8-1 FOOTINGS: CLASSIFICATION AND PURPOSE

A footing carrying a single column is called a spread footing, since its function is to "spread" the column load laterally to the soil so that the stress intensity is reduced to a value that the soil can safely carry. These members are sometimes called single or isolated footings. Wall footings serve a similar purpose of spreading the wall load to the soil. Often, however, wall footing widths are controlled by factors other than the allowable soil pressure since wall loads (including wall weight) are usually rather low. Foundation members carrying more than one column are considered in Chapters 9 and 10. Concrete is almost universally used for footings because of its durability in a potentially hostile environment and for economy.

Spread footings with tension reinforcing may be called two-way or one-way depending on whether the steel used for bending runs both ways (usual case) or in one direction (as is common for wall footings). Single footings may be of constant thickness or either stepped or sloped. Stepped or sloped footings are most commonly used to reduce the quantity of concrete away from the column where the bending moments are small and when the footing is not reinforced. When labor costs are high relative to material, it is usually more economical to use constant-thickness reinforced footings. Figure 8-1 illustrates several spread footings.

Footings are designed to resist the full dead load delivered by the column. The live load contribution may be either the full amount for one- or two-story buildings or a reduced value.

---

1 This chapter will retain some Fps units as a reader convenience. This text is widely used as a reference work, and in remodeling/remedial work access to Fps units may be necessary. Also this chapter uses the standard American Institute of Steel Construction (AISC) terminology for rolled sections as given in their AISC (1989) publication for metric shapes based on the ASTM A 6M (SI) standard. For example, a W 360 x 79 is a rolled Wide flange shape of nominal 360-mm depth (actual depth = 354 mm), has a mass of 79 kg/m, and is usually used as a column.
Figure 8-1 Typical footings. (a) Single or spread footings; (b) stepped footing; (c) sloped footing; (d) wall footing; (e) footing with pedestal.

as allowed by the local building code for multistory structures. Additionally the footing may be required to resist wind or earthquake effects in combination with the dead and live loads. The footing loads may consist of a combination of vertical and horizontal loads (inclined resultant) or these loads in combination with overturning moments. The current ACI\(^2\) Code strength design procedure uses reduced load factors for the several transient loading conditions in lieu of increasing the allowable material stresses.

A pedestal (Fig. 8-1e) may be used to interface metal columns with spread or wall footings that are located at the depth in the ground. This prevents possible corrosion of metal through direct contact with the soil.

8-2 ALLOWABLE SOIL PRESSURES IN SPREAD FOOTING DESIGN

The allowable soil pressure for footing design is obtained as the worst case of bearing capacity and settlement as in Example 5-9. Where settlements control, the reported value is the net

\(^2\)American Concrete Institute Building Code 318-. This code is revised every four to eight years. The metric version is designated 318M-. The latest (as of 1995) was issued in 1989 and revised in 1992 [the metric version being designated ACI 318RM-89 (Revised 1992)].
increase in soil pressure that can be allowed. The reason is that settlements are caused by
increases in pressure over that currently existing from overburden.

The allowable bearing capacity furnished to the structural designer by the geotechnical
engineer will have a suitable factor already applied. The safety factor ranges from 2 to 5 for
cohesionless materials depending on density, effects of failure, and consultant caution. The
value may range from 3 to 6 for cohesive materials, with the higher values used where con-
solidation settlements might occur over a long period of time. Note that these safety factors
are larger than those cited in Table 4-9. Geotechnical caution should not be viewed as poor
practice unless it results in a different type of foundation that is several times more expensive.
In general, reduction of $q_a$ from, say, 500 to 300 kPa will result in larger spread footings, but
the percent increase in total building cost will be nearly negligible. This can be considered in-
surance, since a foundation failure requires very expensive remedial measures and structural
repairs, whereas a superstructure failure may be localized and easily repaired.

The geotechnical consultant is not usually aware that the footing will be subjected to ec-
centric load and/or moment, so the allowable bearing pressure may not be found using the
$B'$ analysis of Chap. 4. Also if settlement controls, there is no reliable method to account for
eccentricity. In these cases the best approach is to avoid any large differential pressure across
the base of the footing. Any footing rotation will have a marked effect on the column base
moment when the columns are rigidly attached to the footing. The footing rotation will be in
a direction to reduce the base moment and may, in fact, reduce it to zero. Equation (5-17) can
be used to estimate moment loss due to footing rotation as in Example 5-8.

Any increase in allowable soil pressure for transient load conditions should be verified
with the geotechnical consultant. Increasing $q_a$ by one-third as commonly found in design
codes for other materials may not be appropriate. Factors such as frequency of overload, soil
state, climatic conditions, and type of structure may disallow any large deviation from the
recommended $q_a$.

8-3 ASSUMPTIONS USED IN FOOTING DESIGN

Theory of Elasticity analysis [Borowicka (1963)] and observations [Schultze (1961), Bar-
den (1962)] indicate that the stress distribution beneath symmetrically loaded footings is not
uniform. The actual stress distribution depends on both footing rigidity and base soil. For
footings on loose sand the grains near the edge tend to displace laterally, whereas the interior
soil is relatively confined. This difference results in a pressure diagram qualitatively shown
in Fig. 8-2a. Figure 8-2b is the theoretical pressure distribution for the general case of rigid
footings on any material. The high edge pressure may be explained by considering that edge
shear must occur before any settlement can take place. Since soil has a low rupture strength,
and most footings are of intermediate rigidity, it is not very likely that high edge shear stresses
are developed. The edge stress also depends on the thickness $H$ of compressible soil as shown
in Fig. 8-2b.

The pressure distribution beneath most footings will be rather indeterminate because of
the interaction of the footing rigidity with the soil type, state, and time response to stress. For
this reason it is common practice to use the linear pressure distribution of Fig. 8-2c beneath
spread footings. The few field measurements reported indicate this assumption is adequate.

Spread footing design is based almost entirely on the work of Richart (1948) and Moe
(1961). Richart’s work contributed to locating the critical section for moments; critical
sections for shear are based on Moe's work. The ACI, AASHTO, and AREA specifications for footing design are identical for locations of critical sections. AASHTO and ACI use the same design equations and factors for strength design. AREA uses the alternative design method for footings but allowable concrete strengths are about 10 percent less than those allowed by ACI. Because of the similarity in the several codes the ACI code will be the primary reference in this and the following two chapters.

8-4 REINFORCED-CONCRETE DESIGN: USD

The latest revision of the ACI Standard Building Code Requirements for Reinforced Concrete (ACI 318-), hereinafter termed the Code, places almost total emphasis on ultimate strength-design (USD) methods. The older procedure, termed the Alternate Design Method (ADM), is still allowed, and the basic elements are given in Appendix A of the ACI Code. The AASHTO bridge code gives about equal emphasis to both the alternate and the strength design methods. For spread footings, even though the design is reasonably direct, the ADM procedure is simpler to use but produces a more conservative design. When one compares designs by the two methods the ADM will consistently compute a concrete footing thickness on the order of 15 to 25 mm larger and reinforcing bar areas 30 to 50 percent larger. For these two reasons AASHTO gives more emphasis to the ADM than does ACI.

This text uses the ADM for the retaining wall design of Chap. 12—still a widely used procedure in practice—since the ACI code procedure does not give greatly different results.

---

Figure 8-2 Probable pressure distribution beneath a rigid footing. (a) On a cohesionless soil; (b) generally for cohesive soils; (c) usual assumed linear distribution.

3ACI = American Concrete Institute, AASHTO = American Association of State Highway and Transportation Officials, AREA = American Railway Engineering Association.
and there is much uncertainty with that design. We will use the USD for spread footing design; however, footing depth equations [Eqs.(8-5)-(8-9)] are also applicable for the ADM. The only difference is whether column loads are factored (USD) or unfactored (ADM).

If you have difficulty factoring column moments for a spread footing design you should use the ADM method. You should also use the ADM where the column loads are not well-defined. The basic procedure is given as previously stated in Appendix A of ACI 318-; select parts and most of the methodology are given in Sec. 12-16 [basic design equations and allowable stresses (in Tables 12-1 and 12-2)].

All notation pertaining to concrete design used in this text will conform to the ACI Code. Where this conflicts with notation previously used, the reader should take note. Strength design requires converting working design dead (D) and live (L) loads (see Table 4-10) to ultimate loads through the use of load factors as

\[ P_u = 1.4D + 1.7L \]  
\[ = 0.75(1.4D + 1.7L + 1.7W) \]  
\[ = 0.9D + 1.3W \]  

(alternative with wind) (c)

For earthquake loading substitute \( E \) for \( W \) (wind) as applicable. Other load combinations may be used, but the user is referred to Art 9.2 of the Code for their application.

The ultimate concrete strength \( f'_c \) in USD is reduced for workmanship and other uncertainties by use of \( \phi \) factors (Art 9.3) as follows:

<table>
<thead>
<tr>
<th>Design consideration</th>
<th>( \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment, without axial load</td>
<td>0.90</td>
</tr>
<tr>
<td>Two-way action, bond, and anchorage</td>
<td>0.85</td>
</tr>
<tr>
<td>Compression members, spiral</td>
<td>0.75</td>
</tr>
<tr>
<td>Compression members, tied</td>
<td>0.70</td>
</tr>
<tr>
<td>Unreinforced footings</td>
<td>0.65</td>
</tr>
<tr>
<td>Bearings on concrete</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Concrete strain at ultimate stress is taken as 0.003 according to Art. 10.3.2, and the yield strength \( f_y \) of reinforcing steel is limited to 550 MPa (80 ksi) per Art. 9.4. The most popular grade of reinforcing steel in current use has \( f_y = 400 \) MPa (Grade 400 or 60 ksi).

**ELEMENTS OF USD.** For the partial development of the USD equations that follow, refer to Fig. 8-3.

From Fig. 8-3b the summing of horizontal forces, \( \sum F_H = 0 \), yields \( C = T \), and, taking the compressive stress block as a rectangle of dimensions shown,

\[ C = 0.85 f'_c b a \]

The tensile force in the steel reinforcement \( T \) is

\[ T = A_s f_y \]

Equating the latter quantities yields an expression for the depth of the compression block as

\[ a = \frac{A_s f_y}{0.85 f'_c b} \]  

(8-1)
For beams, \( b = \) width; for footings \( b = 1 \) unit (m or ft). From statics and summing moments at a convenient point (either \( T \) or \( C \)) we obtain

\[
T \left( d - \frac{a}{2} \right) = M_u = C \left( d - \frac{a}{2} \right)
\]

and solving for the ultimate resisting moment on a section and inserting the work quality factor \( \phi \), we have

\[
M_u = \phi A_s f_y \left( d - \frac{a}{2} \right) \tag{8-2}
\]

Alternatively, if steel ratio terms \( p \) and \( q \) are defined as follows,

\[
p = \frac{A_s}{bd} \quad q = \frac{p f_y}{f'_c}
\]

Eq. (8-2) can be written as

\[
M_u = \phi bd^2 f'_c q(1 - 0.59q) \tag{8-2a}
\]

The steel ratio at a cross section has been defined as \( p = A_s/bd \) and the ratio at balanced design will be designated as \( p_b \). To ensure a tensile failure rather than a sudden concrete compression failure \( p_d \) is taken as not over 0.75\( p_b \) (Art. 10.3.3) where the balanced reinforcement ratio is computed based on the concrete strain at ultimate stress of 0.003 and \( E_s = 200,000 \) MPa or \( 29 \times 10^6 \) psi as

\[
\text{SI: } p_b = \frac{0.85 \beta_1 f'_c}{f_y} \left( \frac{600}{f_y + 600} \right) \quad \text{Fps: } p_b = \frac{0.85 \beta_1 f'_c}{f_y} \left( \frac{87,000}{f_y + 87,000} \right) \tag{8-3}
\]

The factor \( \beta_1 \) in the preceding equation is defined as follows:

\[
\text{SI: } \beta_1 = 0.85 - 0.008(f'_c - 30 \text{ MPa}) \geq 0.65 \\
\text{Fps: } \beta_1 = 0.85 - 0.05(f'_c - 4 \text{ ksi}) \geq 0.65
\]

Footings for buildings seldom use \( f'_c > 21 \) MPa (3 ksi); for bridge footings \( f'_c \) is not likely to exceed 30 MPa (4 ksi), so the factor \( \beta_1 \) will, in nearly all cases, be 0.85. The lower-strength concrete is somewhat less costly per cubic meter but, more importantly, will produce a more rigid footing as it will have to be made thicker (larger \( D_c \) of Fig. 8-3a). Table 8-1 provides values for \( \beta_1 \) for a range of \( f'_c \), which may be of use here and for mat design (Chap. 10), where
TABLE 8-1
Maximum allowable steel ratio $\rho_d$*

Note: ASTM 615M and 615 now define only two grades of rebars: Grade 300 (40 ksi) and Grade 400 (60 ksi)

<table>
<thead>
<tr>
<th>$f_c$, MPa (ksi)</th>
<th>$\beta_1$</th>
<th>Grade 300 (40 ksi)</th>
<th>Grade 400 (60 ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 (3.0)</td>
<td>0.85</td>
<td>0.028</td>
<td>0.016</td>
</tr>
<tr>
<td>24 (3.5)</td>
<td>0.85</td>
<td>0.032</td>
<td>0.020</td>
</tr>
<tr>
<td>30 (4.0)</td>
<td>0.85</td>
<td>0.041</td>
<td>0.024</td>
</tr>
<tr>
<td>35 (5.0)</td>
<td>0.81</td>
<td>0.047</td>
<td>0.028</td>
</tr>
<tr>
<td>40 (6.0)</td>
<td>0.77</td>
<td>0.054</td>
<td>0.033</td>
</tr>
</tbody>
</table>

*Table ratios shown are 0.75$p_b$ for ensuring a tensile rebar failure per ACI Art. 10.3.3.

Values are slightly approximate for Fps units.

Higher-strength concrete may be used on occasion. Also given in Table 8-1 are the several values of $0.75p_b$ (limiting percentage of steel at a cross section), which as shown above depend on both $f_c'$ and $f_y$. Adequate concrete-to-rebar adhesion (termed bond) is provided for in Art. 12.2 by specifying the minimum length of embedment $L_d$ for reinforcing bars in tension depending on diameter or area as follows:

<table>
<thead>
<tr>
<th>Bar code number or diameter</th>
<th>$L_d \geq 300$ mm (12 in.)</th>
<th>SI</th>
<th>Fps</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 35 (35 mm) and smaller</td>
<td>$C_1A_{p_b}f_y/\sqrt{f_c'}$</td>
<td>$C_1 = 0.02$</td>
<td>0.04</td>
</tr>
<tr>
<td>No. 45 (No. 14)</td>
<td>$C_2f_y/\sqrt{f_c'}$</td>
<td>$C_2 = 25.0$</td>
<td>0.085</td>
</tr>
<tr>
<td>No. 55 (No. 18)</td>
<td>$C_3f_y/\sqrt{f_c'}$</td>
<td>$C_3 = 40.0$</td>
<td>0.125</td>
</tr>
</tbody>
</table>

Note: $f_c' = \text{MPa or psi}$, $A_{p_b} = \text{mm}^2$ or in$^2$, $L_d = \text{mm or in}$. Max. $\sqrt{f_c'} = \frac{25}{f_y}$ MPa or 100 psi (Art 12.1.2)

These development lengths should be multiplied by the following factors as applicable:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top rebars with more than 300 mm or 12 in. of concrete below bar</td>
<td>1.3</td>
</tr>
<tr>
<td>Lightweight concrete (seldom used for footings)</td>
<td>$1.3 \geq 1.0$</td>
</tr>
<tr>
<td>If bar spacing is at least $5d_{p_b}$ on centers and has at least $2.5d_{p_b}$ of side cover</td>
<td>0.8</td>
</tr>
<tr>
<td>If reinforcement is in excess of that required</td>
<td>$A_{p_b,\text{reqd}}/A_{p_b,\text{sum}}$</td>
</tr>
<tr>
<td>In all cases the embedment depth</td>
<td>$L_d \geq 300$ mm (12 in.), or $0.375d_{p_b}f_y/\sqrt{f_c'}$ mm or $0.03d_{p_b}f_y/\sqrt{f_c'}$ in</td>
</tr>
</tbody>
</table>

The development length for bond (Art. 12.3) for compression bars is the largest of the following:
where $A_b = \text{bar area, mm}^2 \text{ or in.}^2$

$d_b = \text{bar diameter, mm or in.}$

$f_y = \text{yield strength of steel, MPa or psi}$

$f'_c = \text{28-day compressive strength of concrete, MPa or psi}$

Standard hooks can be used to reduce the required value of $L_d$ from the preceding equations but are not usually used for footings. Hook requirements are given in Art. 12.5 of the Code.

Shear often governs the design of spread footings. The ACI Code allows shear to be computed as

$$v_u = \frac{V_u}{bd}$$  \(8-4\)

where $V_u$ is the ultimate shear force (factored working loads) and $bd$ is the resisting shear area of width $b$ and effective depth $d$ to center of tension steel. The nominal computed value of shear $v_u$ is compared with the allowable values, which are wide-beam and two-way action shear defined on Fig. 8-4. The allowable values of $v_c$ are as follows:

<table>
<thead>
<tr>
<th>$\phi$ = 0.85</th>
<th>SI</th>
<th>Fps</th>
<th>ACI Code Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wide-beam</td>
<td>$\phi \sqrt{f'_c / 6}$</td>
<td>$2\phi \sqrt{f'_c}$</td>
<td>Art. 11.3.1.1</td>
</tr>
<tr>
<td>Two-way action when $\beta \leq 2$</td>
<td>$\left(1 + \frac{2}{\beta}\right) \phi \sqrt{f'_c / 6}$</td>
<td>$\left(2 + \frac{4}{\beta}\right) \phi \sqrt{f'_c}$</td>
<td>Art. 11.12.2.1</td>
</tr>
<tr>
<td>but not more than:</td>
<td>$v_c = \frac{\phi \sqrt{f'_c}}{3}$ MPa</td>
<td>$4\phi \sqrt{f'_c}$ psi</td>
<td></td>
</tr>
<tr>
<td>$\beta = \frac{\text{Col. length}}{\text{Col. width}}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f'_c = \text{MPa psi}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In most practical design cases the columns have $L_c/B_c \leq 2$ (are often square or round with $L_c/B_c = 1$) so that $v_c = \phi \sqrt{f'_c / 3}$ (or $4\phi \sqrt{f'_c}$). The ACI Code allows shear reinforcement in footings and it is also obvious that a higher $f'_c$ concrete would reduce or eliminate the need for shear reinforcement. Neither of these alternatives is much used; rather, the effective footing depth $d$ is increased to satisfy shear requirements. This decision has the beneficial effect of increasing the footing rigidity, so the assumption of uniform base pressure is more likely to be obtained, as well as somewhat reducing settlement.

A minimum area of dowels of 0.005$A_{\text{col}}$ is required to anchor the column to the footing according to Art. 15.8.2.1. Dowels are sometimes required to transfer column stress into the footing, particularly if the column concrete is substantially stronger than the footing concrete.

---

4This was formerly called diagonal tension or punching shear.
Dowels are required if the column contact stress exceeds the following:

\[ f_c = 0.85 \phi f'_c \sqrt{\frac{A_2}{A_1}} \]

The ratio \( A_2/A_1 \leq 2 \) and the \( \phi \) factor is 0.7. The area \( A_1 \) is the column contact area \((b \times c)\) or \( \pi a^2/4 \); the area \( A_2 \) is the base of the frustum that can be placed entirely in the footing as shown in Fig. 8-4c and defined in Art 10.16.

Table 8-2 gives allowable wide-beam and two-way action shear values for several \( f'_c \) values. Table 8-3 summarizes the principal ACI Code requirements particularly applicable to concrete foundation elements (spread footings, mats, retaining walls).

8-5 STRUCTURAL DESIGN OF SPREAD FOOTINGS

The allowable soil pressure controls the plan \((B \times L)\) dimensions of a spread footing. Structural (such as a basement) and environmental factors locate the footing vertically in the soil. Shear stresses usually control the footing thickness \( D \). Two-way action shear always controls the depth for centrally loaded square footings. Wide-beam shear may control the depth for rectangular footings when the \( L/B \) ratio is greater than about 1.2 and may control for other \( L/B \) ratios when there are overturning or eccentric loadings.

The depth of footing for two-way action produces a quadratic equation that is developed from Fig. 8-4b, c using

\[ \sum F_v = 0 \]

on the two-way action zone shown. Noting the footing block weight cancels, we have—valid
TABLE 8-2
Allowable limiting two-way action and wide-beam shear \( v_c \) by ACI 318 Code for several concrete strengths \( f'_c \) for \( \beta \leq 2.0 \) and the \( \phi \) factor of 0.85 (ACI Art. 9.3.2.3)

<table>
<thead>
<tr>
<th>( f'_c, \text{MPa (psi)} )</th>
<th>( \phi = 0.85 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>(3000)</td>
</tr>
</tbody>
</table>

Two-way action (ACI 11.12.2-)

\[
\phi \frac{\sqrt{f'_c}}{3} \quad (\text{MPa})
\]

\[
4\phi \frac{\sqrt{f'_c}}{6} \quad (\text{ksi})
\]

Wide-beam shear* (ACI 11.3.1.1)

\[
\frac{\phi \sqrt{f'_c}}{6} \quad (\text{MPa})
\]

\[
2\phi \sqrt{f'_c} \quad (\text{ksi})
\]

*For two-way action the ACI Code allowable shear stress (MPa) is the smallest of the above and the following two equations:

\[
\left(1 + \frac{2}{\beta_c}\right)\phi \frac{\sqrt{f'_c}}{6} \quad [\text{ACI (11-36)}]
\]

\[
\left(\alpha_s d + 2\right)\phi \frac{\sqrt{f'_c}}{12} \quad [\text{ACI (11-37)}]
\]

where \( \beta_c = \) ratio of long column side over short column side (and must be \( \beta_c > 2 \) to become the smallest allowable \( v_c \)).

\( \alpha_s = \) 40 for interior, 30 for edge, and 20 for corner columns.

\( b_o = \) two-way action perimeter defined using column dimensions + \( \frac{1}{2}d \) distance as appropriate from column face(s).

\( d = \) effective depth of member.

for either USD or ADM (select elements of the ADM are given in Sec. 12-16)

\[
P_u = 2dv_c(b + d) + 2dv_c(c + d) + (c + d)(b + d)q
\]

Substitution of \( P_u \) or \( P_d = BLq \) and using either the USD or the ADM shear stress \( v_c \) gives

\[
d^2(4v_c + q) + d(2v_c + q)(b + c) = (BL - cb)q \quad (8-5)
\]

For a square column \( c = b = w \) we obtain

\[
d^2 \left( v_c + \frac{q}{4} \right) + d \left( v_c + \frac{q}{2} \right)w = (BL - w^2)\frac{q}{4} \quad (8-6)
\]

For a round column, \( a = \) diameter, the expression is

\[
d^2 \left( v_c + \frac{q}{4} \right) + d \left( v_c + \frac{q}{2} \right)a = (BL - A_{col})\frac{q}{\pi} \quad (8-7)
\]
### TABLE 8-3
Summary of ACI 318 Code reinforced concrete foundation requirements

<table>
<thead>
<tr>
<th>Design factor</th>
<th>Art. number</th>
<th>General requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing (column on footing)</td>
<td>10.15</td>
<td>( q_{	ext{req}} \leq \psi 0.85 f'_c ) ( \psi \leq 2 )</td>
</tr>
<tr>
<td>Design load combinations</td>
<td>9.2</td>
<td>E.g., as ( 1.4 \times \text{dead load} + 1.7 \times \text{live load} )</td>
</tr>
<tr>
<td>Footings</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Column-to-base stress transfer</td>
<td>15.8</td>
<td>With dowels, ( w/A_s \geq 0.005 A_g )</td>
</tr>
<tr>
<td>Location of moment</td>
<td>15.4.2</td>
<td>See Fig. 8-5</td>
</tr>
<tr>
<td>Location of shear</td>
<td>15.5</td>
<td>See Fig. 8-4</td>
</tr>
<tr>
<td>Minimum edge thickness</td>
<td>15.7</td>
<td>150 mm above reinforcement; 300 mm above pile heads</td>
</tr>
<tr>
<td>Round columns on</td>
<td>15.3</td>
<td>Use equivalent square of same area</td>
</tr>
<tr>
<td>Grade beams</td>
<td>14.7</td>
<td>Use walls with grade beams</td>
</tr>
<tr>
<td>Load factors ( \phi )</td>
<td>9.3.2</td>
<td>See table in textbook (Sec. 8-4)</td>
</tr>
<tr>
<td>Minimum wall thickness</td>
<td>14.5.3.2</td>
<td>Generally 190 mm</td>
</tr>
<tr>
<td>Modulus of elasticity ( E_c )</td>
<td>8.5</td>
<td>( E_c = 4700 \sqrt{f'_c} ) MPa*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( = 57000 \sqrt{f'_c} ) psi</td>
</tr>
<tr>
<td>Reinforcement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development length</td>
<td>12.2, 12.6</td>
<td>See equations in text or code</td>
</tr>
<tr>
<td>Lap splices in</td>
<td>12.14.2</td>
<td>Not for bars &gt; No. 35</td>
</tr>
<tr>
<td>Limits in compression</td>
<td>10.9</td>
<td>( 0.01 \leq A_{\mu}/A_g \leq 0.08 )</td>
</tr>
<tr>
<td>Maximum ratio</td>
<td>10.3.3</td>
<td></td>
</tr>
<tr>
<td>Minimum ratio</td>
<td>10.5.1</td>
<td></td>
</tr>
<tr>
<td>Minimum cover</td>
<td>7.7.1</td>
<td>Cast-in-place use 70 mm; with forms 50 mm</td>
</tr>
<tr>
<td>Rectangular footings, for</td>
<td>15.4.4</td>
<td></td>
</tr>
<tr>
<td>Spacing of</td>
<td>7.6</td>
<td>Not less than ( D ) or 25 mm or 1.33 \times \text{max. aggregate size}; not more than ( 3 \times D_c ) or 500 mm</td>
</tr>
<tr>
<td>Temperature and shrinkage</td>
<td>7.12</td>
<td></td>
</tr>
<tr>
<td>Shear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two-way action</td>
<td>11.12.1:2</td>
<td>( v = V_w/bd )</td>
</tr>
<tr>
<td>( v_c = \left( 1 + \frac{2}{\beta} \right) \frac{\phi \sqrt{f'_c}}{6} \leq \frac{\phi \sqrt{f'_c}}{3} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta = \frac{\text{Column length}}{\text{Column width}} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wide-beam</td>
<td>11.12.1</td>
<td>( v_c = 2\phi \sqrt{f'_c} ) psi</td>
</tr>
<tr>
<td>Reinforcement allowed</td>
<td>11.12.3</td>
<td></td>
</tr>
<tr>
<td>( \beta_1 ) factor</td>
<td>10.2.7.3</td>
<td>( \beta_1 = 0.85 ) ( f'_c \leq 30 \text{ MPa} ); reduced by ( 0.008 ) for each ( 1 \text{ MPa in excess of } 30 \text{ MPa but } \beta_1 \geq 0.65 )</td>
</tr>
</tbody>
</table>

*\( E_c = \gamma_c^{1.5} 44 \sqrt{f'_c} \) MPa for \( 14 \leq \gamma_c \leq 25 \text{ kN/m}^3 \)

Values shown of 4700 and 57 000 are for "normal" weight concrete.

If we neglect the upward soil pressure on the diagonal tension block, an approximate effective concrete depth \( d \) can be obtained for rectangular and round columns as

\[
\text{Rectangular: } \quad 4d^2 + 2(b + c)d = \frac{BLq}{v_c} = \frac{P_v}{v_c} \quad (8-8)
\]
The approximate formulas will result in a \( d \) value seldom more than 25 mm or 1 in. larger than the "exact" formulas of Eqs. (8-5) and (8-7).

Always use Eq. (8-7) or (8-9) for round columns to obtain the effective footing depth \( d \) since using an equivalent square column and Eq. (8-5) gives a smaller value.

Steps in square or rectangular spread footing design with a centrally loaded column and no moments are as follows:

1. Compute the footing plan dimensions \( B \times L \) or \( B \) using the allowable soil pressure:

   **Square:**
   \[
   B = \sqrt{\frac{\text{Critical load combination}}{q_a}} = \sqrt{\frac{P}{q_a}}
   \]

   **Rectangular:**
   \[
   BL = \frac{P}{q_a}
   \]

   A rectangular footing may have a number of satisfactory solutions unless either \( B \) or \( L \) is fixed.

2. Convert the allowable soil pressure \( q_a \) to an ultimate value \( q_{\text{ult}} = q \) for use in Eqs. (8-5) through (8-9) for footing depth

   \[
   \frac{P_u}{BL} = q = \frac{P_{\text{ult}}}{P_{\text{design}}} q_a
   \]

   Obtain \( P_u \) by applying appropriate load factors to the given design loading.

3. Obtain the allowable two-way action shear stress \( v_c \) from Table 8-2 (or compute it) and, using the appropriate Eqs. (8-5) through (8-9), compute the effective footing depth \( d \).

4. If the footing is rectangular, immediately check wide-beam shear. Use the larger \( d \) from two-way action (step 3) or wide-beam.

5. Compute the required steel for bending, and use the same amount each way for square footings. Use the effective \( d \) to the intersection of the two bar layers for square footings and if \( d > 305 \text{ mm or 12 in.} \). For \( d \) less than this and for rectangular footings use the actual \( d \) for the two directions. The bending moment is computed at the critical section shown in Fig. 8-5. For the length \( l \) shown the ultimate bending moment/unit width is

   \[
   M_u = \frac{ql^2}{2}
   \]

In Eq. (8-2) use \( M \) equals \( M_u \) if \( q = q_{\text{ult}} \) to obtain the amount of reinforcing bar steel/unit width. Check the steel ratio \( p \) to satisfy Temperature and Shrinkage (\( T \) and \( S \)) and to verify that the maximum steel ratio of Table 8-1 is not exceeded. You should be aware that the ACI 318 has specific \( T \) and \( S \) requirements for slabs but is somewhat ambiguous about \( T \) and \( S \) requirements for footings. The commentary R7.12.1 states, "... the provisions of these sections are intended for structural slabs only; they are not intended for 'slabs on grade.'"

Some designers would routinely put \( T \) and \( S \) steel in spread footings or mats if the top is not covered with earth. Where the top of the footing is covered by about 400 to 500 mm
of earth, there is enough insulation provided that changes in temperature are not wide. Large temperature variations tend to produce tension cracks unless restrained by $T$ and $S$ reinforcement. Regardless of the code, which tends to give minimum requirements, one can always overdesign, that is, exceed any minimum code requirements.

6. Compute column bearing and use dowels for bearing if the allowable bearing stress is exceeded. In that case, compute the required dowels based on the difference between actual and allowable stresses $\times$ column area. This force, divided by $f_y$, is the required area of dowels for bearing.

   It is necessary always to use a minimum of $0.005A_{col}$ of dowel steel regardless of the bearing stress.

   If dowels are required to transfer any column load, the length must be adequate for compression bond. The ACI 318 covers the required length in Art. 12.3. If the footing does not have a sufficient $d$ you can put them in a spiral encasement and reduce the required length 25 percent. If that is not adequate you will have to increase the effective footing depth $d$. The use of $90^\circ$ bends (whether required or not) is common, as it allows easy attachment of the column dowels to the footing reinforcement by wiring.

7. Detail the design. At least provide enough detail that a draftsperson (or a CAD operator) can produce a working drawing for the construction personnel.

The current ACI Code procedure as outlined in the preceding steps is based primarily on tests by Richart (1948), which showed larger bending moments at the column face for column strips and lesser values on other strips. Bowles (1974a, Chap. 7), using finite-difference and finite-element analytical procedures, found that, whereas the bending moment is higher in the column area, for finite-difference methods the average bending moment across the footing at the section taken in Fig. 8-5 is the same as the Code procedure. The maximum computed moment will exceed the average moment by about 30 percent for the finite-difference method and by more than 40 percent using the finite-element method, and assuming column fixity, which is close to reality for concrete columns attached by the Code requirement to the footing as shown later in Example 10-4. It is implicit that readjustment will take place to reduce the cracking effect of the column-zone moment. It may be questionable whether the 40 percent
larger moment can be adequately readjusted without possible cracking and long-term corrosion effects. This problem was less severe when the alternative design method was more popular than at present. The problem is such that one should consider the use of larger load factors than $1.4D$ and $1.7L$ for footings based on the USD method. It is, of course, always permissible to use larger factors since any code provides only minimum values. Alternatively, one could compute the total steel required for the side and put, say, 60 percent in a column zone with a width of about $w + 2d$ and the remainder in the two end zones—similarly for the orthogonal direction.

**Example 8-1.** Design a plain (unreinforced) concrete spread footing (see Fig. E8-1a) for the following data:

\[
DL = 90 \text{ kN} \quad LL = 100 \text{ kN}
\]

Column: W 200 × 31.3 resting on a 220 × 180 × 18 mm base plate

[Rolled structural shape dimensions are available in ASTM A 6M or AISC (1992).] Also:

\[
f'_c = 21 \text{ MPa}
\]

Allowable soil pressure $q_a = 200 \text{ kPa}$

**Solution:** Note the following:

1. Plain concrete footings must be designed using ACI 318.1 "Building Code Requirements for Structural Plain Concrete." The SI version is ACI 318.1M.
2. Unreinforced footings are only practical and economical for small column loads as in this example.
3. We could step or taper the footing to reduce the volume of concrete slightly but at current labor costs for the additional formwork and shaping; it is usually more economical to use a constant depth footing.
4. Wall footings are very commonly made of plain concrete.

With these comments we will now proceed with the footing design.

**Step 1.** Size the footing:

\[
B^2 q_a = P = 90 + 100 = 190
\]

\[
B = \sqrt{\frac{190}{200}} = 0.97 \text{ m} \quad \text{Use } B = 1 \times 1 \text{ m}
\]
Step 2. Find the footing depth. For plain footings the moment requirement is usually critical, so we will find the depth to satisfy moment and then check shear.

Convert $q_a$ to a pseudo $q_{ult}$ so we can use USD:

$$P_{ult} = 1.4DL + 1.7LL$$

(as one load combination given in ACI 318- (Art. 9.2), which we assume controls in this example)

$$P_{ult} = 1.4(90) + 1.7(100) = 296 \text{ kN} \quad q_{ult} = \frac{P_{ult}}{A_{fg}} = \frac{296}{1^2} = 296 \text{ kPa}$$

For flexure the maximum tensile stress is $f_t = 0.4\sqrt{f'_c}$ (ACI 318.1M, Art. 6.2.1).

For all cases the $\phi$ factor = 0.65 for plain concrete (Art. 6.2.2). Thus,

$$f_t + 0.4(0.65)(21)^{1/2} = 1.19 \text{ MPa}$$

The critical section is defined at $\frac{1}{2}$ distance from edge of base plate to column face (Figs. 8-5 and 8-6b), which will be taken as $\frac{1}{2}$ distance to center of web that gives the largest moment arm $L_m$.

Referring to Figs. E8-1c and 8-6b the distance is

$$L_m = \frac{B}{2} - \frac{0.180}{2} + \frac{0.180}{4} = 0.455 \text{ m}$$

$$M_u = \frac{q_{ult}L_m^2}{2} = \frac{296(0.455)^2}{2} = 30.64 \text{ kN} \cdot \text{m/m}$$

Equating allowable stress $f_t \times$ section modulus $S = M_u$ and for a rectangle

$$S = bd^2/6$$

Here we will use $b = \text{unit width} = 1 \text{ m}$ giving $f_t S = f_t d^2/6 = M_u = 30.64$.

$$f_t d^2/6 = 30.64$$

$$d = \sqrt{\frac{30.64(6)}{1.19 \times 1000}} = 0.393 \text{ m}$$

To this thickness $d$ we must add 50 mm according to Art. 6.3.5 (of 318-1M) for concrete in contact with ground, or

$$D_c = d + 0.050 = 0.393 + 0.050 = 0.443 \text{ m} \quad \text{Use 450 mm}$$

![Figure E8-1b](image-url)

This depth not effective (ACI 318-1M, Art.6.3.5)
Step 3. Check two-way action using \( d = 450 - 50 = 400 \) mm effective depth.

\[
v_c = \left(1 + \frac{2}{\beta}\right) \phi \frac{\sqrt{f_c}}{6} \leq \frac{\sqrt{f_c}}{3} \quad \text{(Art. 6.2.1c or ACI 318-, Art. 11.12.2.1)}
\]

\[
\beta = \frac{\text{Col. length}}{\text{Col. width}} = \frac{210}{90} = 2.33 \quad \text{(using “effective” width)}
\]

\[
v_c = \left(1 + \frac{2}{2.33}\right) \frac{0.65(21)^{1/2}}{6} = 0.92 \text{ MPa} < \frac{0.65(21)^{1/2}}{3}
\]

The average shear perimeter \( p \) at \( d/2 \) from the column with average column dimensions of depth = \( (220 + 210)/2 = 215 \) mm and width = \( 180/2 = 90 \) mm (see Fig. E8-1c) is

\[
p = 2(0.215 + 0.400 + 0.090 + 0.400) = 2.21 \text{ m}
\]

The shear resistance (neglecting the upward soil pressure on this area) is

\[
R = pdv_c = 2.21(0.40)(0.92 \times 1000) = 813 \text{ kN} \gg 296 (= P_{ult})
\]

Step 4. We should check wide-beam shear at distance \( d \) from the critical column face.

Critical \( L' = L_m - d \) (by inspection of Fig. E8-1c)

\[
L' = 0.455 - 0.400 = 0.055 \text{ m (negligible)}
\]

For a shear force \( V = 0.055q_{ult} \), wide-beam shear is not critical.

Step 5. Draw a final design sketch as in Fig. E8-1c. A question may arise of whether this plain concrete base should contain temperature and shrinkage (T and S) steel. Strictly, the ACI Code is not
clear on this point; however, if we check Art. 2.1 of 318.1, it defines plain concrete as either unreinforced or containing less reinforcement than the minimum specified in ACI 318. Some authorities are of the opinion that concrete placed in the ground does not require temperature and shrinkage steel since the temperature differentials are not large. For footings, one must make a judgment of effects of temperature and shrinkage cracks. For this and other plain concrete footings a more conservative solution is obtained by using T and S steel both ways. For this problem, and referring to ACI Sec 7.12.2.1, use

\[ T \text{ and } S \text{ reinforcement both ways} = 0.002(0.4 \times 1) \times 10^6 = 800 \text{ mm}^2 \text{ each way} \]

From Table inside front cover try four No. 15 (16 mm diam.) bars each giving

\[ A_s = 4 \times 200 = 800 \text{ mm}^2 \]

Four equally spaced bars satisfy maximum spacing requirements.

Example 8-2. Design a spread footing for the average soil conditions and footing load given in Fig. E8-2a. Note the geotechnical consultant provided \( q_a \) in Example 8-1; however, in this case the designer preferred to select the allowable soil pressure from a soil profile provided by the geotechnical engineer.

\[ DL = 350 \text{ kN} \quad LL = 450 \text{ kN} \quad f' = 21 \text{ MPa} \]

Use grade 400 rebars \( f_y = 400 \text{ MPa} \)

The column has dimensions of 0.35 \( \times \) 0.35 m and uses four No. 30 bars (diam. = 29.9 mm, see inside front cover).
Solution.

Step 1. From the soil profile find \( q_a \). To start, we readily obtain \( q_a = q_u \) from the average \( q_u \) (SF = 3 as in Example 4-4). Estimate \( \gamma_{clay} \approx 18.00 \text{kN/m}^3 \). So, we can include the \( \bar{q}N_q \) term (and \( N_q = 1.0 \)):

\[
q_a = 200 \text{kPa} + 1.2(18.00)(1) \approx 220 \text{kPa} \quad \text{(Use 200 kPa)}
\]

Step 2. Find tentative base dimensions \( B \) using a square footing, or

\[
P = 350 + 450 = 800 \text{kN} \quad \text{and} \quad B^2 q_a = P
\]

\[
B = \sqrt{\frac{800}{200}} = 2.00 \text{ m}
\]

Step 3. Check the immediate settlement. Consolidation settlement is not a problem since the water table is at the top of the sand at \(-12 \text{ m}\). Take

\[
E_s = 1000s_u \text{ since clay is stiff; } s_u = q_u/2 = 100 \text{kPa}
\]

\[
E_s = 1000(100) = 100000 \text{kPa}
\]

For the sand we must convert \( N_{70} \) to \( N_{55} \) in order to use Table 5-5. Use a conservative value of \( E_s = 500(N_{55} + 15) \):

\[
\text{Above GWT: } E_s = 500[25(70/55) + 15] = 23409 \text{kPa}
\]

\[
\text{Below GWT: } E_s = 500[30(70/55) + 15] = 26590 \text{kPa}
\]

The depth of influence is taken as \( 5B = 10 \text{ m} \), which is \( 2 \text{ m} \) above the \( 12 \text{ m} \) depth of the boring. Also estimate Poisson's ratio \( \nu = 0.35 \) (for the clay).

Use a weighted average \( E_s \) for the influence depth below the footing base of \( 8.8 \text{ m} \), based on stratum thickness:

\[
E_s = \frac{(6-1.2)100000 + (9-6)23409 + (10-9)26590}{8.8} = \frac{576817}{8.8} = 65500 \text{kPa (rounding down slightly)}
\]

For \( 10/B' = 10/(2/2) = 10 \), we obtain (using Table 5-2)

\[
I_s = 0.498 + 0.016\frac{[1 - 2(0.35)]}{[1 - 0.35]} = 0.505
\]

For \( D/B = 1.2/2 = 0.6 \) estimate the Fox embedment factor as

\[
I_F = 0.75 \quad \text{(using Fig. 5-7)}
\]

Using Eq. (5-16a), we see that

\[
\Delta H = q_o B' \frac{1 - \mu^2}{E_s} m I_s I_f \quad \text{(and with } m = 4)
\]

\[
= \frac{800}{2} \frac{1 - 0.35^2}{2} \frac{4(0.505)(0.75)}{65000}
\]

\[
= 0.00406 \text{ m} \rightarrow 4.06 \text{ mm} \quad \text{(clearly } \Delta H \text{ is not a problem.)}
\]

We can now proceed with the footing design using

\[
B \times B = 2 \times 2 \text{ m} \quad \text{and} \quad q_o = 800/4 = 200 \text{kPa} < q_a
\]

We have made no allowance for soil displaced by concrete, but recall that \( q_a \approx 220 \text{kPa} \), which should be sufficient.
Step 4. First find the pseudo $q_{ult}$:

$$q_{ult} = \frac{1.4(350) + 1.7(450)}{2} = \frac{1255}{4} = 313.8 \text{ kPa}$$

Ratio = $\frac{q_{ult}}{q_a} = \frac{313.8}{200} = 1.57$

Step 5. Find the depth for two-way action shear using Eq. (8-6):

$$d^2 \left( v_c + \frac{q}{4} \right) + d \left( v_c + \frac{q}{2} \right) w = (B^2 - w^2) \frac{q}{4}$$

Allowable concrete shear stress $v_c = \phi \sqrt{f_c/3} = 1.30 \text{ MPa}$. Substituting values $q = 313.8 \text{ kPa}$; $v_c = 1300 \text{ kPa}$, and $w = 0.35 \text{ m}$ into Eq. (8-6), we obtain

$$1378.45d^2 + 509.9d = 304.2$$

$$d^2 + 0.37d - 0.2207 = 0$$

$$d = \frac{-0.37 \pm \sqrt{0.37^2 - 4(1)(-0.2207)}}{2} = \frac{-0.37 + 1.01}{2} = 0.32 \text{ m}$$

The approximate effective depth by Eq. (8-8) is

$$4d^2 + 2(w + w)d = \frac{BLq}{v_c}$$

Substituting values, we obtain

$$4d^2 + 4(0.35)d = 4(313.8)/1300$$

$$d^2 + 0.35d - 0.241 = 0$$

$$d = 0.346 \text{ m} (346 \text{ mm vs. 320 mm by "exact" method})$$

For a square, centrally loaded footing it is never necessary to check wide-beam shear and since column $w = d$ it is not necessary to check ACI Eq. (11-37).

Step 6. Find the required steel for bending. We will take $d$ as 0.32 m to the intersection of the bottom of the top bars and the top of the bottom bars, for they will go both ways and will likely be wired together in the shop so that either side of the resulting grid can be the "top" (refer to Fig. 8-2c). Refer to Fig. 8-2b for the moment arm as defined by the ACI 318.

$$L_m = \frac{(B - w)}{2} = \frac{(2.00 - 0.35)}{2} = 0.825 \text{ m}$$

$$M_u = \frac{qL_m^2}{2} = \frac{(313.8)(0.825)^2}{2} = 106.8 \text{ kN} \cdot \text{m}$$

$$M_u = \phi A_s f_y \left( d - \frac{a}{2} \right)$$ (8-2)

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s(400)}{0.85(21)(1)} = 22.4 A_s$$

Rearranging Eq. (8-2) and substituting, we have

$$A_s \left( 0.32 - \frac{22.4}{2} \right) A_s = \frac{106.8}{0.9 \times 400 \text{000}}$$
Solving, we obtain

\[-11.2A_t^2 + 0.32A_s = 0.000297\]

\[A_s = \frac{0.0286 \pm \sqrt{0.0286^2 - 4(1)(0.0000265)}}{2(1)}\]

\[= 0.00096 \text{ m}^2/\text{m} \rightarrow 960 \text{ mm}^2/\text{m} \quad \text{[always use largest (+) value]}\]

Use five No. 15 bars/m to provide \(5 \times 200 = 1000 \text{ mm}^2/\text{m}\) of steel at a spacing of \(1000/4 = 250 \text{ mm}\). We could use a lesser number of bars:

- four No. 20 giving \(4(300) = 1200 \text{ mm}^2/\text{m}\)
- two No. 25 giving \(2(500) = 1000 \text{ mm}^2/\text{m}\)

This latter value sets the spacing at 1000 mm which is greater than 500 mm allowed by ACI.

\[A_s,\text{total} = 2 \text{ m} \times 1000 \text{ mm}^2/\text{m} = 2000 \text{ mm}^2/\text{m} \quad \text{(and each way)}\]

Use 10 No. 15 bars at spacing \(s: 9s + 2(70) + 16 = 2000; s = 205 \text{ mm}\) with \(10 \times 200 = 2000 \text{ mm}^2/\text{m}\) steel area. Now check steel ratio:

\[p = \frac{1000}{(320)(1000)} = 0.00312 > 0.002 \text{ O.K.}\]

\(< 0.016 \text{ Table 8-1 also O.K.}\]

**Step 7.** Check if the furnished \(L = 0.825 - 0.07 \text{ m}\) (clear cover requirement of Art. 7-7.1) = \(0.755 \text{ m} \leq L_d\) of Art. 12.2.2:

\[L_{db} = \frac{0.02A_h f_s}{\sqrt{f_c}} = \frac{0.02(200)(400)}{721}\]

\[= 349 \text{ mm} > 300 \text{ (minimum length in any case)}\]

\(< 755 \text{ mm furnished}\]

There are no multipliers to increase this computed \(L_{db}\) so it will not be larger than the 0.755 m provided by the footing. Thus, the tension bar anchorage is adequate.

**Step 8.** Check column bearing on the footing per ACI Arts. 10.15 and 15.8. In general allowable bearing pressure is

\[f_c = \phi(0.85)(f'c)\Psi\]
where \( \Psi = \sqrt{\frac{A_2}{A_1}} \leq 2 \)

\( A_1 \) = column contact area
\( A_2 \) = area of column spread through depth \( d \) using the distribution shown in Fig. 8-4c.

Inserting values, we compute the allowable bearing stress \( f_c \) as

\[
f_c = 0.70(0.85)(21)(2) = 25 \text{ MPa}
\]

Check the column capacity based on a gross concrete section. If that is adequate, a refined check is not required.

\[
P_{\text{comp}} = 0.35^2(25 \times 1000) = 3062 \text{ kN}
\]

\[
P_u = 1.4(350) + 1.7(450) = 1255 \text{ kN} \ll 3062 \quad \text{O.K.}
\]

**Step 9.** Design dowels. ACI 318 requires a minimum area of dowels of \( 0.005A_{\text{col}} \) (Art. 15-8.2.1) unless a larger amount is needed to transfer compressive forces or moments. In this case the minimum controls:

\[
A_{s,\text{dowels}} = 0.005(0.35^2) = 0.0006125 \text{ m}^2 = 612.5 \text{ mm}^2
\]

Set four column reinforcing bars with right-angle bends onto the footing reinforcing bars and wire them into position:

\[
A_{s,\text{furn}} = 4(700) = 2800 \text{ mm}^2 \gg 612.5 \text{ required}
\]

Use column reinforcing bar lengths so they either do not have to be spliced in the column zone or will extend above the top of the footing so that the splice length of Art. 12-14 can be satisfied.

**Step 10.** Make a design sketch as in Fig. E8-2c.

It will be necessary to provide at least a 70-mm clear cover from the bottom of the lower reinforcing bar (No. 15 of diam. = 16 mm) to the bottom of the footing. This gives a total depth of

\[
D_c = 320 \text{ mm} + 16 \text{ mm} + 70 \text{ mm} = 406 \text{ mm} \rightarrow 410 \text{ mm}
\]

Note that the top layer of reinforcing bars requires slightly more than 960 mm\(^2\) (actually, 1000 mm\(^2\)) and the lower layer requires slightly less than 960 mm\(^2\). This methodology is standard practice, however, since it is seldom that one can obtain a bar schedule that exactly produces the computed (or required) \( A_s \). It is not good practice to mix bar sizes to obtain exactly the required amount of steel area.

We did not check the actual and allowable soil pressures. First, we designed the base on the basis of using 200 kPa when we could have used about 220 kPa. This base is thin (at 406 mm), so soil-concrete displacement pressure is negligible (about 2.3 kPa).

It will be useful to compare any cost savings by using the approximate base depth equation [Eq. (8-8)] versus the exact equation. See the next example.

**Example 8-3.**

**Given.** The footing and foundation data of Example 8-2.

**Required.** Compute the required reinforcement and compare this to Example 8-2.
**Solution.** All data are exactly the same except \( d \). The approximate value of \( d \) computed in Example 8-2 (see Step 5) is \( d = 346 \text{ mm} \rightarrow \text{use 350 mm} \); similarly, \( a = 22.4A^2 \) and constant = 0.000 0297.

**Step 1.** Substitute values and obtain

\[-11.2A^2 + 0.35A_s = 0.000 297\]

Dividing through by 11.2 and solving the resulting quadratic equation, we have [and again use largest (+) value]

\[A_s = \frac{\sqrt{0.03125 \pm 0.031 25^2 - 4(1)(0.000 0265)}}{2(1)}\]

\[= (0.03125 - 0.0295)/2 = 0.000 872 \text{ m}^2/\text{m} = 872 \text{ mm}^2/\text{m}\]

For \( B = 2 \text{ m} \) the required total is

\[A_s = 2(872) = 1744 \text{ mm}^2\]

Use six No. 20 bars giving \( 6(300) = 1800 \text{ mm}^2 \) each way:

- Spacing \( \approx 2000/5 = 400 \text{ mm} < 3(35) \)
  - \( < 500 \text{ mm} \) (Art. 7-6)
- Diam. of No. 20 bar = 19.5 mm \( \rightarrow \) use 20 mm
- Total depth \( D_c = 350 + 20 + 70 = 440 \text{ mm} \)

**Step 2.** Steel mass \( = 490 \text{ lb/ft}^3 = 490(3.2808^3)(0.453) = 7840 \text{ kg/m}^3 \)

From Example 8-2,

\[L_s = 2000 - 2(70) = 1860 \text{ mm} \quad \text{(clear cover = 70 mm)}\]

\[A_s = 2000 \text{ mm}^2 \text{ each way}\]

Vol. of steel \( V_s = 2(2000)(1860) = 7440000 \text{ mm}^3\]

\[= 0.00744 \text{ m}^3\]

Mass of steel \( M_1 = 7840(0.00744) = 58.3 \text{ kg/footing} \)

\( D_c \) of Example 8-2 = 410 mm

Vol. of concrete \( V_c = 2 \times 2 \times 0.410 = 1.64 \text{ m}^3\)

For Example 8-3,

\[L_s = 1860 \text{ mm}\]

\[A_s = 1800 \text{ mm}^2 \text{ each way}\]

Vol. of steel \( V_s = 2(1800)(1860) = 6696000 \text{ mm}^3\]

\[= 0.0067 \text{ m}^3\]

Mass of steel \( M_2 = 7840(0.0067) = 52.5 \text{ kg/footing} \)

Vol. of concrete \( V_c = 2 \times 2 \times 0.44 = 1.76 \text{ m}^3\)

**Summarizing,**

<table>
<thead>
<tr>
<th>Item</th>
<th>Exact</th>
<th>Approx</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{\text{concrete}} ), m(^3)</td>
<td>1.64</td>
<td>1.76</td>
<td>0.12</td>
</tr>
<tr>
<td>( D_c ) mm</td>
<td>410.</td>
<td>440.</td>
<td>30.</td>
</tr>
<tr>
<td>Mass of steel, kg</td>
<td>58.3</td>
<td>52.6</td>
<td>5.8</td>
</tr>
</tbody>
</table>

The "approximate" depth footing is probably about $10 (US) more economical and certainly a small amount stiffer than the "exact" depth footing. Foundations of this type are usually bid on
the basis of volume (m$^3$) of in-place concrete (currently around $200$ to $225$ per m$^3$ in-place). There is much to recommend using Eq. (8-8) over the "exact" Eq. (8-7). This consideration will come up later in this chapter for footings with moment.

8-6 BEARING PLATES AND ANCHOR BOLTS

Metal column members, including various tower-type elements, require a base plate to spread the very high metal stresses in the small column/tower contact area at the footing interface to a value that the footing or pedestal concrete can safely carry. The bearing plate is cut to size in the steel fabricating shop from rolled plate stock and either shop-welded or field-bolted to the column member. Holes 2- to 5-mm larger in diameter than the anchor rods/bolts are shop-punched in the base plate for later attachment to the footing.

The anchor rods are usually set in nearly exact position in the wet concrete and become fixed in place. The slightly oversized holes allow a small amount of anchor rod misalignment when placing the base plate into position. The plate is then carefully aligned horizontally and to elevation, and nuts are added and tightened to attach the column firmly to the footing.

The AISC (1989) specification provides general guidance in the design of base plates. There is little available design material for anchor bolts aside from that provided by the several manufacturers, which usually is limited to suggested embedment depth and allowable anchor rod force.

8-6.1 Base Plate Design

Base plates can be designed using the AISC specification for axial-loaded columns as follows:

When the base plate covers the concrete support (typically the base plate of a pedestal is the same size as a pedestal) the allowable bearing stress $F_p$ is

$$ F_p = 0.35 f_c' $$

(\text{a})

When the base plate covers less than the supporting concrete surface (typical for spread footings carrying steel columns fitted with a base plate), the allowable bearing stress $F_p$ is

$$ F_p \leq 0.35 f_c' \sqrt{\frac{A_2}{A_1}} \leq 0.7 f_c' $$

(\text{b})

where $F_p =$ allowable concrete stress; must be greater than the actual bearing stress defined as $f_p = P/A_1$, where $P =$ sum of column loads acting on footing

$A_1 =$ area of base plate in consistent units

$A_2 =$ area of supporting member; is area of pedestal when the base plate is on the pedestal; is area of footing for other cases

A limitation is that $\Psi = \sqrt{A_2/A_1} \leq 2.$

\text{Baseplate methodology has changed with the last three editions of the American Institute of Steel Construction (AISC) Allowable Stress Design manual.}
If we substitute for $F_p$ in Eq. (b), note the limitation on $\sqrt{A_2/A_1}$ and square both sides, we obtain

$$\left(\frac{P}{0.35 f'_c}\right)^2 \leq A_1^2 \left(\frac{A_2}{A_1}\right) \leq 4A_1^2 \quad (c)$$

From the left two terms of Eq. (c) we obtain the base plate area as

$$A_1 = \frac{1}{A_2} \left(\frac{P}{0.35 f'_c}\right)^2 \quad (8-10)$$

The minimum pedestal dimensions $A_2$ are obtained from the right two terms of Eq. (c) to give

$$A_2 = 4A_1$$

which can be written by making substitution in Eq. (8-10) for $A_1$ as

$$A_2 = \frac{P}{0.175 f'_c} \quad (8-10a)$$

In this equation the area $A_2 = \text{both minimum and optimum size of the pedestal.}$

We may summarize the steps in designing a base plate by the AISC (1989) specifications as follows:

1. Find plate area $A_1$ as the larger of

$$A_1 = \frac{1}{A_2} \left(\frac{P}{0.35 f'_c}\right)^2 \quad \text{and} \quad A_1 = \frac{P}{0.7 f'_c}$$

You may first have to find area $A_2$ using Eq. (8-10a) if a pedestal is being used.

2. Find the base plate dimensions (refer to Fig. 8-6 for identification of dimensions) $B \times C \geq A_1$ and use multiples of 5 mm (integers of inches for Fps) for dimensions $B, C$. Also try to make $m \approx n$ to minimize plate thickness $t_p$. For $m, n$ use the following:

$$m = \frac{C - 0.95d}{2} \quad n = \frac{B - 0.80b_f}{2}$$

3. Compute a dimension $^6 n'$ as follows:

   a. Define $L = d + b_f$.
   b. Define $X = \frac{4P_o}{L^2 F_b}$ with Bowles' approximations of $P = P_o; F_b = F_p$ (plate is heavily loaded if $X \approx 0.64$).
   c. Define $\lambda = \min\left[1, 0.5 \sqrt{\frac{2}{1 + \sqrt{1-x}}} \right]$. Note that $\lambda \leq 1$. If you have a negative square root, $\lambda$ is 1.
   d. Compute $n' = 0.25 \sqrt{db_f}$.

---

$^6$Here the author deviates from the AISC (1989) ninth edition manual and uses a modification proposed by one of the AISC committee members involved with the Manual [Thornton (1990)]. Except for using $\lambda$ and $n'$ the computations are exactly as in the AISC manual.
Figure 8-6  Base plate design according to the current AISC design specifications. Symbols are consistent with AISC (1989).

e. Extract the maximum \( v = \max[m, n, (\lambda n')] \)

f. Compute actual bearing stress \( f_p = P/(B \times C) \).

4. Compute the base plate thickness \( t_p \) as

\[
t_p = 2v \sqrt{\frac{f_p}{F_y}} \quad \text{(units of } v) \quad (8-11)
\]

Essentially the AISC specification requires sizing the base plate to satisfy the actual bearing pressure \( f_p \). Next the plate thickness is computed based on an allowable bending stress of 0.75\( F_y \) (\( F_y \) = yield stress of base plate steel) using a cantilever moment arm of \( v \) and a unit width strip of 1-m or (1-inch) equivalent. After computing plate thickness \( t_p \) select a final thickness that is available or round up to the next available plate thickness.

When there is a column moment in addition to the axial load, you must use a form of computations as

\[
f_p \leq \frac{P}{B \times C} + \frac{M_C}{I} \quad \text{(8-12)}
\]

This problem is not addressed directly by AISC so you must use engineering judgment. When there is a column moment, the base plate must be adequately attached to both the column and the foundation. Few steel columns transmit moments to isolated spread footings, but moments into mat foundations are fairly common.

Refer to Example 8-1 and Fig. 8-6 for the shear and moment locations for columns with base plates. It is suggested that the approximate equation for shear depth [Eq. (8-8)] be used for a footing supporting a base plate because of the approximation for locating the critical section.

The previous discussion will be illustrated by a design example.

Example 8-4. Design a reinforced concrete footing with a steel W250 \times 67 column (see Fig. E8-4) using the design data of Example 8-2.
General data: $D = 350$ kN  
$L = 450$ kN  
$q_a \approx 220$ kPa (used 200)  
$f'_c = 21$ MPa  
$F_y = 250$ MPa (for column)  
Rebar $F_y =$ Grade 400  
(400 MPa or 60 ksi)  

From rolled section tables [AISC (1992)] obtain for a $W250 \times 67$: 
$d = 257$ mm (depth)  
$b_f = 204$ mm (width)  
$t_w = 8.9$ mm (web)  
$t_f = 15.7$ mm (flange thickness)

**Solution.**

**Step 1.** Find footing area. Since loads and soil pressure are the same as in Example 8-2 we have $B = 2$ m.

**Step 2.** Since dimensions are same, use the depth $d = 350$ mm and the overall design of steel and $D_c = 440$ mm of Example 8-3.

**Step 3.** Thus, we need only to size the base plate.

a. Since the base plate is clearly smaller than the footing, it is evident that the ratio $\Psi = A_2/A_1 = 2$ and we have $A_1$ computed as

$$A_1 = \frac{P}{0.7f'_c} = \frac{800}{0.7 \times 21 \times 1000} = 0.0544 \text{ m}^2$$

The baseplate must fit the column footprint with about 12 mm overhang on all sides in case it is fillet-welded to the column. Thus, tentatively try the following:

$B = 204 + 25 = 230$ mm (rounded) and  
$C = 257 + 25 = 285$ mm (rounded to 5 mm)

These values yield

$$A_1 = 0.230(0.285) = 0.0655 \text{ m}^2 > 0.0544 \quad \text{O.K.}$$

Use $B = 230$ mm $\times C = 285$ mm.
b. Find dimensions \( m \) and \( n \):

\[
m = \frac{285 - 0.95(257)}{2} = 20.4 \text{ mm}
\]

\[
n = \frac{230 - 0.80(204)}{2} = 33.4 \text{ mm}
\]

To obtain \( \lambda n' \) we must do some side computations:

\[
F_p = 0.35 f'_c \Psi = 0.35(21)(2) = 14.7 \text{ MPa} = (0.7 f'_c)
\]

\[
L = (d + b_f) = 257 + 204 = 461 \text{ mm} = 0.461 \text{ m}
\]

\[
X \approx \frac{4P}{L^2 F_p} = \frac{4(800)}{0.461^2 \times 14.7 \times 1000} = 1.024 \text{ m}
\]

\[
\lambda = \min \left(1.0, \frac{2 \sqrt{X}}{1 + \sqrt{1 - X}}\right)
\]

Since \( X = 1.024 \) > 1 we have a negative root so use \( \lambda = 1.0 \)

\[
n' = 0.25 \sqrt{257 \times 204} = 57.24 \text{ mm} \rightarrow \lambda n' = 1(57.24) = 57.24 \text{ mm}
\]

\[
v = \max(20.4, 33.4, 57.24) = 57.24 \text{ mm}
\]

\[
t_p = 2v \sqrt{f_p/F_y} = 2(57.24) \sqrt{\frac{12.7}{250}} = 25.8 \text{ mm}
\]

Prior to the 8th ed. of the AISC manual, \( t_p = 2(33.4) \sqrt{f_p/F_y} = 15.1 \text{ mm} \). Use \( t_p = 22 \text{ mm} \) (\( \approx 1.5 \times 15.1 \), or next larger available plate thickness).

c. Complete the design by selecting anchor bolts. Since there is no moment we can probably use two anchor bolts of minimum dimension.

\[8.6.2 \text{ Interfacing Base Plate to Footing}\]

So far we have considered the idealized base plate. It still must be interfaced to the footing, the surface of which may be rough or at least rough enough that some base plate leveling is required. Base plate leveling can be accomplished in several ways. One way is to use shims (small, thin strips of tapered steel), which are driven between the plate and footing. Any space remaining is grouted (see Fig. 8-7). Grouting of base plates and machinery has received much attention; and ACI has a committee for this purpose, with the latest report being ACI 351 (1992).

It is not an easy task to grout this gap so that the base plate fully bears on grout—often there is uneven contact from grout shrinkage and trapped air. Holes may be drilled in the base plate to eliminate trapped air. Once grout exits the hole there is no underside cavity.

Another method to level base plates is to use thin metal plates on the order of 5–6 mm thick and slightly larger than the base plate with holes precut for the anchor bolts. These are stacked as required to bring the base to the correct elevation. Again it may be necessary to use a leveling course of grout beneath the first leveling plate for horizontal alignment.

In another method leveling nuts are used, requiring a minimum of four anchor bolts. Leveling is accomplished by putting a nut on each of the anchor bolts and installing the base plate. By adjusting the nuts vertically the base plate can be brought to level. The top nuts are then installed and tightened. The space between the base plate and footing is then grouted.
Figure 8-7  (a) Grout space to be filled when frame alignment is complete. Note that an attempt has already been made to grout space but subsequent realignment has created a new grout gap. (b) Base plate grouted using an enclosure to hold grout in position. Some excess grout can be seen around vertical bolt in foreground and between wood containment and base plate.
Anchor bolts are required to attach the base plate firmly to the footing or pedestal. Figure 8-8 displays several types of anchor bolts. A number of proprietary types (not shown) are available that work on similar principles but their advantages are mainly to provide additional vertical adjustments and thread protection during concrete placement. Most columns and tower-type structures as well as larger machinery use anchor bolts of the type shown in Fig. 8-8.

Anchor bolts are usually of A307 bolt material grade A (A-36 steel of $F_{\text{ult}} \approx 400$ MPa and $F_y = 250$ MPa) or grade B($F_{\text{ult}} \approx 690$ MPa). High-strength bolt material in A325 and A490 grades is usually not required since pullout/bond generally controls the design. Anchor bolts
**TABLE 8-4**

Ultimate tensile strength of selected A307 bolts in diameters most commonly used for base plate anchors.*

<table>
<thead>
<tr>
<th>Bolt diameter and pitch, mm</th>
<th>Net tensile stress area, $A_t$, mm$^2$</th>
<th>Tensile force† $T_u$, kN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Grade A</td>
<td>Grade B</td>
</tr>
<tr>
<td>16P2</td>
<td>157</td>
<td>63</td>
</tr>
<tr>
<td>20P2.5</td>
<td>245</td>
<td>98</td>
</tr>
<tr>
<td>24P3</td>
<td>353</td>
<td>141</td>
</tr>
<tr>
<td>30P3.5</td>
<td>561</td>
<td>224</td>
</tr>
<tr>
<td>36P4</td>
<td>817</td>
<td>327</td>
</tr>
<tr>
<td>42P4.5</td>
<td>1120</td>
<td>448</td>
</tr>
<tr>
<td>48P5</td>
<td>1470</td>
<td>588</td>
</tr>
<tr>
<td>56P5.5</td>
<td>2030</td>
<td>812</td>
</tr>
<tr>
<td>64P6</td>
<td>2680</td>
<td>1072</td>
</tr>
<tr>
<td>72P6</td>
<td>3460</td>
<td>1384</td>
</tr>
<tr>
<td>80P6</td>
<td>4340</td>
<td>1736</td>
</tr>
<tr>
<td>90P6</td>
<td>5590</td>
<td>2236</td>
</tr>
<tr>
<td>100P6</td>
<td>6990</td>
<td>2796</td>
</tr>
</tbody>
</table>

*From American National Standards Institute (ANSI) SR 17 (it is also ASTM STP 587, dated 1975).

Notes: 16P2 is a nominal bolt diameter of 16 mm with a thread pitch $P = 2$ mm (see inset sketch).

$$A_t = 0.7854(Diam. - 0.9382P)^2$$

Grade A = 400 MPa ($f_u$); $f_y = 250$ MPa

Grade B = 690 MPa; $f_y = 400$ MPa

For 16P2:

$$A_t = 0.7854(16 - 0.9382 \times 2)^2 = 157 \text{ mm}^2$$

$$T_u = \frac{400}{1000} \times 157 \text{ mm} = 63 \text{ kN}$$

†For design divide the ultimate tensile force $T_u$ to obtain $T_d = T_u/SF$. Use a SF of about 4.

in A307 material are available from $\frac{1}{2}$- to 4-in. diameter. Most structural applications will fall in the 25- to 100-mm bolt diameter range. Table 8-4 gives selected bolt properties for design use.

In practice the anchor bolt, with the nut(s) and washers attached to avoid loss and to protect the threads, is set in the wet concrete with a sufficient length of the threaded end above the concrete to adjust the baseplate elevation, provide a space to place a grout bed, and allow the nut to be fully effective. To do so, the distance must be large enough for the bolt to elongate while being tightened. Since stress always produces strain, if the anchor bolt were fixed at the top of the concrete and only had an elongation length of the base plate + nut, it might pull apart during the tightening operation.

What is usually done is to slip an oversized cardboard or metal sleeve over the anchor rod so the upper 75 to 90 mm of shaft is not bonded to the hardened concrete. During tightening

---

7 When this textbook went to print ASTM had not converted the A307 bolt standard to SI. It will be necessary to soft convert values as necessary.
this length, plus the thickness of the base plate, allows elongation so that the plate can be securely fastened. The sleeve will also allow the smaller-diameter anchor bolts to be bent to fit the predrilled holes in the base plate if there is slight misalignment.

If a sleeve is used, it may or may not be filled with grout after the base plate is attached and the anchor nut tightened. There are major differences of opinion on this:

1. Some think the sleeve should not be grouted so that stress reversals will produce strain changes over a length of bolt rather than locally.
2. Some think that after the bolt is tightened to a proof load (about 70 percent of yield) no strains of any magnitude are developed unless the moment is large enough to separate the base plate from the grout bed.

In any case, if the sleeve is grouted, the distance to develop subsequent strains is limited to roughly the thickness of the base plate. The question is of little importance where no stress reversals occur because the sleeve is used only for alignment in this case and the nut is usually made only snug-tight (about one-fourth turn from tight).

Anchor studs are available that are screwed into expanding sleeves that have been placed in predrilled holes in the footing to a depth of 75 to 300 mm. The studs may expand the sleeve against the concrete, or the sleeve may be driven down over a steel wedge to produce expansion, after which the anchor is screwed in place. Anchor studs can only be tightened a limited amount since the elongation distance is the base plate thickness. They are primarily used for anchoring equipment into permanent position.

Base plate anchor bolts are designed for any tension and/or shear forces that develop when overturning moments are present. Both bolt diameter and depth of embedment require analysis, although the latter is not specifically indicated in most (including ACI) building codes. Where a column has no moment a pair of anchor bolts is used, with the size being somewhat arbitrarily selected by the designer. Some additional information on anchor bolts may be found in Ueda et al. (1991, with references).

8-7 PEDESTALS

A pedestal is used to carry the loads from metal columns through the floor and soil to the footing when the footing is at some depth in the ground. The purpose is to avoid possible corrosion of the metal from the soil. Careful backfill over the footing and around the pedestal will be necessary to avoid subsidence and floor cracks. If the pedestal is very long, a carefully compacted backfill will provide sufficient lateral support to control buckling. The ACI (Art. 7.3 and 318.1) limits the ratio of unsupported length $L_u$ to least lateral dimension $h$ as

$$\frac{L_u}{h} \leq 3$$

for pedestals. The problem is to identify the unsupported length $L_u$ correctly when the member is embedded in the soil.

The code allows both reinforced and unreinforced pedestals. Generally the minimum percentage of steel for columns of $0.01A_{col}$ of Art. 10.98 should be used even when the pedestal

---

8The ACI Code specifies gross column area—that is, no area reduction for column reinforcing. The symbol often used is $A_g$, but this text uses $A_{col}$. 