rightarrow V_{o,avg} = \frac{t_{on}}{T_s}V_d + \frac{T_s - t_x}{T_s}E = \frac{1.25}{2.5} \times 110 + \frac{2.5 - 1.94}{2.5} \times 40 = 64\text{V}$$

$$\text{step3} \Rightarrow I_{o,avg} = \frac{V_{o,avg} - E}{R} = \frac{64-40}{0.25} = 96\text{A}$$

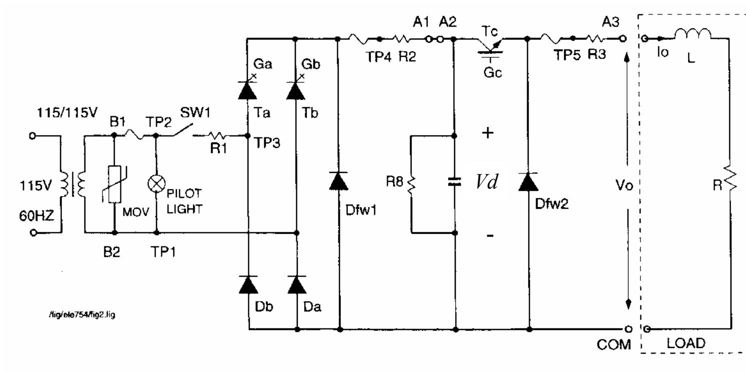


## 1.4 Project 1 dc-dc converter

- Power Converter Module



# 1.4 Project 1 dc-dc converter




$V_d$ – dc voltage, from SCR rectifier, A1(A2) respect to COM

$R8$ – discharge resistor for safety

Tc – IGBT switching device

R2, R3 – current sensing resistor, 1Ω

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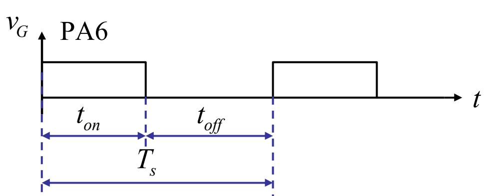
# 1.4 Project 1 dc-dc converter




- Duty cycle control
  - $f_s$ is fixed $\rightarrow T_s$ is fixed
  - $D$ is variable $\rightarrow$ $t_{on}$ is variable

$$[\text{TIME\_HI}] = \frac{t_{on}}{T_{E-cycle}} = t_{on} \times f_{E-cycle} = DT_s f_{E-cycle}$$

$$\text{if } T_s = \frac{1}{f_{sw}} = \frac{1}{500} = 2000\mu s,\ f_{E-cycle} = 1\text{MHz}$$

$$[\text{TIME\_HI}] = D \times 2000 = (100 \times D) \times 20$$

$$[\text{TIME\_LO}] = (1 - D) \times 2000 = [100 - (100 \times D)] \times 20$$

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## 1.4 Project 1 dc-dc converter

- Programming
  - Main program
    - All non real-time subroutines
    - Use subroutines (easy to debug, easy to read)
  - ISR (interrupt service routine)
    - Real-time running
    - As short as possible
    - Optimize the code to save CPU time

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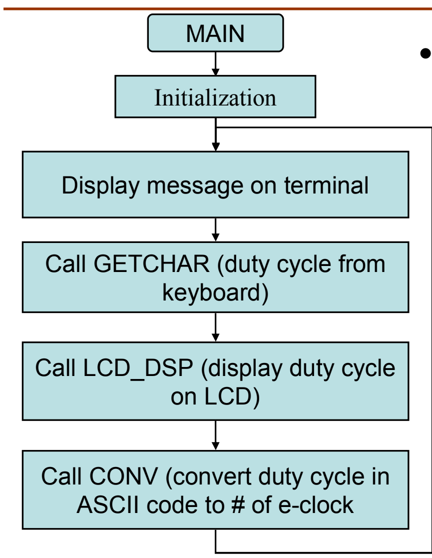
## 1.4 Project 1 dc-dc converter




- Main program flow chat

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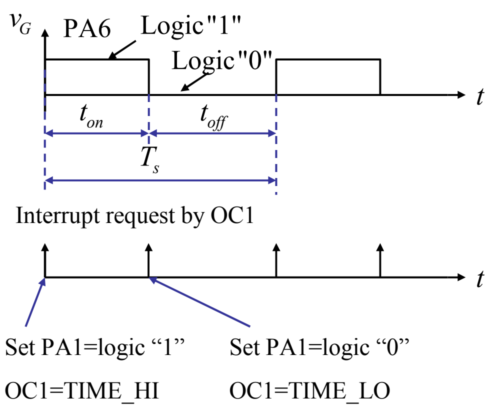
## 1.4 Project 1 dc-dc converter

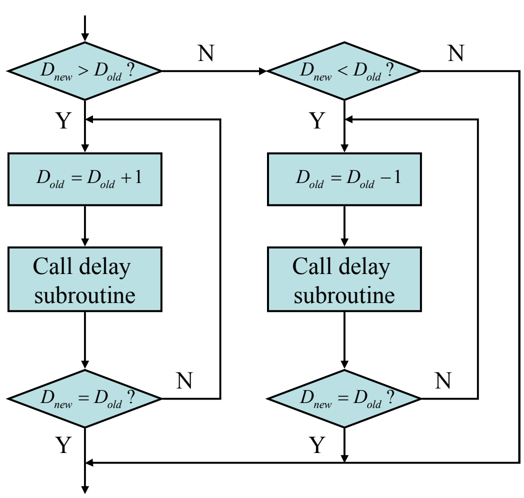



- ISR
  - Set the output port PA6 to logic "0" or "1"
  - Set output compare register OC1 (timer) for next interrupt
  - Return to main program



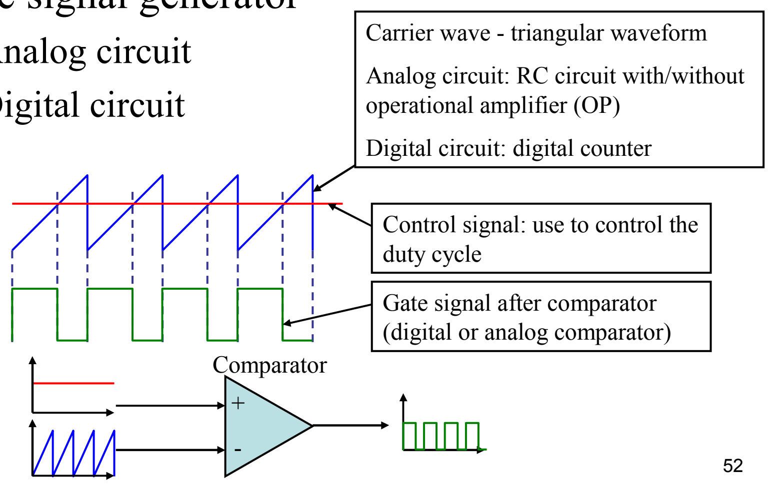
## 1.4 Project 1 dc-dc converter

- Ramp function flow chart



# 1.4 Project 1 dc-dc converter

- Gate signal generator
  - Analog circuit
  - Digital circuit

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# 1.5 Fourier Analysis of 1Q Chopper

- Theorem
  - Fourier series

$$f(t) = f_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

where $f_0 = \frac{1}{2\pi}\int_0^{2\pi} f(t)d(\omega t) = \frac{1}{T}\int_0^T f(t)dt$ ← dc component

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\cos(n\omega t)d(\omega t) = \frac{2}{T}\int_0^T f(t)\cos(n\omega t)dt$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\sin(n\omega t)d(\omega t) = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)dt$$

} nth order harmonics

also :

$$f(t) = f_0 + \sum_{n=1}^{\infty} c_n \sin(n\omega t + \theta_n) \text{ where } c_n = \sqrt{a_n^2 + b_n^2} \text{ and } \theta_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

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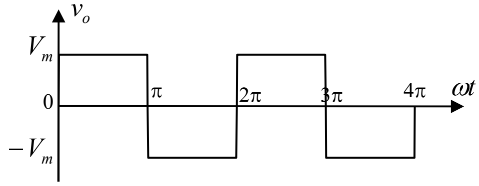
## 1.5 Fourier Analysis of 1Q Chopper

– Example

$$v_o = \begin{cases} V_m & 2k\pi \le t < (2k+1)\pi \\ -V_m & (2k+1)\pi \le t < (2k+2)\pi \end{cases}, k = 0, 1, 2, \ldots$$

Fourier series :

$$v_o = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_m}{n\pi} \sin(n\omega t) = \frac{4V_m}{\pi}\left(\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \ldots\right)$$

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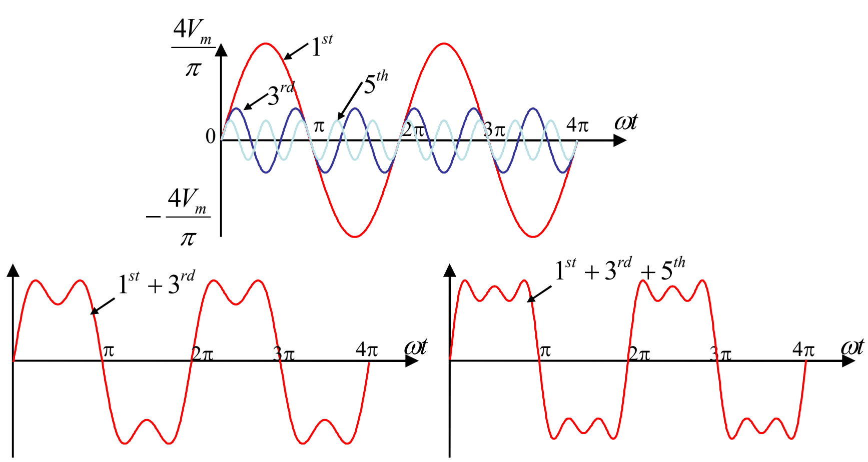
## 1.5 Fourier Analysis of 1Q Chopper

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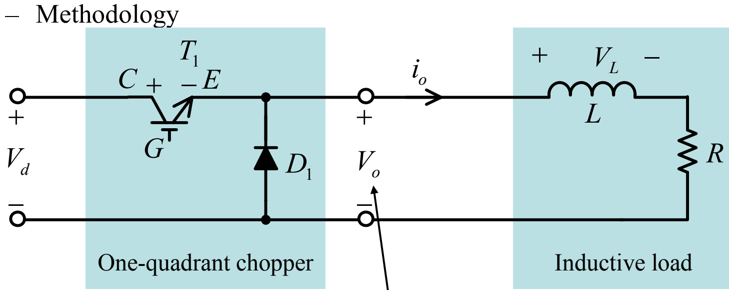
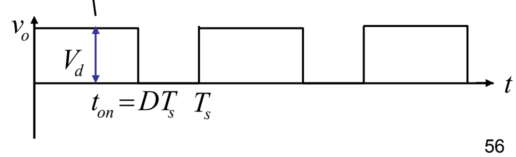
# 1.5 Fourier Analysis of 1Q Chopper

- Fourier Analysis of 1Q Chopper
  - Methodology




$$v_o(t) = \begin{cases} V_d & 0 \le t < DT_s \\ 0 & DT_s \le t < T_s \end{cases}$$






# 1.5 Fourier Analysis of 1Q Chopper

Fourier analysis:

$$V_{o,dc} = \frac{1}{T_s}\int_0^{T_s} v_o(t)dt = \frac{1}{T_s}\left(\int_0^{DT_s} V_d dt + \int_{DT_s}^{T_s} 0dt + \right) = \frac{V_d}{T_s} t\Big|_0^{DT_s} = DV_d$$

$$a_n = \frac{2}{T_s}\int_0^{T_s} v_o(t)\cos(n\omega t)dt = \frac{2}{T_s}\frac{1}{n\omega}\int_0^{DT_s} V_d \cos(n\omega t)d(n\omega t)$$

$$= \frac{2V_d}{n\omega T_s}\sin(n\omega t)\Big|_0^{DT_s} = \frac{2V_d}{n\omega T_s}\sin(n\omega DT_s) = \frac{V_d}{n\pi}\sin(2\pi n D)$$

where $\omega = 2\pi f = \frac{2\pi}{T_s}$

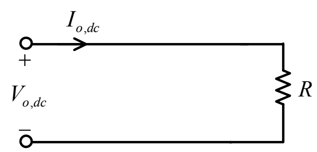
$$b_n = \frac{2}{T_s}\int_0^{T_s} v_o(t)\sin(n\omega t)dt = \frac{2}{T_s}\frac{1}{n\omega}\int_0^{DT_s} V_d \sin(n\omega t)d(n\omega t)$$

$$= \frac{2V_d}{n\omega T_s}\left[-\cos(n\omega t)\right]_0^{DT_s} = \frac{2V_d}{n\omega T_s}\left[1 - \cos(n\omega DT_s)\right] = \frac{V_d}{n\pi}\left[1 - \cos(2\pi n D)\right]$$

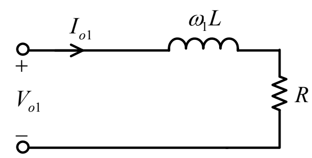
$$\therefore v_o(t) = DV_d + \frac{V_d}{n\pi}\sum_{n=1,2,3\ldots}^{\infty}\sin(2\pi n D)\cos(n\omega t) + \frac{V_d}{n\pi}\sum_{n=1,2,3\ldots}^{\infty}\left[1 - \cos(2\pi n D)\right]\sin(n\omega t)$$

# 1.5 Fourier Analysis of 1Q Chopper

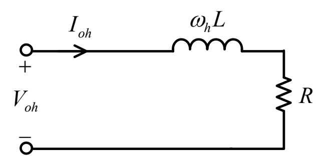
– High frequency equivalent circuit




dc equivalent circuit




fundamental equivalent circuit




$h^{th}$ order harmonic equivalent circuit

where $\omega_1 = 2\pi f = \dfrac{2\pi}{T_s}$

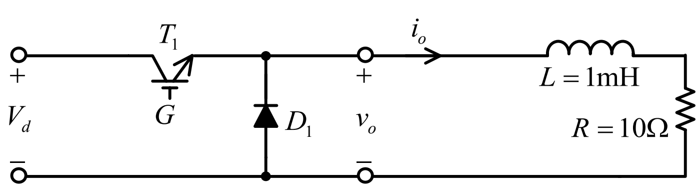
$$\omega_1 = 2\pi hf = \frac{2\pi}{T_s}h$$



# Example IV

Given: 1Q chopper with RL load. $V_d$=100V, $D$=0.4, $R$=10Ω and $L$=1mH, the switching frequency is 1KHz.

Find: (1) The harmonic components of $v_o$, up to 5th order harmonic
(2) The harmonic components of $i_o$, up to 5th order harmonic




Use following formulas:

$$V_{o,dc} = DV_d,\ a_n = \frac{V_d}{n\pi}\sin(2\pi nD),\ b_n = \frac{V_d}{n\pi}\left[1-\cos(2\pi nD)\right]$$

# Example IV

Solution (1) :

$$a_1 = \frac{V_d}{\pi}\sin(2\pi D) = \frac{100}{\pi}\sin(2\pi\times 0.4) = 18.71,$$

$$b_1 = \frac{V_d}{n\pi}[1-\cos(2\pi nD)] = \frac{100}{\pi}[1-\cos(2\pi\times 0.4)] = 57.58$$

$$V_{o1} = \sqrt{a_1^2+b_1^2} = \sqrt{18.71^2+57.58^2} = 60.55\text{V (peak)}$$

$$a_2 = \frac{V_d}{2\pi}\sin(2\pi\times 2D) = -15.15,\ b_2 = \frac{V_d}{2\pi}[1-\cos(2\pi\times 2D)] = 11.00$$

$$V_{o2} = \sqrt{a_2^2+b_2^2} = 18.71\text{V (peak)}$$

$$a_3 = 10.09, b_3 = 7.33, V_{o3} = 12.47\text{V (peak)}$$

$$a_4 = -4.68, b_4 = 14.40, V_{o4} = 15.14\text{V (peak)}$$

$$a_5 = 0, b_5 = 0, V_{o5} = 0\text{V}$$

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# Example IV

Solution (2) :

$$V_{o1} = 60.55, Z_1 = \sqrt{(2\pi fL)^2+R^2} = \sqrt{(2\pi\times 10^3\times 1\times 10^{-3})^2+10^2} = 11.81$$

$$I_{o1} = \frac{V_{o1}}{Z_1} = \frac{60.55}{11.81} = 5.13\text{A (peak)}$$

$$V_{o2} = 18.71, Z_2 = \sqrt{(2\pi\times 2fL)^2+R^2} = \sqrt{(2\pi\times 2\times 10^3\times 1\times 10^{-3})^2+10^2} = 16.06$$

$$I_{o2} = \frac{V_{o2}}{Z_2} = \frac{18.71}{16.06} = 1.17\text{A (peak)}$$

$$V_{o3} = 12.47, Z_3 = 21.34, I_{o3} = 0.58\text{A (peak)}$$

$$V_{o4} = 15.14, Z_4 = 27.05, I_{o4} = 0.56\text{A (peak)}$$

$$V_{o5} = 0, Z_5 = 32.97, I_{o5} = 0\text{A}$$

Q: How to calculate rms value?

A: for $h^{th}$ order harmonic,

$$\text{rms value} = \frac{\text{peak value}}{\sqrt{2}}$$

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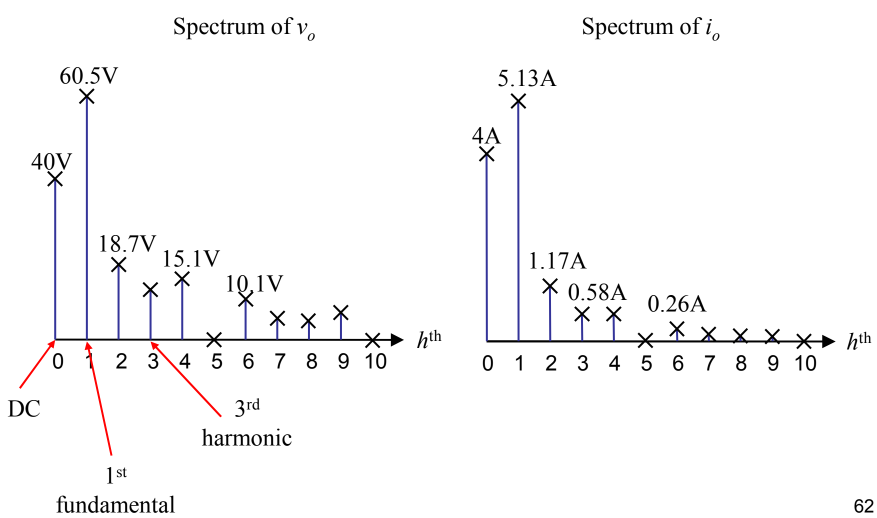
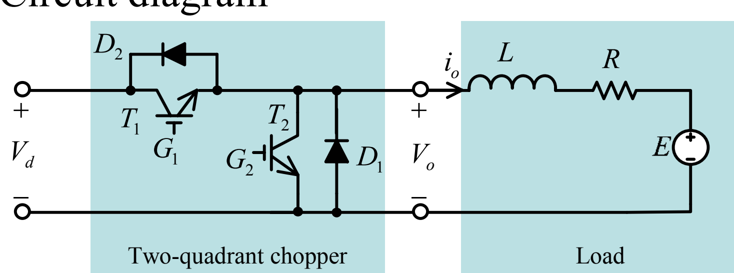
## Example IV



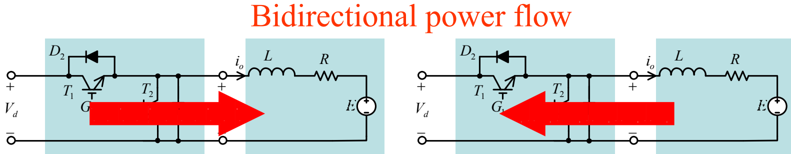



## 1.6 Two Quadrant Chopper

- Circuit diagram



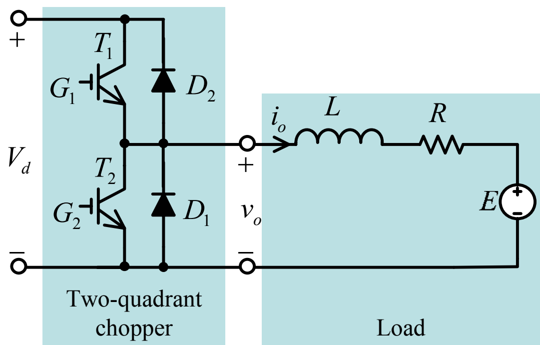

Bidirectional power flow



$I_o > 0$ and $V_o > 0$ $\qquad$ $I_o < 0$ and $V_o > 0$

# 1.6 Two Quadrant Chopper

- Circuit diagram



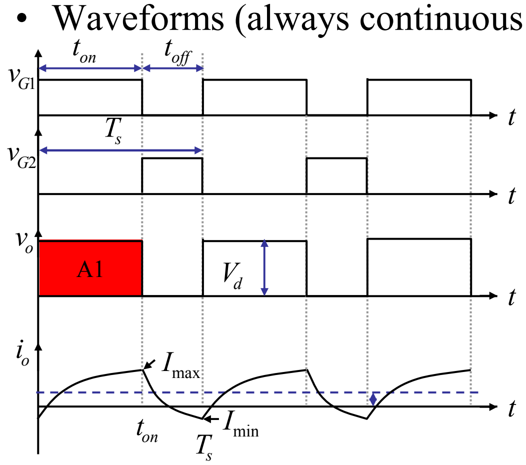

Applications:

- Battery control: charge or discharge the battery.
- DC motor drives: provide fast dynamic braking
- Any dc-dc applications need bidirectional power flow.



# 1.6 Two Quadrant Chopper

- Waveforms (always continuous current mode)




$$V_{o,avg} = \frac{\text{Area A1}}{T_s} = \frac{V_d t_{on}}{T_s} = V_d \frac{t_{on}}{T_s} = DV_d$$

when $T_1 / D_2$ is on :

$$v_o = V_d = L\frac{di_o}{dt} + Ri_o + E \rightarrow$$

$$i_o(t) = \frac{V_d - E}{R}\left(1 - e^{-\frac{t}{\tau}}\right) + I_{min} e^{-\frac{t}{\tau}}$$

when $T_2 / D_1$ is on :

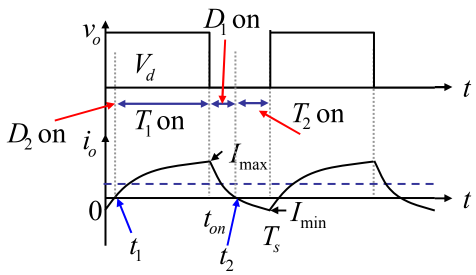
$$v_o = 0 = L\frac{di_o}{dt} + Ri_o + E \rightarrow$$

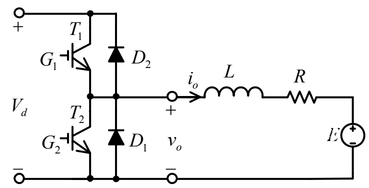
$$i_o(t) = -\frac{E}{R}\left(1 - e^{-(t-t_{on})/\tau}\right) + I_{max} e^{-(t-t_{on})/\tau}$$

where $\tau = L/R$ is time constant of the load

# 1.6 Two Quadrant Chopper

- Current waveforms




$0 \le t < t_1$ : The energy stored in $L$ is releasing to $V_d$, $D_2$ is ON

at $t_1$ : The energy stored in $L$ is fully released

$t_1 \le t < t_{on}$ : The energy from $V_d$ is stored in $L$, $i_o \uparrow$, $T_1$ is ON

$t_{on} \le t < t_2$ : The energy stored in $L$ is releasing to $R$ and $E$, $D_1$ is ON

at $t_2$ : The energy stored in $L$ is fully released

$t_2 \le t < T_s$ : The energy from $E$ is stored in $L$, $|i_o| \uparrow$, $T_2$ is ON

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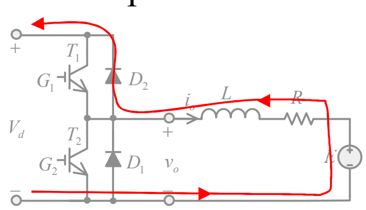
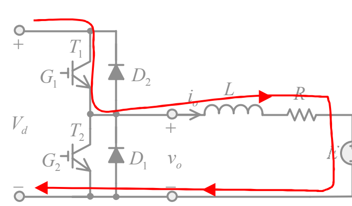
# 1.6 Two Quadrant Chopper

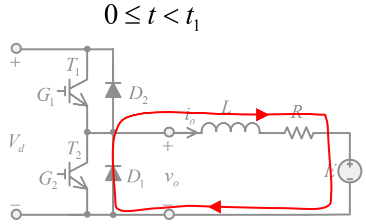
- Current paths



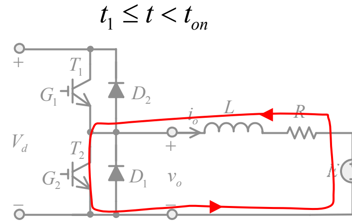
$0 \le t < t_1$



$t_1 \le t < t_{on}$



$t_{on} \le t < t_2$



$t_2 \le t < T_s$

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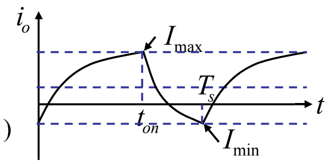
# 1.6 Two Quadrant Chopper

- $I_{max}$ and $I_{min}$




From current waveform

$$i_o(t) = \frac{V_d - E}{R}\left(1 - e^{-\frac{t}{\tau}}\right) + I_{min} e^{-\frac{t}{\tau}}, 0 \le t < t_{on} \qquad (1)$$

$$i_o(t) = -\frac{E}{R}\left(1 - e^{-(t-t_{on})/\tau}\right) + I_{max} e^{-(t-t_{on})/\tau}, t_{on} \le t < T_s \qquad (2)$$

From Eq.(1) At $t = t_{on}, i_o(t_{on}) = I_{max} \rightarrow I_{max} = \frac{V_d - E}{R}\left(1 - e^{-\frac{t_{on}}{\tau}}\right) + I_{min} e^{-\frac{t_{on}}{\tau}}$ (3)

From Eq.(2) At $t = T_s, i_o(T_s) = I_{min} \rightarrow I_{min} = -\frac{E}{R}\left(1 - e^{-(T_s-t_{on})/\tau}\right) + I_{max} e^{-(T_s-t_{on})/\tau}$ (4)

Solve Eq.(3) and Eq.(4)

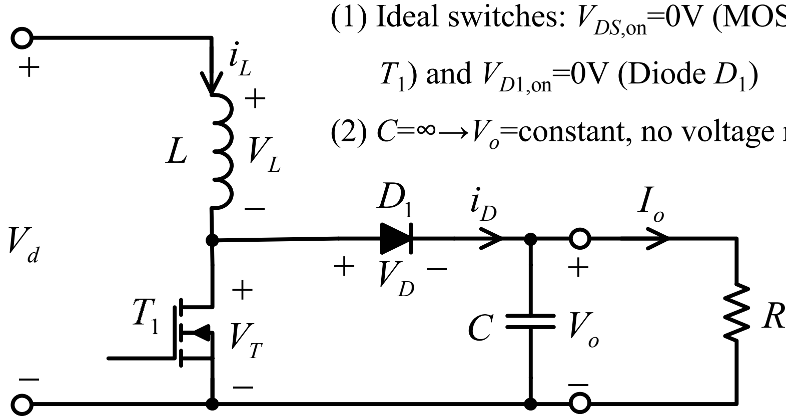
$$I_{max} = \frac{V_d}{R}\frac{1 - e^{-t_{on}/\tau}}{1 - e^{-T_s/\tau}} - \frac{E}{R} \text{ and } I_{min} = \frac{V_d}{R}\frac{e^{t_{on}/\tau} - 1}{e^{T_s/\tau} - 1} - \frac{E}{R}$$

**Note: always continuous current mode!**



# 1.7 Boost Converters

- Topology



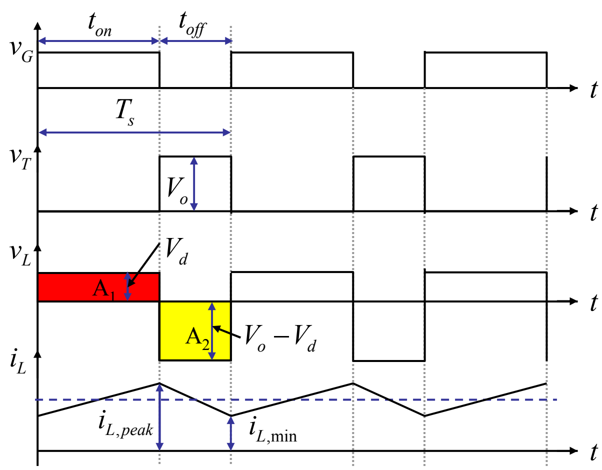

Assumption:

(1) Ideal switches: $V_{DS,on}$=0V (MOSFET $T_1$) and $V_{D1,on}$=0V (Diode $D_1$)

(2) $C=\infty \rightarrow V_o$=constant, no voltage ripple

# 1.7 Boost Converters

- Waveforms (continues current mode)




Area A1 = Area A2.

$\rightarrow V_d t_{on} = (V_o - V_d) t_{off}$

$$\rightarrow V_o = \frac{V_d}{1-D}$$

$0 \le D \le 1 \rightarrow V_o \ge V_d$

$\rightarrow$ boost converter, or step-up converter

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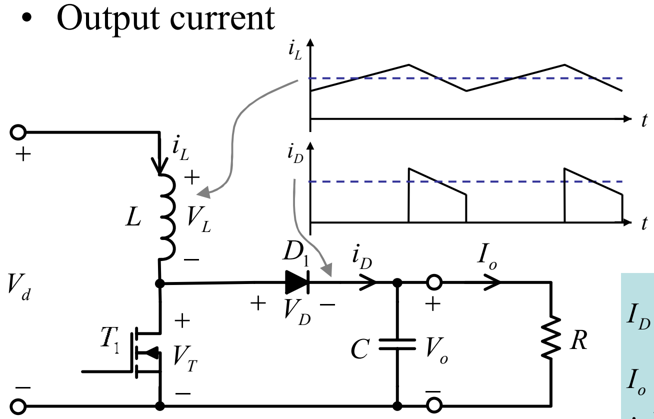
# 1.7 Boost Converters

- Output current




$$I_D = \frac{t_{off}}{T_s} I_L = (1-D) I_L$$

$$I_o = I_D$$

$$\therefore I_o = (1-D) I_L$$

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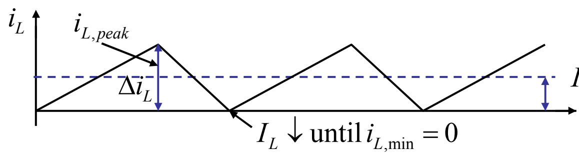
# 1.7 Boost Converters

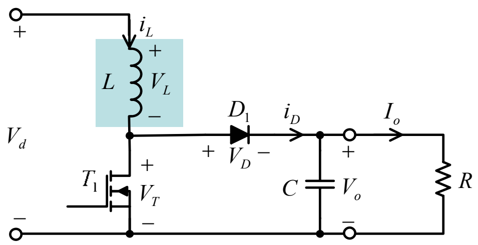
- Boundary between continuous and discontinuous current mode

$i_L$ continuous →continuous current mode

$i_L$ discontinuous →discontinuous current mode




$$I_{LB} = \frac{I_{OB}}{(1-D)}$$




$$v_L = L\frac{di_L}{dt} \rightarrow v_L = L\frac{\Delta i_L}{\Delta t} \rightarrow \Delta i_L = \frac{v_L}{L}\Delta t$$

During $t_{on}$, $v_L = V_d$

$$I_{LB} = \frac{1}{2} i_{L,peak} = \frac{1}{2}\frac{v_L}{L}\Delta t = \frac{1}{2}\frac{V_d}{L}t_{on} = \frac{1}{2L}V_d D T_s$$

$$\therefore I_{LB} = \frac{T_s V_o}{2L} D(1-D)$$

$$I_{OB} = \frac{T_s V_o}{2L} D(1-D)^2$$



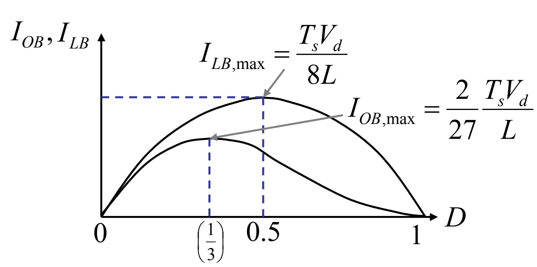
# 1.7 Boost Converters

- Identify the operation mode of buck converter

$$I_{LB} = \frac{T_s V_o}{2L} D(1-D) \text{ or } I_{OB} = \frac{T_s V_o}{2L} D(1-D)^2$$

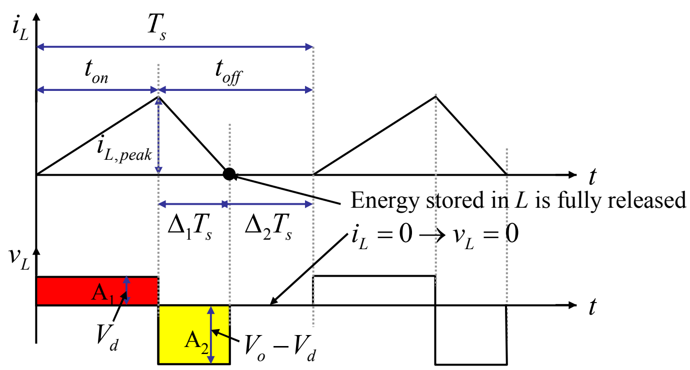
If $I_L > I_{LB}$ or $I_o > I_{OB}$ → continuous current mode

If $I_L < I_{LB}$ or $I_o < I_{OB}$ → discontinuous current mode



# 1.7 Boost Converters

- Discontinues current mode
  - Waveforms






# 1.7 Boost Converters

– Output voltage $V_o$=?

Area $A_1$ = Area $A_2$

$$\rightarrow V_d D T_s = (V_o - V_d)\Delta_1 T_s \rightarrow V_o = \frac{D + \Delta_1}{\Delta_1} V_d \quad (1)$$

$$\Delta i_L = \frac{V_L}{L}\Delta t \rightarrow i_{L,peak} = \frac{V_o - V_d}{L}\Delta_1 T_s \rightarrow \Delta_1 = \frac{L i_{L,peak}}{(V_o - V_d) T_s} = \frac{I_o}{4 I_{LB,\max} D} \quad (2)$$

$$(2) \rightarrow (1)\ V_o = \frac{D^2 + \frac{4}{27}\left(I_o / I_{OB,\max}\right)}{\frac{4}{27}\left(I_o / I_{OB,\max}\right)} V_d \quad \text{for discontinuous current mode}$$

if $V_o$ is constant

$$D = \sqrt{\frac{4}{27}\left(\frac{V_o}{V_d} - 1\right)\frac{I_o}{I_{OB,\max}}}, \text{ where } I_{OB,\max} = \frac{2}{27}\frac{T_s V_d}{L}$$

## Summary

- Topologies: buck converter, one-quadrant chopper, two-quadrant chopper, boost converter, etc.
- Waveforms: current and voltage
- Operating mode and boundary (inductor current): continuous current mode or discontinuous current mode.
- DC component of voltage and current in two operating modes.
- Ripple current and ripple voltage
- Fourier analysis and high frequency equivalent circuits.

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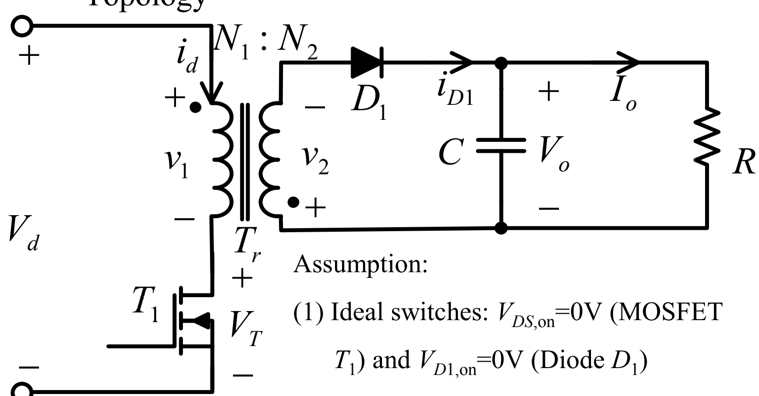
## 1.8 Isolated Switch Mode Power Supplies

- Flyback converter
  - Topology




Assumption:

(1) Ideal switches: $V_{DS,on}$=0V (MOSFET $T_1$) and $V_{D1,on}$=0V (Diode $D_1$)

(2) $C=\infty \rightarrow V_o$=constant, no voltage ripple

(3) No leakage inductance of the transformer

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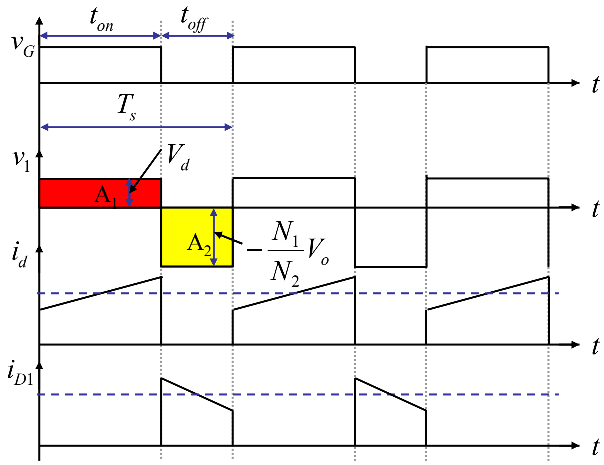
# 1.8 Isolated Switch Mode Power Supplies

– Waveforms (continues current mode)




Area A1 = Area A2.

$$\rightarrow V_d t_{on} = \frac{N_1}{N_2} V_o t_{off}$$

$$\rightarrow V_o = \frac{N_2}{N_1}\frac{D}{1-D}V_d$$

Compare with:

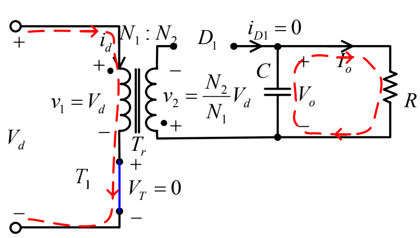
Buck-boost converter

$$V_o = \frac{D}{1-D}V_d$$

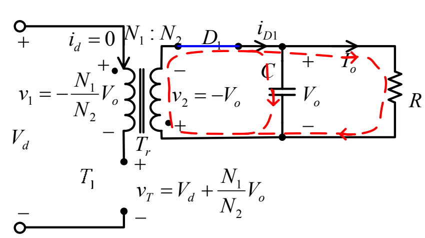


# 1.8 Isolated Switch Mode Power Supplies

– Analysis




$t_{on}$ interval




$t_{off}$ interval

Note:

Energy is stored in the transformer

$i_d\uparrow$, $D_1$ is off, the load is supplied by $C$

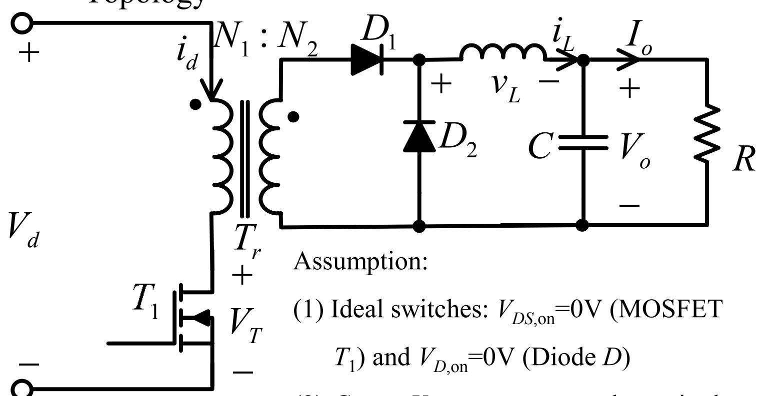
Note:

Energy stored in the transformer is released to $C$ and load

$i_{d1}\downarrow$, $D_1$ is on.

## 1.8 Isolated Switch Mode Power Supplies

- Forward converter
  - Topology




Assumption:

(1) Ideal switches: $V_{DS,on}$=0V (MOSFET $T_1$) and $V_{D,on}$=0V (Diode $D$)

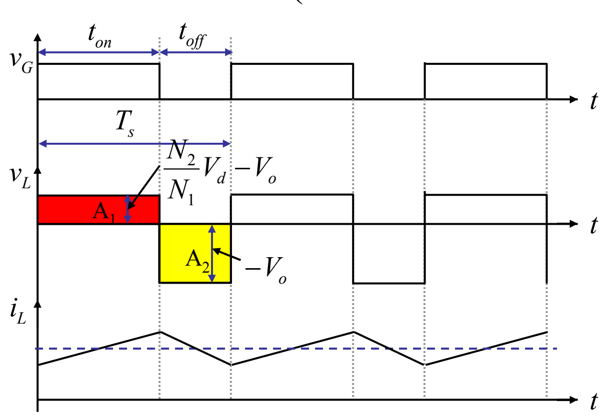
(2) $C=\infty \rightarrow V_o$=constant, no voltage ripple

(3) Ideal transformer



## 1.8 Isolated Switch Mode Power Supplies

- Waveforms (continues current mode)




Area A1 = Area A2.

$$\rightarrow \left(\frac{N_2}{N_1}V_d - V_o\right)t_{on} = V_o t_{off}$$

$$\rightarrow V_o = \frac{N_2}{N_1}DV_d$$

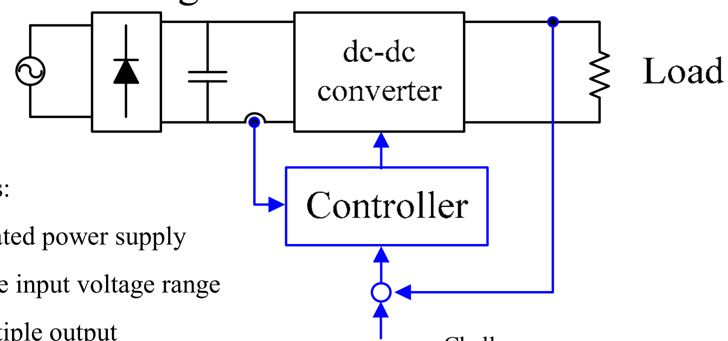
Compare with:

Buck converter

$V_o = DV_d$

## 1.8 Isolated Switch Mode Power Supplies

- Practical design




Features:

(1) Isolated power supply

(2) Wide input voltage range

(3) Multiple output

(4) Low utilization of the transformer

Challenges:

(1) Improve efficiency

(2) Compact size: >50W/in$^3$

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