### 13-6 FINITE-ELEMENT ANALYSIS OF SHEET-PILE WALLS

The finite-element method presented in the following material is the most efficient and rational method for the design of sheet-pile wall design/analysis currently available. The same program is applicable for both cantilever (Fig. 13-1a) and anchored (Fig. 13-1b) walls and, with some adjustment, can be used for the braced walls of Fig. 13-1c. It directly gives the lateral displacement profile (valid for that set of soil parameters and pile stiffness) as well as nodal pressures in the passive zone in front of the wall, bending moments at nodes, and force(s) in the anchor rod(s). Multiple anchor levels can be as readily accommodated as a single anchor; and parametric studies for optimum anchor location can be made very easily, for data copies can be made and edited with the new location.

Any wall material can be analyzed—we are not limited to sheet piles as given in Sec. 13-2. You can use the program for composite sheet piles (part is one material with $E_1$ and part has an $E_2$). In this case it is only necessary to adjust the input so that the program computes $EI$ correctly. For example, if $E_1$ is the base material and you use a second material of $E_2$, simply adjust the moments of inertia $I_m$ so that you have

$$E_1 I_m = E_2 I_2 \rightarrow I_m = \frac{E_2 I_2}{E_1}$$

where $I_m = \text{adjusted value of actual moment of inertia for material } m$.

The FEM analysis finds the center of pressure to sustain the wall in a soil-pile interaction mode rather than making arbitrary assumptions about passive pressure as in the classical methods. Another particular advantage is that the same method of developing the stiffness matrix used for the beam-on-elastic foundation of Sec. 9-8 can be used for sheet-pile walls, so very little new material has to be learned.

The finite-element method uses the same equations as given in Chap. 9 and repeated here for convenience:

$$P = AF \quad e = A^TX \quad F = Se$$

and substituting, we obtain

$$F = SA^TX \quad P = ASA^TX \quad X = (ASA^T)^{-1}P$$

which are the wall deflections consisting of translations and rotations of the several nodes. With the deflections at each node known, the bending moments are computed using the element $ESA^T$ as

$$F = ESA^TX$$

The element shear is computed from the element bending moments, but the node reactions and anchor rod force are directly computed using the spring equation of

$$F = K(I)X(I)$$

Study Fig. 13-8 carefully, for it illustrates the sheet-pile wall and $P-X$ coding, the element forces, soil node springs, and the sign convention—this last is absolutely essential to interpret output. The problem is actually the beam-on-elastic-foundation problem turned $90^\circ$ with the soil springs removed above the dredge line.

Anchor rods are allowed for by considering that an anchor rod will consist in a member of cross section $A$, modulus of elasticity $E$, and some length $L$. Now, the axial displacement in
Finite-element model for either a cantilever or anchored (including multiple anchors) sheet-pile wall. Both soil \( k \) and anchor rod springs are input as nodal entries. Here the anchor is identified with node 2, and the program computes soil springs (\( K_3 \) shown) for nodes 5 through 9, which are then added at NP locations of 4 (anchor rod), 10, 12, 14, 16, and 18 (soil springs) in the stiffness matrix \([\text{STIFF(I)}]\).

This type of member is similar to the bar of a truss and is given in any Mechanics of Materials text as

\[
e = \frac{FL}{AE} = X \quad \text{and obtain} \quad F = \frac{P}{L} = \frac{AE}{L}X
\]

where \( X \) = nodal displacement computed from inverting the \( \text{ASA}^T \) matrix

\( P \) = anchor rod force

To obtain the anchor rod force, one must place the anchor rod at a node. The anchor rod spring(s) \( \frac{AE}{L} \) (where force/length = units of a spring) are part of the input data required by the program. Earth anchors used in tieback construction (Chap. 14) usually slope from the horizontal, but we can input the horizontal component of the spring and obtain the horizontal component of force. It is then a trivial computation to obtain the axial force in the tieback or anchor rod. Since we always analyze a unit width of wall, the \( \frac{AE}{L} \) of the anchor rod is prorated based on the anchor rod spacing \( s \) [which is often more than a unit width—say, 1.5 to 2.0 m (5 to 6 ft)]. For spacing \( s \) (Fig. 13-9c) and defining \( \eta = \) slope with horizontal (Fig. 13-9e), the input anchor rod spring is

\[
K(I)_{ar} = \frac{AE}{sL} \cos \eta
\]

(13-5)
For each anchor rod the preceding computation for its spring is made by hand and input into the data set. An input program parameter identifies the number of springs used, and other input identifies their node locations.

Soil springs are computed by the program in a subroutine and saved into an array for recycling as necessary. Similarly the program builds the banded stiffness matrix (always four entries wide \( \times \) NP).

**GENERAL PROGRAM OVERVIEW.** This program uses a large number of subroutines so that any program modifications can be isolated for easier debugging.

The first subroutine that might be used is a universal subroutine DATAIN, which allows you to create a data file that is always saved to disk on program exit. Since you have only the compiled program the first time you use this program, to create a data file you should do a series of `<PRT-SCREEN>` keypresses to obtain a paper listing of the several lines of input data. You do not have to use this routine if you already have a data file on disk.

There is a USERMANL.DOC file on your diskette that both identifies and gives the order of input for selected data for applicable programs; you should print and file this for a convenient reference when using that program. You should print one data set and write in the variable names so that, when you want to do parameter studies, you can quickly identify the applicable control parameters. The element and most other data are readily identified by looking at the data set.

In subroutine INPUT the element lengths are read from the input file, and on any recycling the element lengths below the dredge line are increased by the input parameter DEPINC. This approach allows us to find the optimum embedment depth by starting with a small value of DEMB and incrementing it using DEPINC. In previous versions of this program the elements were of constant length below the dredge line—this version allows variable lengths initially but all added increments are the value of DEPINC.

The node NPs are computed in subroutine INPUT, so they do not have to be input by hand.

**Incrementing depth of embedment.** When the depth is incremented, the program increases NP by 2. Under these initial conditions,

\[
\begin{align*}
\text{DEPINC} & = 0.3 & \text{DEMB} & = 1.5 \text{ m} \\
\text{15 elements (16 nodes)} & \quad \text{and} \quad \text{NP} & = 32
\end{align*}
\]

the first depth increment gives

\[
\begin{align*}
\text{DEMB} & = 1.5 + 0.3 = 1.8 \text{ m} & \quad \text{16 elements (17 nodes)} \\
\text{NP} & = 32 + 2 = 34 \quad \text{and so on}
\end{align*}
\]

If an equation is used to compute \( k_s \) and additional nodes are created using DEPINC, the program automatically computes \( k_s \) for the new node. If you input values of \( k_s \) for each node, you must assume that the program may increment the depth NCYC times. Thus, it is necessary to input sufficient *additional* node \( k_s \) values so that there is a value for any new node produced. The number NK of \( k_s \) to input is as follows:

\[
\begin{align*}
\text{NCYC} = 1: & \quad \text{NK} = \text{NM} - \text{JTSOIL} + 2 \\
\text{NCYC} > 1: & \quad \text{NK} = \text{NM} - \text{JTSOIL} + \text{NCYC} + 2
\end{align*}
\]

If you do not input enough \( k_s \) values according to the preceding, the program will output a message and stop. If this happens, use your DOS editor to recover the disk file and insert the additional \( k_s \) entries as required.
Subroutine LOAD allows us to input the node pressures from the top node to the first node below the dredge line (1 to JTSoIL + 1). One must input a value of 0.0 for the first node below the dredge line, as the program uses the pressure profile illustrated in Fig. 13-8d to compute node forces at nodes 1 through JTSoIL + 1 using the average end area method. This subroutine also allows input of node P matrix entries using NNZP > 0, so a strut/anchor rod can be modeled as either a force or a spring. The load (P) matrix is saved for reuse when NCYC > 1.

Subroutine SPRING computes both the node $k_s$ (if an equation is used) and the soil springs below the dredge line. If an equation is used for the soil below the dredge line to obtain node $k_s$ the program allows the use of two reduction factors, FAC1 and FAC2. Factor FAC1 is used to reduce the $k_s$ values as follows:

$$SK(JTSoIL) = FAC1 \times SK(JTSoIL)$$
$$SK(JTSoIL+1) = FAC2 \times SK(JTSoIL+1)$$

In earlier editions of this text a single factor REDFAC was used to reduce the dredge line soil spring for driving and other disturbance. It has since been found that it is more realistic to reduce the soil modulus. The preceding reductions will affect the top three node springs by varying amounts. Since FAC1, FAC2 are not specified, they can be 1.0, but their relationship must be FAC2 $\geq$ FAC1. Usually, take FAC1 on the order of 0.6 to 1.0 and FAC2 on the order of 0.7 to 1.0.

If node $k_s$ values are input, any reductions for dredge line damage or for other causes are made before their entry so that the control parameter to input soil node springs will be input as NRC = 0.

It is in this subroutine that the anchor rod springs are input using IAR = number of anchor rods. We input all springs via node identification ($J$) and spring value (SPRNG).

Subroutine BSTIF is then called to build the element stiffness matrix ESAT and EASAT for each element in turn. This routine calls subroutine BANDM to band the global ASAT. The result is a rectangular matrix four columns wide $\times$ NP rows, a particularly attractive feature of this program over a finite-difference method. This is saved in a single array STIFF(I) to save memory. The ESAT is saved in this routine so that it can be used later to compute the element bending moments. This routine is used each time the depth is incremented.

Subroutine MODIF is next called so that the stiffness matrix can be modified to add the previously computed soil (and anchor) springs to the appropriate diagonal nodes. This routine also allows input of boundary conditions based on NZX > 0. Those NP values that have known displacements (say, zero translation and/or rotation) are input in array NXZERO(I) and the known displacements input in array XSPEC(I) and in the same order. This procedure allows us, for example, to fix the top of the wall (or any other node location). Although the program allows nonzero XSPEC(I) values, seldom will we know any boundary displacements other than zero.

LISTB allows you to write the band matrix so that you can check whether the boundary cases were correctly identified. At a boundary location there should be a 1.0 in the first column (the diagonal) and three horizontal and diagonal 0.0s from that position.

Subroutine SOLVI is called next to reduce the band matrix, and in the process it replaces the P matrix with the displacement matrix. This is the reason for saving the original P matrix when NCYC > 1.

Subroutine CONVER is called if NCYC > 1, and in this case the program always does at least two cycles so that the current and previous dredge line displacements can be compared.
When two successive values are within the range of the input value CONV (usual range of 0.002 to 0.003 m) recycling stops.

In case convergence is not obtained in NCYC iterations, inspect the last output (also check for any input errors) and/or increase the initial depth of embedment DEMB and rerun.

Subroutine CHECK is called after dredge line convergence if NONLIN > 0 to see if any dredge line displacements X(I) > XMAX(I). For a valid check, short element lengths should be used in a zone near the dredge line, since it is nodes in that zone that will have any X(I) > XMAX(I). When a node displacement is larger than XMAX(I), a node force is computed as follows:

\[ F = -K(I) \times XMAX(I) \]  

This force (with sign) is inserted into the P matrix and spring K(I) is set to 0.0, and the problem is recycled until the number of springs set to zero equals the number required to be set to zero. More than one spring may be zeroed on any cycle.

Setting a spring to zero can produce a substantial discontinuity in the soil-pile model. The effect is reduced by using closer-spaced nodes.

It is also necessary to recycle when making a nonlinear check. Since the negative force K(I) * XMAX(I) is less than was required by the previous analysis, the node displacement will increase. This change may result in the next lower node having X(I) > XMAX(I), and so on; if this were to occur for all the nodes the system would be unstable and you might get a halt in execution with an exponent overflow error reported. Otherwise, you get very large displacements in the soil below the dredge line, which indicate a shear failure.

When the program recycles for nonlinear effects the embedment depth is not incremented.

The nonlinear check is a reasonably realistic procedure for this model and usually produces three items of considerable interest:

1. Dredge line zone displacements are increased.
2. Bending moments in the pile are slightly increased.
3. Anchor rod force increases.

Subroutine FORCE is called when convergence and displacement criteria (or NCYC = 1) are met. By using the last-computed displacement matrix, the element end moments, shears, the anchor rod force, and the soil spring reactions (forces) \( R(I) \) and the soil pressures \( Q(I) \) are computed. These soil values are computed as

\[ R(I) = X(I) \times K(I) \quad \text{and} \quad Q(I) = X(I) \times SK(I) \]

The program uses SK(I) for \( k_s \); Q(I) for soil pressure \( q \); and I for the translation NP-value.

**Steps in a sheet-pile wall design.** Steps in making a finite-element solution should include at least the following:

1. Assemble available site (importance factor) and soil information.
2. Draw the soil-wall system to a reasonable scale and decide on node locations. Tentatively locate anchor rod node(s) since a search may be made for a best node location. Both bending moments and rod forces are sensitive to location of the anchor rod node.
Locate nodes where soil stratum changes occur and at the GWT. Try to keep the ratio of adjacent element lengths under 5 and preferably under 3. Element lengths do not have to be constant, as for the finite-difference method. Where output is not limited (as it is in the following text examples, to save text pages) lengths should be on the order of 0.4 to 0.6 m with some as short as 0.3 m in critical regions.

3. Compute the lateral soil pressure from ground surface to the dredge line using $K_a$ from the Coulomb (preferably) or Rankine equations; however, it may be appropriate to use a larger value if site conditions warrant. Where the strata change, use an average pressure value, which will introduce a small computational error for the node force unless the two contributing elements are equal in length. There can be much busywork in this step, so a program such as B-25 is recommended.

4. Estimate $k_s$ below the dredge line. For depths up to about 5 m there should not be a great difference between the value for the dredge line and that for lower nodes; for clay a constant value based on the upper soil may be adequate. For sand there would be a small increase with depth.

5. Locate any nodes where you will input soil springs ($NRC > 0$) to replace program-computed values. These may be where marked differences in adjacent strata occur, soft lenses or thin strata of poor soil have been identified, cavities are known, and similar. Note: It may be preferable to input all node $k_s$ values (an equation would probably not apply in this case anyway). That way, node springs that would give an incorrect soil pressure would not have to be input.

6. Select a tentative wall section and obtain the moment of inertia/unit width and section modulus/unit width so that the output moments can be checked for actual bending stresses. It is a trivial task to edit a copy of the data file to use a stiffer (or less stiff) section.

7. Select a tentative anchor rod cross section $A$, length $L$, and spacing $s$ so the anchor rod spring can be computed using Eq. (13-5). It is a trivial task to edit the data file to input a larger- (or smaller-) diameter anchor rod. It is possible, however, to use other sections such as double angles, small I or W sections, square rods, etc. for the anchor “rod.” Rods are usually more practical.

8. You have the option of either inputting an anchor rod force or a spring—the spring is usually preferable. You also have the option of inputting either node forces or node earth-pressure values—pressure is usually preferable. If you input a node force for the anchor rod, use $IAR = 0$ (no rod) and use $NNZP = 1$ to input the force. If you input node forces in lieu of the wall pressures input $IPRESS = 0$ and $NNZP = IPRESS$.

9. Check the output for overstress or excessive displacements. The largest element moment, anchor rod force, and soil pressure are checked by the user for

$$f_s = \frac{M}{S} \leq f_a$$
$$f_s = \frac{F_{arr}}{A} \leq f_a$$
$$Q(I) \leq q_a$$

You will have allowable stress values $f_a$ for the piling and anchor rod but you may not have a $q_a$ for the soil. Even so, you can still check if the node soil pressures are reasonable or possible. They probably should not exceed the vertical bearing capacity or the passive earth pressure (or force using program WEDGE) at about the middepth of embedment.

Finally, check the node displacements below the dredge line. If they are all forward and sufficiently large, it is evident that a slip failure has formed. For example, if the
bottom node has a +X displacement of 0.002 m (about 2 mm) this is negligible; however, if the +X displacement is 0.003 to 0.010 m this may be large enough for a slip failure in the base soil. The embedment depth should be increased and the entire design recycled. Depending on anchor rod location, wall height, and stiffness, one or more of the nodes above the top anchor rod may have a (−) displacement, indicating the development of passive pressure. You might approximate this by rerunning the data set with the active pressure entries increased by using a small surcharge whose magnitude depends on the type of backfill and the (−) displacement—perhaps 10 kPa for a dense sandy backfill when the (−) X is on the order of 0.006 m.

10. The overall wall stability must be checked when a design has been produced for which statics are satisfied, none of the elements are overstressed, and displacements are not deemed excessive. The overall stability is considered in some detail in Sec. 13-9.

13-7 FINITE-ELEMENT EXAMPLES

The following examples will illustrate the FEM in a general manner and can be reproduced using program FADSPABW (B-9) on your diskette with the included data sets. Expertise can only be gained by making a number of computations in parametric studies, which are beyond the scope of a textbook. Also it is not possible to show the iterations necessary to optimize any of these examples because there is too much output for a textbook. The data sets are included so that you can do this without much effort.

Example 13-1. Anchored sheet-pile walls (or anchored bulkheads).

Given. the soil-wall system in a silty cohesionless material as in Fig. E13-1a. The initial location of the wall line is such that about half the depth shown is initially retained and material is to be dredged from the front. This location allows the piling to be driven and the anchors set. Then the remaining front soil is excavated and mixed with imported sand to produce a backfill with properties estimated as shown. The top will be paved so that boats can load and unload. We will account for those activities with a 25 kPa surcharge. The soil below the dredge line is a silty clay with some sand, and the average of consolidated-undrained tests on several tube samples gives the properties shown.

Required. Find a suitable rolled sheet-pile section and anchor rod for the system.

Solution. Estimate \( \gamma = 16.50 \text{ kN/m}^3 \) above and below the water line. The angle of internal friction \( \phi \) may be on the order of 34 to 36°, but we will conservatively use \( \phi = 30° \) since the dredged soil will be somewhat loose—at least initially.

With the same water level on both sides of the wall (the interlocks are seldom watertight unless sealed as noted in Sec. 13-2.3) the water pressure is ignored. We must, however, use \( \gamma' = 6.70 \text{ kN/m}^3 \) in computing the lateral earth pressure below the water line. The dredge line \( \gamma_{sat} \) is obtained by trimming a sample and performing a direct measurement as in Example 2-1.

Step 1. Draw Fig. E13-1a based on the given data and tentative node locations; plot Fig. E13-1b to keep track of the initial P–X coding. Plot the lateral earth-pressure profile of Fig. E13-1c for reader convenience. The pressure profile uses the Coulomb \( K_a = 0.3 \) shown since there was little variation for any reasonable \( \delta \) angle. The information to plot the node forces of Fig. E13-1d was obtained from outputs of an initial trial program execution. They can also be computed from the pressure profile of Fig. E13-1c using the average end area method, and nodes 1 and 2 are hand-computed (for illustration) as follows:

Node 1:

\[
P(2) = \frac{7.5 \times 1.2}{2} + (13.4 - 7.5) \times \frac{1.2}{2} \times \frac{1}{3} = 5.7 \text{ kN}
\]
Figure E13-1a–e

(a) Wall profile.
(b) P–X coding.
(c) Pressure profile.
(d) Node forces.
(e) XMAX(I) definition.
Required. Find a suitable rolled sheet-pile section and anchor rod for the system.

Node 2: \[ P(4) = \frac{12}{2} (7.5 + 13.4 + 13.4 + 19.4) = 16.1 \text{ kN} \]

Figure E13-1e illustrates the significance of using a different XMAX(I) for the top several nodes below the dredge line.

Comments

1. The node spacing above the dredge line is as shown to save space. Ideally a node spacing of 0.6 (instead of 1.2) and 0.5 (instead of 1.0) would be used. The element length transition ratio at the dredge line is 1.0/0.3 = 3.333, and 0.5/0.3 would be much preferred.

2. As part of this design, additional nodes are shown below the 1.8-m initial embedment depth. You will see their purpose later.

3. The node forces of Fig. E13-1d are from computer output sheets and would not usually be shown like this. A better location is on the output sheets beside their listing—or not at all.

Step 2. From the initial \( P-X \) coding and general node configuration for the total depth of \( 9.0 + 1.8 = 10.8 \text{ m} \), obtain the following initial values (the text output sheets will use slightly different values in some cases):

\[
\begin{align*}
\text{NM} &= 14 \quad \text{(initial count of 8 above and 6 below the dredge line)} \\
\text{NP} &= 30 \quad [2 \times (\text{NM} + 1)] \\
\text{There are no input forces} &\Rightarrow \text{NNZP} = 0 \\
\text{Use 1 load case} &\Rightarrow \text{NLC} = 1 \\
\text{For a sheet-pile wall, ITYPE} &= 1 \\
\text{We do not need a listing of the band matrix} &\Rightarrow \text{LISTB} = 0 \\
\text{Recycle limit} &\Rightarrow \text{NCYC} = 5 \\
\text{No soil springs to input} &\Rightarrow \text{NRC} = 0
\end{align*}
\]

The foregoing eight parameters are the first line of input in the given sequence after the project \text{TITLE}. The next line in order is as follows:

\[
\begin{align*}
\text{Dredge line soil starts at node 9 by count} &\Rightarrow \text{JTSOIL} = 9 \\
\text{Activate the nonlinear routine} &\Rightarrow \text{NONLIN} = 1 \\
\text{Anchor rod at node 2} &\Rightarrow \text{IAR} = 2 \\
\text{No known displacements} &\Rightarrow \text{NZX} = 0 \\
\text{There are JTSOIL + 1 pressure entries} &\Rightarrow \text{IPRESS} = 10 \\
\text{We are using SI units} &\Rightarrow \text{IMET} = 1
\end{align*}
\]

The next line of input contains the following (in order):

\[
\begin{align*}
E &= \text{modulus of elasticity of steel pile} = 200\,000 \text{ MPa} \\
\text{DEMB} &= \text{initial embedment depth} = 6 \times 0.3 = 1.8 \text{ m} \\
\text{CONV} &= \text{dredge line displacement convergence} = 0.002 \text{ m (2 mm)} \\
\text{DEPINC} &= \text{depth increment for recycling} = 0.3 \text{ m} \\
\text{BSHP} &= \text{width used (usually 1 unit)} = 1.0 \text{ m}
\end{align*}
\]
Sheet-pile and anchor rod sections must be selected and later revised as necessary. For the initial trial let us use an anchor rod with these properties:

- **Diam** = 55 mm
- **Spacing** \( s = 1.83 \text{ m} \)
- **Length** \( L = 10.83 \text{ m} \)
- **Steel grade** = 250 (A-36 with \( f_y = 250 \text{ MPa} \))
- \( f_{a,ar} = 0.6 f_y = 0.6(250) = 150 \text{ MPa} \)

The anchor rod area is

\[
A = 0.7854(0.055)^2 = 2.3758 \times 10^{-3} \text{ m}^2
\]

and the spring [using Eq. (13-5) with \( \eta = 0^\circ \) is

\[
ARSPG = \frac{AE \cos \eta}{sL} = \frac{2.3758 \times 10^{-3} \times 200 \times 10^{-3}}{1.83 \times 10.83} = 23974 \text{ kN/m}
\]

Try a PZ32 pile section using A-328 steel (Grade 250) with \( f_y = 250 \text{ MPa} \) and

\[
f_{a,p} = 0.6 f_y = 0.6(250) = 150 \text{ MPa}
\]

and convert table values for \( I \) and \( S \) for pile width to values per 1 meter of wall width, giving

- **Moment of inertia** \( I = \frac{I_{\text{table}}}{\text{Width}} = \frac{283.7 \times 10^{-6}}{0.575} = 0.4934 \times 10^{-3} \text{ m}^4/\text{m} \)
- **Section modulus** \( S = \frac{S_{\text{table}}}{\text{Width}} = \frac{1.498 \times 10^{-3}}{0.575} = 2.605 \times 10^{-3} \text{ m}^3/\text{m} \)

We will use an approximate equation for \( k_s \). From Table 4-4 we obtain Hansen bearing-capacity factors of 8.34, 2.5, and 0.4 at \( \phi = 10^\circ \) and compute

\[
\begin{align*}
AS &= 40[cN_c + 0.5\gamma(1)(N_y)] \\
&= 40[20 \times 8.34 + 0.5(17.0 - 9.81)(1)(0.4)] = 6673 \\
BS &= 40(7.19)(2.5) = 719
\end{align*}
\]

Round and use  \( AS = 7000 \)  \( BS = 1000 \)  (equation is approximate)

Since the dredge line will be excavated, use soil modulus reduction factors (but not for the case of lost dredge line depth) as

- **FAC1** = 0.80
- **FAC2** = 0.90

We do not want \( k_s \) to increase much with depth, so use the following equation form (a program option) instead of \( Z^4 \):

\[
k_s = SK(I) = 7000 + 1000 \tan^{-1}(Z/D)
\]

where \( D \) = embedment depth on any cycle

\( Z \) = depth from dredge line to the current node

Referring to Fig. E13-1a, b, and c, we will make a program execution using \( \text{DEMB} = 1.8 \text{ m} \); \( \text{NCYC} = 5 \); \( \text{NONLIN} = 1 \); and setting all \( XMAX(I) = 0.5 \text{ m} \).

From this output the nodal displacements \( XMAX(I) \) are revised to those shown on the output sheets (0.010, 0.015, 0.020, 0.022, and the remainder at 0.025). Their significance is shown on Fig. E13-1e.

Let us somewhat arbitrarily make some additional executions using \( \text{NCYC} = 1 \); \( \text{NONLIN} = 1 \); and for \( \text{DEMB} = 1.8, 2.4, 3.0, \) and \( 3.6 \text{ m} \). These executions are summarized in the following table:
From the output sheets for each trial, the maximum moment occurs at node 6; $q_{\text{max}}$ occurs at node 10. From the preceding table tentatively select an embedment depth DEMB = 3.0 m. This gives a reasonable driving depth, and we will consider in the stability analysis a loss of dredge line of 0.6 m (leaving only 2.4 m—for a 1.8-m initial depth the dredge line converged at 2.1 m, which is very close to 2.4 m). We will also consider the possibility of the surcharge somehow becoming doubled (from 25 kPa as used above to 50 kPa). Part of this effect might derive from an actual surcharge increase that increases the lateral pressure; another possibility is that the active pressure might not fully develop if the anchor rod spring does not stretch sufficiently.

With these considerations a copy of the initial data file is made and named EX131.DTA, (on your diskette). It was edited for depth of embedment DEMB = 3.0, NM = 16 (two bottom elements of 0.6 m added), and NCYC = 1 (we do not want to increment since we already know from using the 1.8-m depth that convergence is obtained on the first cycle). Use NONLIN = 1 (we do want to check the dredge line for possible $X(9) > X_{\text{MAX}}(9) = 0.010$ m (10 mm). This set of output is shown as Fig. E13-1f.

We make copies of file EX131.DTA as EX131A.DTA and EX131B.DTA (all on your diskette) and edit them. EX131A.DTA is edited for a 0.6-m loss of dredge line so that for DEMB = 3.0 - 0.6 = 2.4, we use FAC1, FAC2 = 1.0 (not 0.8 and 0.9 of EX131.DTA). We must recompute the dredge line soil pressure and include nodes 10 and 11. The clay below node 9 produces a discontinuity as shown in Fig. E13-1c and the two values are “averaged.” The other two nodes have values as shown.

For the surcharge increase from 25 to 50 kPa we edit file EX131B.DTA for the new pressure profile (not shown; but at node 1 it is 15.0 instead of 7.5 kPa). Refer to the data file for the pressure profile if you wish to check it—actually, all values merely increased by 7.5 kPa.

These files were executed and the data are summarized in the following table:

<table>
<thead>
<tr>
<th>For: Design</th>
<th>D.L. loss of 0.6 m</th>
<th>Surcharge = 50 kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>JTSOIL = 9</td>
<td>IPRESS = 10</td>
<td>JTSOIL = 9</td>
</tr>
<tr>
<td>DMB = 3.0</td>
<td></td>
<td>IPRESS = 10</td>
</tr>
<tr>
<td>Value Increase, %</td>
<td>Value Increase, %</td>
<td>Value Increase, %</td>
</tr>
<tr>
<td>$\delta_{\text{D.L.}}, \text{mm}$</td>
<td>13.7</td>
<td>16.7</td>
</tr>
<tr>
<td>$\text{Mom}_{\text{max}}, \text{kN} \cdot \text{m/m}$</td>
<td>213.1</td>
<td>270.9</td>
</tr>
<tr>
<td>$F_{ar}, \text{kN}$</td>
<td>106.6</td>
<td>116.9</td>
</tr>
<tr>
<td>$\delta_{ar}, \text{mm}$</td>
<td>4.4</td>
<td>4.9</td>
</tr>
<tr>
<td>$q_{\text{max}}, \text{kPa}$</td>
<td>76.6</td>
<td>102.8</td>
</tr>
<tr>
<td>$\delta_{\text{max}}, \text{mm}$</td>
<td>23.7</td>
<td>31.5</td>
</tr>
<tr>
<td>$D(D/2.1), %$</td>
<td>1.42</td>
<td>1.42</td>
</tr>
</tbody>
</table>
A check of the pile (PZ35) and anchor rod (diam. = 50 mm) stresses yields the following (for anchor rod include the spacing s):

\[
f_{\text{ar}} = \frac{sF}{A} = \frac{1.83 \times 148.2}{2.3758 \times 10^{-3} \times 10^3} = 114.1 < 150 \text{ MPa} \quad \text{(O.K.)}
\]

\[
f_{\text{pile}} = \frac{M}{S} = \frac{286.2}{2.605 \times 10^{-3} \times 10^3} = 109.9 < 150 \text{ MPa} \quad \text{(also O.K.)}
\]

From the stresses this section appears somewhat overdesigned, however, several considerations should be made. First, it is a trivial matter to edit the three data files (EX131.DTA, EX131A.DTA, EX131B.DTA) to use a different section (perhaps a PZ27). Second, note the maximum node displacement above the dredge line from the design case of 23.7 (say, 24 mm or 1 in.) is 31.5 (say, 31 mm or 1.25 in.). These displacements are below the water line but may be noticeable. From the information tabulated, one can say with certainty, without changing sections and making additional trials, that the displacements would increase with a smaller pile section.

What one should do is to create a more realistic P-X coding using 0.6-m and 0.5-m elements above the dredge line, and try moving the anchor rod to the new node 4 or 5, and make new executions.

One might try using either a 35- or 40-mm diameter anchor—but a small diameter rod will increase the lateral displacements above the dredge line. This modification clearly has merit, since the current rod elongation of 4.4 mm may not be enough to allow active earth pressure, using as a guide that the wall should translate about 0.001H, giving 0.001(9.0 - 2.4) = 0.001(6.6 \times 1000) = 6.6 \text{ mm} > 4.4 \text{ mm}. Be careful when considering anchor rod diameter. If the rod is normally threaded, the actual area is less than the nominal area because the area is calculated to the thread root (see Table 8-4). If the threads are upset, the actual rod area can be safely used, but a rod with upset threads costs more.

There is some opinion that the anchor rod force will increase with time as the soil settles from beneath the rod. The rod then becomes a beam supported at the wall and at the anchorage, and in addition to the axial anchor rod load it now carries the depth of soil above + its self-weight as a uniform loading along the rod length. It has been suggested that this long-term loading can nearly double the initial anchor rod force—in this case from 106.6 to 213 kN—and the allowable stress would be exceeded.

For the design case we use FAC1 = 0.8 and FAC2 = 0.9 and calculate the following (note the use of DEMB = D = 3.0 here):

\[
SK(9) = 0.8[7000 + 1000 \tan^{-1}(0.0/3.0)] = 5600. \text{ kN/m}^3
\]

\[
SK(10) = 0.9[7000 + 1000 \tan^{-1}(0.3/3.0)] = 6389.702
\]

\[
SK(11) = 7000 + 1000 \tan^{-1}(0.6/3.0) = 7197.396 \ldots \text{and so on}
\]

For the dredge line loss we use the same equation, but there is a design question of whether it should have been adjusted for the depth lost—I arbitrarily decided not to since FAC1, FAC2 are taken as 1.0. In this case \(k_s\) is computed as

\[
SK(JT\text{SOIL}) = SK(11) = (7000 + 0) = 7000. \text{ kN/m}^3
\]

\[
SK(12) = 7000 + \tan^{-1}(0.3/2.4) = 7124.35
\]

\[
SK(13) = 7000 + \tan^{-1}(0.6/2.4) = 7294.979
\]

\[
\vdots
\]

\[
SK(17) = 7000 + \tan^{-1}(2.4/2.4) = 7785.398
\]

The computer output sheets of Fig. E13-1f show the final design choice using data set EX131.DTA with DEMB = 3.0. Thus, there are several changes from the initial input (different NP, NM). You should identify the changes from the original input data used for the preliminary trial (not shown).
**Example 13.1 of Found Analy & Design 5/E—Use 3.0 m as Design—SI Units**

+++++THIS OUTPUT FOR DATA FILE: EX131.DTA+++++

**Solution for Sheet Pile Wall—Cantilever or Anchored**

<table>
<thead>
<tr>
<th>MEMNO</th>
<th>NP1</th>
<th>NP2</th>
<th>NP3</th>
<th>NP4</th>
<th>LENGTH</th>
<th>INERTIA</th>
<th>NODE KS</th>
<th>SPRINGS</th>
<th>XMAX</th>
<th>NODE Q</th>
<th>NODE P</th>
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</thead>
<tbody>
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<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>1.2000</td>
<td>0.0004934</td>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>7.5000</td>
<td>5.6800</td>
</tr>
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<td>0.00</td>
<td>23974.000</td>
<td>0.0000</td>
<td>13.4000</td>
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<td>7</td>
<td>8</td>
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<td>0.0004934</td>
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<td>0.0004934</td>
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<td>29</td>
<td>30</td>
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<td>0.0004934</td>
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<td>7674.741</td>
<td>4602.478</td>
<td>0.2500</td>
<td>7674.741</td>
</tr>
</tbody>
</table>

* = KS reduced by FAC1 or FAC2

+++NON-LINEAR CHECK: CURRENT CYCLE, ICYC = 0 CURRENT SPRGS ZEROED = 1 PREVIOUS COUNT = 0
CURRENT D.L. X(I) = .01341 PREVIOUS D.L. X(I) = .01341

Figure E13-1f
### Data for Plotting

<table>
<thead>
<tr>
<th>Node</th>
<th>Depth</th>
<th>x</th>
<th>comp x/mm</th>
<th>x/mm</th>
<th>shear v(i,1)/v(i,2)</th>
<th>moment mom(i,1)/mom(i,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.00</td>
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</tr>
</tbody>
</table>

**Figure E13-1f (continued)**
The final design clearly needs refinement but this will not be done here because of space limitations.

Discussion of computer output.

1. The program informs you of any recycling based on both NCYC and NONLIN with adequate identification so you can see what was done.

2. The program puts an * beside any SK(I) that have been reduced (FAC1, FAC2 < 1.0). If FAC1 = 1 then FAC2 should also equal 1.0, but if either value is 1.0 the * is not printed for that node.

3. The program puts an * beside nodes where X(I) > XMAX(I) so you can verify (if desired) that the node reaction is computed as

\[ R = XMAX(I) \times K(I) \quad q = XMAX(I) \times SK(I) \]

4. The revised P matrix is output so you can see the effect of inserting the (−) spring force when X(I) > XMAX(I).

5. The program sums the node soil reactions together with the anchor rod and outputs this value along with a sum of the active earth node forces so you can make a visual check of \( \sum F_h = 0 \).

6. The moment table is output along with the spring forces and other data so you can make a visual check that at the ends the element moment is nearly 0 (Node 1 should always be 0 unless the top is embedded in a concrete slab, as in a pier) and is restrained. Computer round-off error using single precision may give small nonzero values (exactly 0.000 is shown on the output sheet but this is unusual).

You can make an instant visual moment check since the far-end moment of element I should equal the near-end moment of element I + 1 with a sign change. For element 1 the near-end moment = 0.000; the far-end moment = 6.816; the near-end moment of element 2 = −6.816. This means that the \( \sum M \) for node 2 = 0(6.816 − 6.816 = 0) ⋯ and so on.

7. The output sheet lists a table for plotting. These data are saved to a disk file if specified at the beginning of program execution. It is always output, however, so you can plot the displacement profile and superimpose on it the XMAX(I) profile below the dredge line. This file is also useful to make a quick handplot of the shear and moment diagrams as shown in Fig. E13-Ig. These diagrams may require interpretation, but this should not be a problem. You know that between the anchor rod and dredge line the piling bulges outward creating compression on the backfill side.

The shear (and direction) for node 1 is

\[ V = \frac{F_1 + F_2}{L} = \frac{0.000 + 6.816}{1.2} = 5.68 \]

The direction derives from using element moment sign conventions. At node 2 (the anchor rod) we have

\[ V_{\text{top}} = 5.68 \quad \text{and from element 2} \]

\[ V_{\text{bot}} = \frac{-6.816 + (-95.015)}{1.2} = -84.859 \]

Check this statement as

\[ \text{To left } 5.68 + 16.01 - 106.639 = -84.859 \]

This expression says the sum of node forces from the top to node 2—the anchor rod force is the shear. It is much easier, however, to get the shears directly from \( V = (F_1 + F_2)/L \) but you need the sign convention for the \( F_i \) (element moments), which is shown on Fig. 13-8c.
Figure E13-1g

(a) Displacement

(b) Shear

(c) Moment
Example 13-2. It is required to find the embedment depth, anchor rod force, and an adequate sheet pile section if the dredge line of Example 13-1 has the slope $\beta = -15^\circ$ as shown in Fig. E13-2a. The figure has been reversed to look from the left, whereas in Fig. 13-1a we look from the right side of the wall and parallel along it. The view here matches the profile used in program WEDGE.

Solution. From the several trial runs of Example 13-1 we will tentatively try the embedment depth of 3 m, use NCYC = 5, and activate NONLIN = 1. We will use FAC1 = 0.8 and FAC2 = 0.9 for dredge line damage and the same XMAX(I) values.

We must also adjust $k_s$ for the sloping dredge line. For this we will use data sets WDG132A.DTA and WDG132B.DTA, provided on your program diskette, with program WEDGE to obtain $P_{p,h} = 239.6$ kN (horizontal dredge line) and $P_{p,s} = 173.6$ kN (sloping dredge line) shown on Fig. E13-2a as well as the force polygon used to find the passive force $P_p$. Passive force $P_p$ is horizontal since $\phi = 10^\circ$, and for this small angle $\delta = 0$. The resulting reduction factor of 0.725 is
EXAMPLE 13-2  SHEET-PILE WALL OF EXAMPLE 13-1--WITH SLOPING DREDGE LINE

++++++++++++++ THIS OUTPUT FOR DATA FILE: EX132.DTA

SOLUTION FOR SHEET PILE WALL—CANTILEVER OR ANCHORED ++++++++++++++ ITYPE = 1

NO OF NP = 34  NO OF MEMBERS = 16
NO OF LOAD CONDITIONS = 1  NO OF BOUNDARY CONDITIONS, NZX = 0
MAX NO OF ITERATIONS, NCY = 5  NONLIN CHECK (IF > 0) = 1
NO OF NODE MODULUS TO INPUT, NRC = 14  NODE SOIL STARTS, JTSOIL = 9
LIST BAND MATRIX, LISTB (IF >0) = 0  NO OF ANCHOR RODS, IAR = 1
INPUT NODE PressURES, IPRESS = 10  NO OF NON-ZERO P-MATRIX ENTRIES = 0
IMET (SI > 0) = 1

MODULUS OF ELASTICITY = 200000.0 MPA

SHEET PILE AND CONTROL DATA:
WIDTH = 1.000 M
INITIAL EMBED DEPTH, DEMB = 3.000 M
DEPTH INCR FACTOR, DBPINC = .300 M
DREDGE LINE CONVERGENCE, CONV = .003000 M

ANCHOR RODS LOCATED AT NODE NOS = 2

MEMBER AND NODE DATA FOR WALL WIDTH = 1.000 M

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<th>NP1</th>
<th>NP2</th>
<th>NP3</th>
<th>NP4</th>
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<tbody>
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KS REDUCED WHEN YOU INPUT ALL VALUES

CURRENT CYCLE NO = 1  D.L. DEFL: PREVIOUS = .00000  CURRENT = .01559  FOR EMBED DEPTH = 3.000 M

++++ NEW NP = 36
NEW NM = 17

Figure E13-2b
### MEMBER AND NODE DATA FOR WALL WIDTH = 1.000 M

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<th>NP4</th>
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<th>INERTIA M M3</th>
<th>KS</th>
<th>SPRINGS NODE</th>
<th>SOIL/A.R.</th>
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<th>NODE Q (KPA)</th>
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KS REDUCED WHEN YOU INPUT ALL VALUES

D. L. DEF. CONVERGED ON CYCLE = 2  
DEFLS ARE: PREVIOUS = .01559  CURRENT = .01565  FOR EMBED DEPTH = 3.300 M

+++NON-LINEAR CHECK: CURRENT CYCLE, ICYC = 2  CURRENT SPRGS ZEROED = 1  PREVIOUS COUNT = 0
CURRENT D.L. X(I) = .01565  PREVIOUS D.L. X(I) = .00000

+++NON-LINEAR CHECK: CURRENT CYCLE, ICYC = 3  CURRENT SPRGS ZEROED = 1  PREVIOUS COUNT = 1
CURRENT D.L. X(I) = .01599  PREVIOUS D.L. X(I) = .01565

MEMBER MOMENTS, NODE REACTIONS, DEFLECTIONS, SOIL PRESSURE, AND LAST USED P-MATRIX FOR LC = 1

**Figure E13-2b (continued)**  Plot file for Fig. E13-2 is not shown.
computed as shown on Fig. E13-2a. From this and other WEDGE trials, for horizontal dredge lines it is evident that the \( \rho \) angle for the passive pressure failure surface is \( \rho_p \neq 45^\circ - \phi/2 \) except for horizontal, cohesionless backfills with wall \( \delta = 0 \). By analogy the active earth-pressure failure surface is only defined by \( \rho_a = 45^\circ - \phi/2 \) for horizontal, cohesionless backfills also with \( \delta = 0 \).

The reduction factor \( RF = 0.725 \) is applied to values 9 through 17 obtained from a listing of \( k_s = SK(I) \) from the output sheet given in Fig. E13-1f. We must input 22 values to allow for \( NCYC = 5 \), so the last five values are computed by hand based on the depth increment \( DINC = 0.3 \, \text{m} \). With this calculation we have the following (edited input):

<table>
<thead>
<tr>
<th>Node</th>
<th>Original ( k_s ), kN/m(^3)</th>
<th>Revised ( k_s ), kN/m(^3)</th>
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<td>9</td>
<td>5600.</td>
<td>( \times 0.725 = 4060 ) (rounded)</td>
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These several node values are input by hand. One could have simply multiplied \( AS = 7000 \times \text{FAC1} \times RF \) and \( BS = 1000 \times \text{FAC2} \times RF \) and used the equation; however, the preceding table illustrates the program option for inputting node values. Actually, considerable efficiency could be obtained by editing a copy of the data set \( \text{EX131.DTA} \) to create the data set \( \text{EX132.DTA} \) for this example.

The output is shown in Fig. E13-2b, from which we can see the dredge line converged on the second cycle of \( NCYC \), producing a final embedment depth of \( 3.0 + 0.3 = 3.3 \, \text{m} \). The nonlinear check also cycled two times: On the first time \( ICYC = NCYC = 2 \), and for the second \( ICYC = 3 \) since only node 9 displaced such that \( X(9) > 0.01 \, \text{m} \) and is marked with an *. The dredge line displacement stabilized at \( X(9) = 15.99 \, \text{mm} > \text{XMAX}(9) = 10 \), which is larger than the value obtained in Example 13-1 of \( X(9) = 13.7 \, \text{mm} \). We would expect that the dredge line displacement would be larger. It would be even larger for a 20° dredge line slope.

Now one can ask, is this a solution? We look at \( F_p \) from the WEDGE program and see it is 173.6 kN. The sum of the node soil reactions that are (+) is written as

\[ 6.41 + 19.91 + 19.34 + \cdots + 5.91 = 107.93 \, \text{kN} < 173.6 \]

This result indicates the embedment depth for this loading case is satisfactory. In fact, it might be satisfactory if the sum of soil reactions were larger than 173.6 kN since the bottom two nodes had (−) displacements (and reactions). The wall could hardly fail in a passive pressure mode (by toe kickout) with negative node displacements. If all the nodes below the D.L. were (+), we would have to look at the toe displacement and, if it were more than 1 or 2 mm, increase the embedment depth \( DEMB \), but this depth appears adequate for this case.

Since the dredge line displacement is larger, the anchor rod force is larger (108.44 vs. 106.64 kN) than in Example 13-1. The maximum bending moment is also larger (−230.3 vs. −221.7 kN \cdot \text{m} \). There is nothing unexpected in this analysis.

Problems occur if there is a loss of dredge line or an increase in backfill surcharge. Both of these situations may call for an embedment depth larger than the current value of 3.0 m. The analysis is left as an exercise for the reader.
Example 13-3.

**Given.** The sheet-pile wall system of Fig. E13-3a, which is supporting 5 m of sand backfill overlying 6 m of clay. Sand data are estimated as shown, and \( q_u \) was obtained from SPT tests. We will use two anchor rods: one is placed above the water level; the lower one uses a drilled tie-back system and can be installed at low tide. From trials not shown, we tentatively choose a PZ40 sheet-pile section.

**Required.** Design the wall (at least the first cycle of the iterative design process).

**Solution.**

**Step 1.** Locate the nodes as in Fig. E13-3b; from these we can readily establish NP, NM, node soil starts JTSoil, etc. as shown.

**Step 2.** Compute the earth-pressure profile using the Coulomb \( K_a \), with \( \delta = 17^\circ \), and \( \beta = 0 \). This calculation gives \( K_a = 0.277 \) (from Table 11-1). The value of \( \delta \) is an engineering estimate and generally ranges from about 0.5 to 0.7\( \phi \). We use a 17\(^\circ\) value here because only the upper 5 m of wall is sand. A larger friction value may not develop because of the deeper clay backfill. Also \( K_a \) only varies from 0.278 to 0.275 as \( \delta \) varies from 16 to 22\(^\circ\), so is not very sensitive in the likely range of wall friction angle.

However, there is wall adhesion in the underlying 6 m of clay, both from its being below the water table and because there is the sand acting as a surcharge to keep the clay squeezed against the wall.

Using the methods given in Chap. 11 for lateral pressure computations, we obtain the pressure profile of Fig. E13-3c with the following supplemental explanation. At the junction of the sand and clay layers at the water line,

\[
\sigma_{a,s} = [20 + 17.9(5)]0.277 = 30.33 \text{ kPa}
\]

In the clay \( K_a = 1.0 \), so we have

\[
\sigma_{a,c} = [20 + 17.9(5)1.0 - 2c \sqrt{1} = 48.5 \text{ kPa}
\]

Averaging for input gives

\[
\sigma_a = \frac{30.3 + 48.5}{2} = 39.4 \text{ kPa}
\]

Below the water line for the rest of clay,

\[
\sigma_{a,c} = 48.5 + \gamma'zK_a = 48.5 + 11.0z
\]

At the dredge line

\[
\sigma_{a,c} = 48.5 + 11(6) = 114.5 \text{ kPa}
\]

**Step 3.** Obtain the moment of inertia per meter of wall width for the PZ40 section. From Appendix A-3a, we find

\[
I = 335.23 \times 10^{-6} \text{ m}^4 \quad \text{for} \ w = 500 \text{ mm}:
\]

\[
I = \frac{335.23}{0.50} = 670.0 \times 10^{-6} \text{ m}^4/\text{m}
\]

\[
E = 200,000 \text{ MPa}
\]

**Step 4.** Compute anchor rod springs per meter of wall width from the spacing of 3 m and using a rod length so the anchor is out of "active" zone (and from Example 13-2 we found we do not really know where this zone is when there is cohesive soil involved). What we will do is use program WEDGE, a wall height from the dredge line to the top of the clay, an adhesion factor of 0.8, and the sand calculated as a surcharge of...
(a) Given condition and selected other date — tentative anchor rods and locations (nodes).

(b) P–X coding.

(c) Pressure profile.

Figure E13-3a–c
We now have a dilemma. If we directly compute the active force, we have

\[ \sigma_a = (q_s + 11z) - 2c \]

Integrating and inserting limits, we have

\[ P_p = \int_{0}^{6} \sigma_a \, dz \]

If we use program WEDGE, we obtain this value exactly at the Rankine \( \rho = 45° + 0°/2 = 45° \). On the other hand, if we use a wall adhesion of 0.8c, we obtain

\[ P_p = 363.93 \text{ kN at } \rho = 37° \]

This latter value is probably more nearly correct and is used to plot the \( \rho \) angle of Fig. E13-3a for the clay. The conventional value of \( \rho = 45° + 32°/2 = 61° \) is used for the sand. This \( \rho \) value is necessary to locate the anchor block.

The anchor block for anchor rod 1 must be located far enough from the wall so that the passive wedge (\( \rho = 45° - 32°/2 = 29° \)) does not intersect the active wedge from the wall. A scaled drawing should be made so that the several control dimensions can be plotted and required distances scaled. This approach tends to reduce computation errors. This plotting is shown on Fig. E13-3a. The design of the anchor block is considered in the next section.

Anchor 2 uses a drilled hole with grout in the zone outside the active wedge zone. The hole diameter and grout length are design parameters taken up in Sec. 13-8.

The rod diameter can be set here. After several trials, we select tentative anchor rod diameters of 40 mm for each. From scaling the drawing one obtains these lengths for the anchors:

No. 1 = 14.2 m
No. 2 = 5.25 m (only the ungrouted length that is free to elongate in the drill hole)

From these lengths and using 40-mm rod diameters (\( A_r = 0.7854(0.040)^2 = 0.001257 \text{ m}^2 \)) we obtain

\[
\text{ARSPRG}(1) = \frac{A_r E}{s L} = \frac{0.001257 \times 200 \times 10^6}{3 \times 14.2} = 5901.4 \text{ kN/m/m}
\]

\[
\text{ARSPRG}(2) = \frac{0.001257 \times 200 \times 10^6}{3 \times 5.25} \cos 20° = 14999.3 \text{ kN/m/m}
\]

Step 5. Take \( k_s = 40(\text{SF})q_a \); it was shown in Chap. 4 that within reasonable accuracy \( q_a = q_u \) with SF = 3. Thus, \( k_s = 40(3)(61) = 7320 \rightarrow 7300 \text{ kN/m}^3 \). Use this value in the equation format of \( \text{AS} = 7300; \text{BS} = 0; \) and arbitrarily use \( \text{FAC1} = 0.70 \) and \( \text{FAC2} = 0.85 \).

Step 6. With these data a file EX133.DTA is built (and on your diskette) and executed to obtain the output shown on Fig. E13-3d.

Perform an output check as follows.

1. Sum of spring forces = 607.41 kN versus input forces computed from the input soil pressures of 607.39 kN \( \rightarrow \sum F_h = 0 \).
2. Dredge line node 10 (JTSOIL), node 11, and node 12 all have displacements as follows:

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<th>Node</th>
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<th>XMAX(I), mm</th>
</tr>
</thead>
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<td>11</td>
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<td>13</td>
<td>15.274</td>
<td>16.0</td>
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DATA FOR EXAMPLE 13-3--ANCHORED WALL PS-40 W/SURCHARGE AND 2 ANCHORS--SI

************** THIS OUTPUT FOR DATA FILE: EX133.DTA

SOLUTION FOR SHEET PILE WALL--CANTILEVER OR ANCHORED ************** ITYPE = 1

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<th>NO OF NP = 36</th>
<th>NO OF MEMBERS = 17</th>
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<td>NO OF LOAD CONDITIONS = 1</td>
<td>NO OF BOUNDARY CONDITIONS, NZX = 0</td>
</tr>
<tr>
<td>MAX NO OF ITERATIONS, NCYC = 1</td>
<td>NONLIN CHECK (IF &gt; 0) = 1</td>
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SHEET PILE AND CONTROL DATA:

- WIDTH = 1.000 M
- INITIAL EMBED DEPTH, DEMB = 4.000 M
- DEPTH INCR FACTOR, DBPINC = .500 M
- DREDGE LINE CONVERGENCE, CONV = .003000 M

ANCHOR RODS LOCATED AT NODE NOS = 3 7

MEMBER AND NODE DATA FOR WALL WIDTH = 1.000 M

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* = KS REDUCED BY FAC1 OR FAC2

NON-LINEAR CHECK: CURRENT CYCLE, ICYC = 0 CURRENT SPRGS ZEROED = 3 PREVIOUS COUNT = 0
CURRENT D.L. X(I) = .02290 PREVIOUS D.L. X(I) = .02290

Figure E13-3d
++NON-LINEAR CHECK: CURRENT CYCLE, ICYC = 1  CURRENT SPRGS ZEROD = 3  PREVIOUS COUNT = 3
CURRENT D.L. X(I) = .02658  PREVIOUS D.L. X(I) = .02290

MEMBER MOMENTS, NODE REACTIONS, DEFLECTIONS, SOIL PRESSURE, AND LAST USED P-MATRIX FOR LC = 1

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SUM SPRING FORCES = 607.41 VS SUM APPLIED FORCES = 607.39 KN
(*) = SOIL DISPLACEMENT > XMAX(I) SO SPRING FORCE AND Q = XMAX*VALUE ++++++++++++++++ 
NOTE THAT P-MATRIX ABOVE INCLUDES ANY EFFECTS FROM X > XMAX ON LAST CYCLE ++++

DATA FOR PLOTTING IS SAVED TO DATA FILE: WALL.PLT 
AND LISTED FOLLOWING FOR HAND PLOTTING

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<th>NODE</th>
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<th>COMP X,M</th>
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<th>MOMENT MOM(I,1),MOM(I,2)</th>
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Figure E13-3d (continued)
Nodes 10, 11, and 12 are marked with an asterisk (*) on Fig. E13-3d for rapid identification that \( X > X_{\text{MAX}}(I) \). The anchor rod forces and pile moments include the effect of this nonlinear check.

3. The maximum sheet-pile moment of 451.10 kN-m occurs at node 9, and the bending stress is computed as

\[
f_s = \frac{M}{S} = \frac{451.10}{S}
\]

\[
S = \frac{1.632 \times 10^{-3}}{0.50} = 0.003264 \text{ m}^3/\text{m}
\]

and inserting values (10^3 converts kN to MN) obtain

\[
f_s = \frac{451.10}{0.003264 \times 10^3} = 138.2 \text{ MPa} < f_a
\]

This stress is satisfactory for A328 steel with \( f_y = 250 \text{ MPa} \) and an allowable bending stress of \( f_a = 0.65f_y = 160 \text{ MPa} \).

4. The anchor rod stresses (based on the 3-m spacing \( s \)) are next checked:

Rod 1:

\[
A_r = 0.001257 \text{ m}^2 \quad P = 39.53s = 39.53 \times 3.0 = 118.59 \text{ kN}
\]

\[
f_s = \frac{P}{A_r} = \frac{118.59}{1.257} = 94.34 \text{ MPa} \quad \text{(since } 10^{-3} \times 10^3 = 1.0)\]

(O.K. for \( f_y = 250 \text{ MPa} \) grade steel)

Rod 2:

\[
P = \frac{P}{\cos 20^\circ} = 346.11 \times 3/\cos 20^\circ = 1105.0 \text{ kN}
\]

\[
f_s = \frac{P}{A_r} = \frac{1105.0}{1.257} = 879.1 \text{ MPa}
\]

The stress in anchor 2 is so high it would require using either a larger rod diameter or using high-strength rods or cables as used for prestressed concrete. If you try a larger rod diameter, you must recompute the spring and rerun the problem.

5. Check the computed soil pressures. The output sheet shows the soil pressures for critical nodes as follows:

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<th>( q, \text{kPa} )</th>
<th>( q_o, \text{kPa} )</th>
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</table>

These soil pressures are not failure values, for \( q_{\text{ult}} \) is theoretically on the order of \( 3 \times 61 = 183 \text{ kPa} \). Also the passive earth force is 415.4 kN, which is greater than the sum of the (+) soil reactions from the dredge line of 234.5 kN. Note that the bottom two nodes kick back \[ (-) X(I) \] into the backfill. Considering these two data items, the wall should be stable for this load case. The only problem is that anchor rod No. 2 may require a larger diameter rod and/or use of prestressed steel cables.

Summary

1. The dredge line soil appears adequate.
2. The sheet-pile section seems satisfactory.
3. Anchor rod No. 2 may be overstressed. It may require a larger diameter rod or use of very-high-strength steel cable. Another possibility is to see if it can be relocated to a lower depth.

4. Anchor rod No. 1 appears overdesigned, but you can check by using a 30-mm diameter rod to see what happens.

The next example is a cantilever retaining wall. The basic difference between the anchored and cantilever wall is that the latter does not use an anchor rod. Another difference is that the cantilever wall is usually limited in height to about 3 to 4 m because without an anchor very large translation $X(I)$ values result, that produce large bending moments. The principal advantage in not using an anchor is economy since the anchor, anchorage, and installation costs are considerable. Adjacent property owners may not allow entry to install anchorage. In those cases where a cantilever wall is higher than 3 to 4 m it may be necessary to use some of the special sections shown in Fig. 13-6. Since some of these built-up sections are more than 1 m in width, it is necessary to divide by their width to obtain the unit width values for use in these analyses.

Example 13-4. Make a tentative design for the cantilever wall shown in Figs. E13-4a, b, and c.

Solution.

Step 1. Do the necessary coding and compute the node pressures to the dredge line as shown in the figures. Note that several preliminary executions were made so that the output could be minimized. From the preliminary trials it appears that a PZ27 section can be used. The resulting moment of inertia is

$$I = \frac{115.0 \times 10^{-6}}{0.460} = 0.2500 \times 10^{-3} \text{ m}^4/\text{m}$$

Step 2. An initial embedment depth $DEMB = 4.0$ m is chosen (based on previous trials) with $NCYC = 1$ and $NONLIN = 1$ so the embedment depth is not increased. The soil below the dredge line is checked for any $X(I) > XMAX(I)$. Most cantilever walls will require an embedment depth $D = \text{height of wall above the D.L.}$

Step 3. Obtain the modulus of subgrade reaction $k_s$. Since the soil is layered it will be best to input node values that are hand-computed. The first two nodes will be reduced by FAC1, FAC2 as shown following.

For the sand, we use the bearing-capacity equation and obtain from Table 4-4 for Hansen's equation the following:

For $\phi = 32^\circ : N_q = 23.2; N_y = 20.8$

$$k_s = 40[16.50(23.2)Z^1 + 0.5(16.5)(1.0)(20.8)]$$

$$k_s = \text{SK}(I) = 6864 + 15312Z^1 \rightarrow 6800 + 15300Z \text{ (rounding)}$$

Using these values we obtain the following (Note: The first two nodes are reduced using FAC1, FAC2):

For node 1: $\text{SK}(1) = 0.7(6800 + 0.) = 4760 \text{ kN/m}^3$

node 2: $\text{SK}(2) = 0.8[6800 + 15300(0.5)] = 11560$

node 3: $\text{SK}(3) = 6800 + 15300(1) = 22100$

node 4: $\text{SK}(4) = 6800 + 15300(1.5) = 29750$

but node 4 is interfaced with the clay.
For the clay use \( k_s = 40(SF)q_a \), but \( q_a = q_u \) with \( SF = 3.0 \). With these values we find

\[
k_s = SK(I) = 40(3)(40) = 4800 \text{ kN/m}^3
\]

We calculate an average \( SK(3) \) as

\[
(29750 + 4800)/2 = 17275 \text{ kN/m}^3
\]

For \( SK(4) \) through end of \( SK(I) \) the value is 4800 kN/m\(^3\).

**Step 4.** With the preceding data for moment of inertia \( XI(I) \), \( SK(I) \), and the control parameters shown on the figure, data file EX134.DTA is created (also on your diskette). The execution gives Fig. E13-4J from which we can make the following observations:

a. The final depth \( D_f = D_i = 4.0 \text{ m} \), which appears adequate. Five nodes have (-) displacement toward the backfill side. Three nodes have a (+) displacement, and node 5 has \( X(5) = 7.9 \text{ mm} > XMAX(5) \) of 6.0 mm—it is marked with an * for rapid notice.

b. The displacement of the top (node 1) is 33.983 mm, which may be noticeable. It can only be reduced by using a stiffer section or by using an anchorage of some type.

c. The maximum bending moment occurs at node 7 (not at dredge line node 5) and is 72.18 kN·m. For \( f_y = 250 \text{ MPa} \) the allowable stress \( f_a = 0.65f_y = 0.65(250) = 162.5 \text{ MPa} \). The section modulus of the PZ27 is

\[
S = \frac{0.742 \times 10^{-3}}{0.460} = 1.620 \text{ m}^3/\text{m}
\]
EXAMPLE 13-4 CANTILEVER SHEET-PILE WALL USING A PZ-27 SECTION 4-M HIGH--SI

++++++ THIS OUTPUT FOR DATA FILE: EX134 DTA

SOLUTION FOR SHEET PILE WALL--CANTILEVER OR ANCHORED ++++++++++++++ ITYPE = 1

NO OF NP = 26 NO OF MEMBERS = 12
NO OF LOAD CONDITIONS = 1 NO OF BOUNDARY CONDITIONS, NZX = 0
MAX NO OF ITERATIONS, NCYC = 1 NONLIN CHECK (IF > 0) = 1
NO OF NODE MODULUS TO INPUT, NRC = 9 NODE SOIL STARTS, JTSoIL = 5
LIST BAND MATRIX, LISTB (IF >0) = 0 NO OF ANCHOR RODS, IAR = 0
INPUT NODE PRESSURES, IPRESS = 6 NO OF NON-ZERO P-MATRIX ENTRIES = 0
IMET (SI > 0) = 1

MODULUS OF ELASTICITY = 200000.0 MPA

SHEET PILE AND CONTROL DATA:
WIDTH = 1.000 M
INITIAL EMBED DEPTH, DEMB = 4.000 M
DEPTH INCR FACTOR, DEPINC = .500 M
DREDGE LINE CONVERGENCE, CONV = .003000 M

MEMBER AND NODE DATA FOR WALL WIDTH = 1.000 M

MEMNO NP1 NP2 NP3 NP4 LENGTH INERTIA KS SPRINGS XMAX NODE Q NODE P
M M*4 NODE KN/M*3 SOIL/A.R. M KPA KN
1 1 2 3 4 1.0000 .0002500 1 .000 .000 .0000 .0000 .8450
2 3 4 5 6 1.0000 .0002500 2 .000 .000 .0000 5.0700 5.0683
3 5 6 7 8 1.0000 .0002500 3 .000 .000 .0000 10.1300 10.1317
4 7 8 9 10 1.0000 .0002500 4 .000 .000 .0000 15.2000 15.1983
5 9 10 11 12 .5000 .0002500 5 4760.000 1756.667 .0060 20.2600 12.6633
6 11 12 13 14 .5000 .0002500 6 11560.000 6091.667 .0100 .0000 1.6883
7 13 14 15 16 .5000 .0002500 7 22100.000 9769.583 .0150
8 15 16 17 18 .5000 .0002500 8 17275.000 8000.000 .0200
9 17 18 19 20 .5000 .0002500 9 4800.000 3439.583 .0250
10 19 20 21 22 .5000 .0002500 10 4800.000 2400.000 .0250
11 21 22 23 24 .5000 .0002500 11 4800.000 2400.000 .0250
12 23 24 25 26 .5000 .0002500 12 4800.000 2400.000 .0250
13 25 26 27 28 .5000 .0002500 13 4800.000 1200.000 .0250

KS REDUCED WHEN YOU INPUT ALL VALUES

+++NON-LINEAR CHECK: CURRENT CYCLE, ICYC = 0 CURRENT SPRGS ZEROED = 1 PREVIOUS COUNT = 0
CURRENT D.L. X(I) = .00759 PREVIOUS D.L. X(I) = .00759

Figure E13-4d
++NON-LINEAR CHECK: CURRENT CYCLE, ICYC = 1  CURRENT SPRGS ZEROED = 1  PREVIOUS COUNT = 1
CURRENT D.L. X(I) = .00793  PREVIOUS D.L. X(I) = .00759

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SUM SPRING FORCES = 45.59 VS SUM APPLIED FORCES = 45.60 KN

(*) = SOIL DISPLACEMENT > XMAX(I) SO SPRING FORCE AND Q = XMAX*VALUE

NOTE THAT P-MATRIX ABOVE INCLUDES ANY EFFECTS FROM X > XMAX ON LAST CYCLE

DATA FOR PLOTTING IS SAVED TO DATA FILE: WALL.PLT
AND LISTED FOLLOWING FOR HAND PLOTTING

Figure E13-4d (continued)
Is this wall overdesigned? If the client will accept a much larger displacement at node 1, it may be possible to use a PZ22. It probably is not necessary to increase the embedment depth. It may be prudent to place a surcharge on the backfill of about 20 kPa and rerun the program to see if the embedment depth and section are still adequate. If they are not it may be necessary to increase the embedment by another 0.5 m and/or use a stiffer section. This latter check is your stability analysis.

13-8 ANCHOR RODS, WALES, AND ANCHORAGES FOR SHEETPILING

This section will consider additional factors in the design of anchored sheet-pile walls.

13-8.1 Anchor Rods

The FEM analysis for the anchored sheet-pile walls of Examples 13-1, 13-2, and 13-3 illustrated that the design of the anchor rods is closely associated with the total design. That is, we must assume some size rod and its length. From this an anchor rod spring \((AE/L)\) is computed as part of the input data.

The program output gives an anchor force for that anchor section used, and the following criteria must be met:

1. The anchor node displacement must be large enough that active earth pressure can develop behind the wall. This \(\delta\) is usually on the order of \(0.001H\), where \(H\) is the free height from the dredge line to the anchor rod node.
2. The allowable tensile stress

\[
fs = \frac{F_{ar}}{A_r} \leq f_a
\]

where \(f_a = 0.6\) to \(0.75f_y\). The factors to reduce \(f_y\) to the allowable stress \(f_a\) are the necessary rod safety factor for that anchor rod force \(F_{ar}\).

The force (and the pile bending moment) also depends on anchor rod location (analyses not shown). Thus, in a design one must first try a given node as in Example 13-1 until a reasonable solution seems to be found. Then one shifts the anchor rod location if this is possible and makes additional trials to attempt to find the lightest pile section and smallest-diameter anchor rod consistent with the given wall specifications + any stability cases checked.

There are several complications to consider in addition to the foregoing two basic considerations:

1. If the soil beneath the anchor settles away from the rod it becomes unsupported and must carry its self-weight + any fraction of the upper soil assigned to the rod as a strip load.

\[\text{Although the term rods is used and implies a round solid bar, in practice the rod may be either a rod or a large wire strand cable.}\]
Usually there is some arching, so the full column of soil over the rod may not bear on it; however, a small-diameter rod in a long span can develop significant bending moments just from self-weight. A small-diameter rod will have a very small section modulus $S$, so the increase in the tension stresses from bending can be substantial.

2. It has been suggested that one should put a negative camber into the rod, using seating blocks (or props), in anticipation of rod sag from item 1. This may be difficult to do since backfill placed over the rod and the several seating blocks would cause settlement of both the underlying soil and the blocks themselves. Seating blocks may be practical in original ground, but this is seldom where the anchor rod is located.

3. Some persons suggest placing the anchor rod in a hollow tube that is supported by the backfill so that the rod is initially unsupported. This method is a solution only if the tube containing the rod does not settle into (or with) the fill.

13-8.2 Wales

Wales are longitudinal members running parallel with and in close contact with the wall, as shown in Fig. 13-9. They may be located on either the front or back face of the wall. The back face location is desirable in certain cases for both appearance and clearance, but it will require both a work space and adequate attachment to the wall by bolting or welding to support the anchor rod pull. Back face wales are most often attached by field-welding.

Bolting is difficult for either face location, since bolt holes shop-drilled in the sheetpiling by the steel fabricator seldom align with the wale after the piles are driven. On the other hand it is very difficult to field-drill large-diameter bolt holes in the driven piles using hand drills.

Wales on the front face are somewhat easier to install but also require a hole through the wall for the anchor rod—usually made by burning with an acetylene torch. Again, shop holes for bolting are not practical; however, here the wale usually covers the hole, so ragged edges of burned holes are not noticeable.

Wales are usually made from a pair of back-to-back channels with spacing for the anchor rod. Sometimes a pair of I beams is used; however, W shapes having wide flanges are not suitable unless the flanges are braced so they do not bend.

It is usually permissible to use large bending stresses—as much as $0.9f_y$ in the wales; however, the wales must be sufficiently rigid to transfer the anchor force laterally over the anchor spacings $s$ (of Fig. 13-9c) to satisfy the mathematical model. If there is very much lateral displacement between anchor spacings, most of the anchor force will be concentrated at the anchor. At best, this effect produces an unsightly wall, but more importantly soil moves into those “bulged” regions and backfill settlements occur. This causes pavement cracks; and if structures are near the wall they may crack and even collapse. Thus, anchor rod spacing $s$ is a significant design parameter.

Since wale fixity is fairly certain only at the anchor points, it is usual to use the assumptions shown in Fig. 13-9c. The wales are assumed to carry a uniform load $w$ of intensity shown, and if we assume an approximate fixed end beam the bending moment at any anchor point (which will be the maximum) is

$$M \approx \frac{ws^2}{10} \quad \text{or} \quad \frac{ws^2}{12}$$

Usually the larger of the two approximations is used.
Web crippling should be checked at the anchor locations as shown in Fig. 13-9b, for very high stresses can develop from the anchor rod force. Web crippling can be checked using the procedure given by the AISC (1989 or later) *ASD* manual.

**Example 13-5.** Tentatively design wales for the lower anchor rod of Example 13-3 assuming the output is satisfactory. Consider a typical wale section on an *interior* span of \( s = 3 \) m as in Fig. E13-5. Try to use a pair of channels back to back with \( f_y = 250 \) MPa (A-36). From the computer output (Fig. E13-3d) the axial anchor rod force per meter was found to be

\[
F_{ar} = \frac{346.11}{\cos 20^\circ} = 368.32 \text{ kN/m}
\]

**Solution.** The anchor rod force per meter is the uniform pressure on the wale. Using the previously given moment approximation, compute the following

\[
M = \frac{ws^2}{10} = \frac{368.32 \times 3^2}{10} = 331.49 \text{ kN \cdot m}
\]
Using an allowable bending stress of $0.75 f_y$ provides a nominal $SF = 1/0.75 = 1.33$ and $f_a = 0.75(250) = 190$ MPa. The required section modulus for two channels is

$$f_a = \frac{M}{S} \rightarrow S = \frac{M}{f_a}$$

$$S_x = \frac{331.49}{190000} = 1.745 \times 10^{-3} \text{ m}^3$$

For a single channel

$$S_x = \frac{1.745 \times 10^{-3}}{2} = 0.8723 \times 10^{-3} \text{ m}^3$$

From tables of rolled sections in metric units in the AISC (1992) manual we find the largest available channel is the only section that can be used:

Use C380 × 74:

- $d = 381$ mm
- $b_f = 94.4$ mm
- $t_w = 18.2$

(C15 × 50):

- $t_f = 16.5$
- $k = 37.0$ mm

$$S_x = 0.882 \times 10^{-3} \text{ m}^3 \quad I_x = 168 \times 10^{-6} \text{ m}^4$$

1. Find the approximate deflection between two anchor points for the wale assuming a fixed end beam with an $L = 3$ m:

$$\Delta_c = \frac{wS^4}{384EI} \quad (\text{AISC (1989) handbook equation})$$

Inserting values ($E = 200,000$ MPa), we find the deflection (using 2 channels) is

$$\Delta_c = \frac{368.32 \times 3^4}{384 \times 200 \times 10^6 \times (2 \times 168 \times 10^{-6})} = 1.16 \times 10^{-3} = 1.16 \text{ mm}$$

This displacement is quite adequate.

2. Check web yielding and crippling under the anchor plate, which is somewhat limited in area. To cover the two channel flanges and leave a 45-mm space for the 40-mm diameter anchor rods assumed in Example 13-3, a cover plate width (Fig. E13-5) will have to be
\[ w_p = 2b_f + 45 = 2 \times 94.4 + 45 = 233.8 \rightarrow 235 \text{ mm} \]

Make the plate length \( L_p = w_p = 235 \text{ mm} \) as well.

For channel web yield, check an end anchor where the contributory length \( s/2 = 3/2 = 1.5 \text{ m} \) and

\[ F_{ar} = 368.32 \times 1.5 = 552.5 \text{ kN} \]

The AISC [9th ed., ASD, Eq. (K1-3)] equation is

\[ \frac{P}{2} = 0.66 f_y t_w (N + 2.5k) \]

Substituting values \( N = w_p = 235 \text{ mm} \); from above, \( k = 37 \text{ mm} \); \( t_w = 18.2 \text{ mm} \); and previously \( f_y = 250 \text{ MPa} \) we obtain

\[ \frac{P}{2} = 0.66 \times 250 \times 10^3 \times 0.0182(0.235 + 2.5 \times 0.037) = 983.5 \text{ kN} > 552.5 \]

Web yielding in the channel is clearly adequate.

3. Check channel web crippling using AISC [9th ed., ASD, Eq. (K1-5)]. The equation is

\[ \frac{P}{2} = C t_w^2 \left[ 1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right] \sqrt{f_y t_f / t_w} \quad \text{(per channel)} \]

where \( C = 89 \) for ends and 176.7 for interior nodes. Since the end node is more critical, use \( C = 89 \) and substitute (1000 kN/MN) to obtain

\[ \frac{P}{2} = 34 \times 0.0182^2 \left[ 1 + 3\left(\frac{235}{381}\right)\left(\frac{18.2}{16.5}\right)^{1.5}\right] \sqrt{250(16.5/18.2) \times 1000} \]

\[ P = 89 \times 0.0182^2[3.14] \times 15.05 \times 1000 \times 2 = 2986 \gg 2(552.5) \]

For interior nodes \( P = 2786(176.7)/89 = 5531 \gg 2(552.5) \).

Web crippling is not a critical design item here.

### 13-8.3 Sheet-Pile Anchorages

Anchorage for sheet piles may be obtained from large cast-in-place concrete blocks (usually square and of necessary length) or precast concrete blocks that are embedded in the soil some depth (Fig. 13-10a). Instead of using a concrete block of some length, a row of sheetpiling that is similar to the supported wall but of shorter length may be driven, as in Fig. 13-10d; alternate pairs may be driven deeper for additional stability. As shown, a wale is used to carry the anchor rod force.

Piles may be driven as in Fig. 13-10b and c, and some authorities suggest these are the most reliable of the several anchorages. A surface paved with concrete may be extended (with edge thickened and reinforced) to provide an encasement for the top node region of the sheet pile instead of using a top anchor. This generally fixes the top against both translation and displacement and is efficiently handled with the FEM program using boundary conditions. Top fixity may reduce the pile bending moments, but the results depend on an interaction of wall height, pile stiffness, and whether the node is both fixed for no rotation and translation or just fixed for no translation.

**TIEBACKS.** One of the most popular anchorage methods presently used is the tieback of Fig. 13-10e. These are essentially small piles oriented at about \( \eta = 15 \text{ to } 25^\circ \) from the horizontal.
By using small slopes the vertical stress component on the wall can be neglected. Tiebacks are constructed by drilling a hole on the order of 150 to 375 mm in diameter using a hollow-stem auger. The anchor cable or rod, with an expandable end plate (or toggle), is pushed or carried in the hollow stem of the auger and at the design depth is extruded. Then the end plate/toggle is expanded. The end plate greatly increases the pull-out resistance of the anchor from the concrete shaft. The auger is slowly withdrawn, and simultaneously concrete or sand-cement grout (with either material containing appropriate admixtures), usually of about $f'_{c} \approx 21$ MPa, is forced through the hollow stem. The concrete/grout is under a pressure of from 75 to 225 kPa so that it expands around the cable/rod for bond and against the soil to produce
an irregular surface for friction/adhesion. A grouting pressure is used to approximate $K^+$ conditions so that the soil-anchor friction angle $\phi \to \phi$ or, if cohesive, an adhesion such that $c_a \to c$.

High-strength steel ($f_y$ on the order of 1000 to 14 000 MPa) tendons or rods are generally used for tieback anchors because they are usually prestressed to a design force computed using methods of the next chapter. High-strength steel is used instead of regular structural steel with an $f_y = 250$ MPa (A36) so that after soil creep and steel relaxation occur there is a substantial holding force remaining in the “tieback.”

Tieback walls are often used in deep excavations where it is essential that lateral wall movements and subsequent perimeter settlements be minimized. An advantage of these walls is they can be constructed from the top down (built as excavation proceeds). Another advantage is they do not produce obstructions in the construction area. Often these walls are left in place and become part of the final construction.

They have the disadvantages that adjacent property owners must give permission and that underground utilities must not be encountered.

Only a part of the drilled depth is backfilled with concrete. A part must be left free so that the anchor cable can elongate (with no length in which to develop $e = PL/AE$, it would pull apart). The force used to develop the prestress is always larger than the design force (the designer knows the soil will creep and the steel will relax), so effectively the anchor is proof tested during installation. If the rod or cable does not pull apart or the assembly pull out, the design is adequate.

The tieback anchor design can be made with reference to Fig. 13-10e as

$$P_{ar} = \pi DL[yd_2K \tan \delta + c_a] \quad (13-7)$$

where

- $D =$ average shaft diameter; compute based on volume of concrete pumped, together with the original and final hole depths, m or ft
- $L =$ length of cement/grout; compute based on original and final hole depth, m or ft
- $K =$ soil coefficient—between $K_a$ and $K_o$
- $d_2 =$ average depth of grouted length $L$, m or ft
- $\delta =$ soil-cement friction coefficient and $\to \phi$
- $c_a =$ adhesion to cement zone—0.7 to 1.0$c$, kPa or ksf

If the anchorage is belled, you can use Eq. (4-25).

Additional details on prestressed anchors may be found in PCI (1974), Ware et al. (1973), and Oosterbaan and Gifford (1972). The methodology is well-established, so there is a scarcity of very recent publications.

**Example 13-6.** Tentatively size the concrete shaft of the tie-back anchor of Example 13-3 for the anchor force of $3(368.32) = 1104.96$ kN (refer to Figs. E13-3a and E13-5 for other data).

**Solution.** Try a 350-mm nominal anchor shaft diameter. Take adhesion as $0.8s_u(s_u = q_u/2 = 61/2 = 30.5$ kPa). Assume that CU conditions will be obtained around the shaft perimeter from the pressure grouting. This state will produce a small angle of internal friction of about $\phi = 20^\circ$. We will also assume the grout pressure produces $K_o$ conditions so that $K = K_o = 1 - \sin 20^\circ = 0.66$ and the friction angle $\delta = \phi = 20^\circ$. 
We are making this design with less than ideal soil data—often the case in practice. In the absence of better data we do the best we can. Proof loading of the anchor will quickly indicate if the design is inadequate. With these estimates we will use Eq. (13-7):

\[ P_{a_{n}} = \pi DL[\gamma d_{2}K \tan \delta + c_{a}] \]

From a scale drawing of Fig. E13-3a we obtain a tentative vertical average distance \(d_{2} \approx 10\) m (we may have to make more than one trial to obtain compatible \(d_{2}\) and embedment length \(L\)). Five meters of this depth is sand to the water line; the remaining 5 m is clay soil below the water line, requiring using \(\gamma' = 20.8 - 9.8 = 11.0\) kN/m\(^3\). Substituting values into Eq. (13-7) we obtain

\[
P_{a_{n}} = L(\pi \times 0.350)[(5 \times 17.9 + 5.0 \times 11)(0.66 \times \tan 20°) + 0.8 \times 30.5]
\]

\[
= L(1.10)[(144.5)(0.2402) + 24.4]
\]

\[
= L(1.10)(34.7 + 24.4) = L(65.01)
\]

Since axial \(P_{a_{n}} = 3(368.32) = 1104.96\) kN, solving for \(L\) gives

\[
L = \frac{1104.96}{65.01} = 17.00\) m
\]

The total anchor rod/cable \(L_{tot} = 5.25 + 17.00 = 22.2\) m. The vertical force/meter of wall \(F_{v} = 368.32\) sin 20° = 126.0 kN. This value of \(L\) is reasonably consistent with \(d_{2}\) used, so we may take this as a valid solution—unless the anchor fails during installation.

BLOCK\(^7\) ANCHORS. The block anchor is a cast-in-place or precast concrete member that may be square or rectangular in section with the necessary length to develop adequate passive resistance for one or more anchor rods/cables attached along its length.

A general equation can be developed for a block anchor using Fig. 13-11b and noting that \(P'_{a}\) may not fully develop unless the anchor translates toward the wall a small amount, and \(P'_{p}\) similarly may not fully develop unless there is sufficient translation. For these reasons the values are given primed superscripts. With this understood, we obtain the general equation as follows:

\[
\sum F_{h} = F_{a_{n}} - L(P'_{p} - P'_{a} + F_{top} + F_{bot})
\]

Solving and including an SF, we obtain

\[
F_{a_{n}} = \frac{L(P'_{p} + F_{top} + F_{bot} - P'_{a})}{SF}
\]  \(\text{(13-8)}\)

Use an SF of about 1.2 to 1.5 in this equation, depending on the importance factor. Assuming that \(P'_{p}, P'_{a}\) should be collinear, we can take \(\sum M_{P_{p}} = 0\), giving

\[
B'LP' + B'LF_{R} + (H - \bar{y})LF_{top} = F_{a_{n}e} + \bar{y}LF_{bot}
\]

Rearranging and solving for vertical corner force \(P'\), we obtain

\[
P' = \frac{F_{a_{n}e}}{BL} + \frac{\bar{y}F_{bot}}{B} - \frac{(H - \bar{y})F_{top}}{B} - F_{R}
\]  \(\text{(13-8a)}\)

---

\(^7\)The block anchor is also called a "deadman." Rather than amending that term to "deadperson," this text will call these members "block anchors."
Figure 13-11 Block anchorage with terms used in Eqs. (13-8) through (13-11). Note \( L \) is perpendicular to paper.

and the force \( P' \) located at point \( b' \) must be

\[
P' \leq (q_{\text{sur}} + \gamma d_1)L
\]

Refer to Fig. 13-11b for identification of terms used in the preceding equations and note that \( F_{\text{ar}} \) = total anchor rod force based on spacing \( s \) and that \( F_L, F_R = \) side friction (\( F_R = P'_a \tan \delta \)) forces. For the preceding equations use \( L = s \) for anchorages that are continuous for the total wall length—the usual case. Earth pressures are usually calculated for a unit width so they must be multiplied by \( L \) to obtain forces consistent with the anchor rod.

When using Eq. (13-8), one should locate the anchorage so that the passive zone of Fig. 13-11d and Example 13-3 is outside the active wedge. Actually the anchorage can be in the reduced efficiency zone of Fig. 13-11d but with a passive pressure computed using \( d_1 \) reduced by the depth of the intersection of the passive and active zones (similar to point \( C \) of Fig. 13-11d). For this case the top and bottom friction/adhesion components must provide the principal anchor rod resistance.

Regardless of anchorage location the anchorage must be carefully backfilled both around the sides and on top so that the assumed passive condition with friction and/or adhesion can develop. There may be a question of using an SF on the active pressure component of Eq. (13-8), but this is a conventional procedure that has generally proved satisfactory.
A few verification tests have been made—primarily on small models but a few on full-scale anchorages [see Smith (1957) and Tschebotarioff (1962)]. From these the following semi-empirical equations were produced:

1. If the anchorage is a short rectangular (or square) block of $L \leq 1.5H$, the anchor resistance can be computed (see Fig. 13-11a or c) as

$$P_{ar} = \frac{C\gamma d_2^2 L K_p + q_u H^2}{\text{SF}}$$

(13-9)

In this equation take $K_p = $ Rankine value from Table 11-4. Use a SF = 1.2 to 1.5. Take $C = 0.65$ for concrete; for steel plates or sheetcpling use $C = 0.5$.

2. For a cohesive soil ($\phi = 0^\circ$) compute the anchorage resistance as

$$P_{ar} = \frac{MHLS_u}{\text{SF}}$$

(13-10)

where $M = 9$ for $\frac{d_2}{H} \geq 3$ (9 = bearing capacity factor for a deep footing)

$$= 9\frac{d_2}{H} \text{ for } \frac{d_2}{H} < 3$$

(using linear interpolation)

$$d_2 = \text{block depth shown on Fig. 13-11a}$$

3. For $\phi - c$ soil and $L > 1.5H$ use Eq. (13-8) with the active and passive earth forces computed using Eqs. (2-54) and (2-55); for short anchor blocks use

$$P_{ar} = \frac{P_p L}{\text{SF}}$$

(13-11)

When the anchor block is very deep, say $d_2/H \geq 6.5$, one may compute the anchor resistance by Eq. (13-9) for all values of $L$.

**Example 13-7.** Design a concrete anchorage for the anchor rod force and its location of Example 13-1.

**Given.**

$F_{ar} = 106.6$ kN on $s = 3$ m (see Fig. E13-1f)

Depth $d_1 = 1.2$ m (see Fig. E13-1a)

$\gamma = 16.50$ kN/m$^3$ $q_{sur} = 25$ kPa (see Fig. E13-1a)

$\phi = 32^\circ$ Concrete: $f_{c'} = 21$ MPa

**Solution.** We know that a soil with $\phi = 32^\circ$ will have a reasonably large passive earth and friction resistance. Let us try a block of $0.6 \times 0.6$ m $\times$ length of the wall; but for any interior anchorage the effective length $L = 3$ m and is 1.5 m for the two ends (but the two ends will also have end friction). We will look at a typical interior section having these properties:

Dimensions = $0.6 \times 0.6 \times 3.0$ m length

Anchor rod force $= sF_{ar} = 3 \times 106.6 = 319.8$ kN

For friction we will use $\delta = 25^\circ$ for top

$\delta = \phi = 32^\circ$ for base
Using a smaller $\delta$ for the block top is justified on the basis that it will not be so rough as the sides, which are cast against the soil; also top will be backfill. With these data we compute block friction resistance as follows:

$$F_{\text{top}} = L(q_{\text{sur}} + \gamma d_1) \tan 25^\circ$$
$$= 3(25 + 16.5 \times 1.2) \tan 25^\circ = 3(20.9) = 62.7 \text{ kN}$$
$$F_{\text{bot}} = 3(25 + 16.5 \times 1.8) \tan 32^\circ = 3(34.2) = 102.5 \text{ kN}$$

Using the Coulomb (same as Rankine) pressure coefficients with $\delta = 0$ and $\phi = 32^\circ$, we obtain, from Table 11-1, $K_a = 0.307$; from Table 11-2, $K_p = 3.25$.

The active and passive earth forces on the block can be computed from the average block pressure as follows:

$$\sigma_{av} = (q + h_{av} \gamma)K_i : \quad \sigma_a = (25 + 1.5 \times 16.5)0.307 = 15.3 \text{ kPa}$$
$$\sigma_p = (25 + 1.5 \times 16.5)3.25 = 161.7 \text{ kPa}$$
$$P_i = L \times \sigma_{i,av} \times H \quad \text{where } L = 3 \text{ m}, H = 0.6 \text{ m}$$
$$P_a = 3 \times 15.3 \times 0.6 = 27.5 \text{ kN}$$
$$P_p = 3 \times 161.7 \times 0.6 = 291.1 \text{ kN}$$

The total resisting force is

$$F_R = F_{\text{top}} + F_{\text{bot}} + P_p' - P_a'$$
$$= 62.7 + 102.5 + 291.1 - 27.5 = 428.8 \text{ kN}$$

and the resulting SF is

$$SF = \frac{F_R}{F_{ar}} = \frac{428.8}{319.8} = 1.34 \quad \text{(probably O.K.)}$$

We do not check the eccentricity of the anchor rod with $P_p'$ but it is probably rather small. Instead, this question is left as a reader exercise (Prob. 13-16).

---

### 13-9 OVERALL WALL STABILITY AND SAFETY FACTORS

A sheet-pile wall can fail in one of four basic modes as shown in Fig. 13-12:

1. **Sheet-pile bending.** Using the maximum design moment $M$ from the analysis with $f_a \approx 0.60$ to $0.65 f_y$ gives an apparent SF = 1.66 to 1.54, which is usually adequate. One may, of course, use a smaller or larger $f_a$ based on site conditions and the importance factor. Safety factors much smaller than 0.5 are not recommended.

   If there is enough lateral displacement (or bending) the pile may pull out of the ground, for it cannot elongate.

2. **Anchor rod or anchorage failure.** This may be by the anchor rod pulling apart either along its length or failing at its anchor point(s). For the anchor rod one should limit the allowable stress so that a SF on the order of 1.5 to 2.0 is obtained.

   Anchorage failure can occur if passive pressure and friction resistance is inadequate. This would occur from placing the block too close to the wall, combined with inadequate backfilling procedures.

3. **A toe (or kickout) failure.** This may occur if the embedment depth is not adequate. This failure mode is usually checked by taking a moment summation about one of the anchor...
rods. When this is done the $\sum M_{af}$ should be for a worst case, not for the basic design case. Actually, this check is not required in the FEM since it is automatically satisfied for any design case checked.

4. A system (see Fig. 13-12b) failure. This failure mode is usually checked using a slope stability program with trial circles located such that they are outside the anchorages for the anchor rods and pass either through or just below the pile tip. A minimum recommended SF for this mode is $1.2^+$. The zone between the active earth-pressure wedge and the anchorage is similar to a reinforced earth system. The major difference is the use of only one or two anchor rods versus a number of reinforcement strips. Thus, it would appear that no slip circles would form in this region.

Some persons suggest that a vertical or “plunging” failure by excessive pile penetration be investigated when the anchor rod slopes. It is not likely that the active pressure would force the piling further into the ground; however, when the anchor uses a prestress tendon that is tensioned to a high value, a fairly large vertical force can be developed. The problem with this type of analysis is that, as the large vertical force develops, there is also an increase in the horizontal force and in the friction component and (depending on the soil) there is additional adhesion, so it is nearly impossible to make any kind of analysis. If plunging is a problem, it will be discovered during the application of the proof load—one can see the wall moving vertically and stop operations for a redesign. Probably the best solution is to increase the embedment depth, since that zone has friction and/or adhesion on both sides of the piling.

**PROBLEMS**

13-1. Use your FEM program FADSPABW (B-9) and find an embedment depth for an HP pile section (see Table A-1 of Appendix A) for the “flagpole” problem summarized in Table P13-1 and illustrated in Fig. P13-1. After the program finds a depth, indicate what you recommend for the depth and your reasons. In this case input IPRESS = 0, NNZP = 1, BSHP = width of HP, m, and the horizontal load $P$ at $NP = 2$. 

![Figure 13-12 Sheet-pile wall failure modes.](image-url)
TABLE P13-1

<table>
<thead>
<tr>
<th>No.</th>
<th>( l ), m</th>
<th>( P ), kN</th>
<th>( \gamma ), kN/m^3</th>
<th>( \phi ), degrees</th>
<th>( c ), kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.0</td>
<td>45</td>
<td>17.30</td>
<td>32</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>6.5</td>
<td>60</td>
<td>17.50</td>
<td>34</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>7.0</td>
<td>40</td>
<td>18.20</td>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

Note: In the following problems, if you input something that does not produce a stable structural system, the program will likely cancel, and you will not get all of the output. If this occurs, you must edit the change to something larger or stiffer or increase the embedment depth.

13-2. Redo Example 13-1 with 0.6- and 0.5-m element lengths above the dredge line and see if there are any major differences in nodal output compared with Fig. E13-1f.

13-3. Make a stability analysis of Example 13-1 using data sets EX131A.DTA and EX131B.DTA. Using the worst-case anchor rod load, check if the anchor rod is adequate. Using the largest moment, check if the bending stress is satisfactory.

13-4. Using data set EX131C.DTA (it is already set to fix node 1 and remove the anchor rod), make an analysis and compare the output to that from using data set EX131.DTA. Draw a sketch showing the bending moment caused by fixity (and sign) and compute the equivalent anchor force produced by fixing the node.

13-5. Redo Example 13-2 (sloping dredge line) but take the dredge line slope as 25°. Use program WEDGE and data sets WDG132A.DTA and WDG132B.DTA (with WDG132B.DTA revised for the new slope angle). Recompute the SK(I) values below the dredge line and make an analysis using program B-9. Compare the output from your analysis with the execution using data set EX132.DTA.

13-6. Redo Example 13-2 using a revised copy of data set EX132.DTA for a surcharge of 50 kPa on the backfill. Check whether the bending moment stress and anchor stress are satisfactory. If they are not try these:
   a. A stiffer sheet-pile section
   b. A larger-diameter anchor rod

13-7. Redo Example 13-2 using a revised copy of data set EX132.DTA. Move the anchor rod to the water line node (be sure that you are using 0.6- and 0.5-m nodes above the dredge line) and see if there is sufficient improvement to warrant movement. Be sure to check the new anchor and bending stresses.

13-8. Redo Example 13-3 using a copy of data set EX133.DTA, edited to use a larger-diameter rod for the top anchor. Compare these results to your execution of the original data set and note whether there is any improvement.
13-9. Redo Example 13-3 using a copy of data set EX133.DTA but with the rod springs reversed (i.e., just switch the two $K$ values). Compare this output to your execution of the original data set.

13-10. Redo Example 13-3 using a copy of data set EX133A.DTA that fixes the top node ($NZX = 2$) and uses only the lower anchor. Compare this output to that from using the original data set. Are any bending moments too large? Is the anchor rod overstressed?

13-11. Redo Example 13-3 using a copy of data set EX133.DTA and increase the surcharge to 40 kPa. Check if the bending and anchor stresses are adequate.

13-12. Redo Example 13-4 using a copy of data set EX134.DTA and the next larger sheet-pile section. How much does this larger section reduce the top node displacement?

13-13. Redo Example 13-4 using a copy of data set EX134.DTA and adding a surcharge of 20 kPa to the backfill. By trial find a section that limits the top node deflection to not more than 35 mm.

13-14. Redo Example 13-5 with a diameter of 375 mm and see if there is any significant change in anchor elongation.

13-15. Design the wales for the anchor rod of Example 13-1 using the data in Fig. E13-1f. You should obtain a regular copy using data set EX131.DTA, which will be easier to read and work with. Use a pair of back-to-back channels with adequate spacing for the anchor rod to fit between in a loose fit.

13-16. For Example 13-6, find the eccentricity $e$ and compute the vertical force $P'$ (at $b'$). Use $F_R = P' \tan 32^\circ$, note there is a surcharge on the backfill, and be sure to include $L$. 