Terzaghi equation. As an exercise let us also use the Terzaghi equation:

\[ N_c = 25.1 \quad N_q = 12.7 \quad N_\gamma = 9.7 \quad \text{(from Table 4-2 at } \phi = 25^\circ) \]

Also, \( s_c = 1.3 \quad s_\gamma = 0.8 \quad \text{(square base).} \)

\[ q_{\text{ult}} = cN_cs_c + \overline{q}N_q + \frac{1}{2} \gamma BN_\gamma s_\gamma \]
\[ = (25)(25.1)(1.3) + 0.3(17.5)(12.7) + \frac{1}{2}(17.5)(2.0)(9.7)(0.8) \]
\[ = 815.8 + 66.7 + 135.8 = 1018.3 \rightarrow 1018 \text{ kPa} \]

\[ q_a = \frac{q_{\text{ult}}}{3} = \frac{1018}{3} = 339 \rightarrow 340 \text{ kPa} \]

Summary. We can summarize the results of the various methods as follows:

<table>
<thead>
<tr>
<th>Method</th>
<th>Ultimate Capacity (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hansen</td>
<td>170</td>
</tr>
<tr>
<td>Vesić</td>
<td>225</td>
</tr>
<tr>
<td>Meyerhof</td>
<td>130</td>
</tr>
<tr>
<td>Terzaghi</td>
<td>340</td>
</tr>
</tbody>
</table>

The question is, what to use for \( q_a \)? The Hansen-Vesić-Meyerhof average seems most promising and is \( q_{a,av} = (170 + 225 + 130)/3 = 175 \text{ kPa} \). The author would probably recommend using \( q_a = 175 \text{ kPa} \). This is between the Hansen and Vesić values; Meyerhof’s equations tend to be conservative and in many cases may be overly so. Here the Terzaghi and Meyerhof equations are not appropriate, because they were developed for horizontal bases vertically loaded. It is useful to make the Terzaghi computation so that a comparison can be made, particularly since the computations are not difficult.

4-7 EFFECT OF WATER TABLE ON BEARING CAPACITY

The effective unit weight of the soil is used in the bearing-capacity equations for computing the ultimate capacity. This has already been defined for \( \overline{q} \) in the \( \overline{q}N_q \) term. A careful inspection of Fig. 4-3 indicates that the wedge term \( 0.5\gamma BN_\gamma \) also uses the effective unit weight for the soil.

The water table is seldom above the base of the footing, as this would, at the very least, cause construction problems. If it is, however, the \( \overline{q} \) term requires adjusting so that the surcharge pressure is an effective value. This computation is a simple one involving computing the pressure at the GWT using that depth and the wet unit weight + pressure from the GWT to the footing base using that depth \( \times \) effective unit weight \( \gamma' \). If the water table is at the ground surface, the effective pressure is approximately one-half that with the water table at or below the footing level, since the effective unit weight \( \gamma' \) is approximately one-half the saturated unit weight.

When the water table is below the wedge zone [depth approximately \( 0.5B \tan(45 + \phi/2) \)], the water table effects can be ignored for computing the bearing capacity. When the water table lies within the wedge zone, some small difficulty may be obtained in computing the

\[ ^4 \text{A major reason the Terzaghi equation is widely used (and often misused) is that it is much easier to calculate than the other equations.} \]
effective unit weight to use in the \(0.5\gamma BN_y\) term. In many cases this term can be ignored for a conservative solution since we saw in Example 4-1 that its contribution is not substantial (see also following Example). In any case, if \(B\) is known, one can compute the average effective weight \(\gamma_e\) of the soil in the wedge zone as

\[
\gamma_e = (2H - d_w) \frac{d_w}{H^2} \gamma_{\text{wet}} + \frac{\gamma'}{H^2} (H - d_w)^2
\]

(4-4)

where \(H = 0.5B \tan(45° + \phi/2)\)
\(d_w = \text{depth to water table below base of footing}\)
\(\gamma_{\text{wet}} = \text{wet unit weight of soil in depth } d_w\)
\(\gamma' = \text{submerged unit weight below water table} = \gamma_{\text{sat}} - \gamma_w\)

Example 4-8. A square footing that is vertically and concentrically loaded is to be placed on a cohesionless soil as shown in Fig. E4-8. The soil and other data are as shown.

![Figure E4-8](image)

**Required.** What is the allowable bearing capacity using the Hansen equation of Table 4-1 and a SF = 2.0?

**Solution.** We should note that \(B\) would, in general, not be known but would depend on the column load and the allowable soil pressure. We could, however, compute several values of \(q_a\) and make a plot of \(q_a\) versus \(B\). Here we will compute a single value of \(q_a\).

**Step 1.** Since the effective soil unit weight is required, let us find these values. Estimate that the "wet" soil has \(w_N = 10\) percent and \(G_8 = 2.68\).

\[
\gamma_{\text{dry}} = \frac{\gamma_{\text{wet}}}{1 + w} = \frac{18.10}{1 + 0.10} = 16.45 \text{ kN/m}^3
\]

\[
V_s = \frac{\gamma_{\text{dry}}}{G_8(9.807)} = \frac{16.45}{2.68(9.807)} = 0.626 \text{ m}^3
\]

\[
V_v = 1.0 - V_s = 1.0 - 0.626 = 0.374 \text{ m}^3
\]

The saturated unit weight is the dry weight + weight of water in voids, or

\[
\gamma_{\text{sat}} = 16.45 + 0.374(9.807) = 20.12 \text{ kN/m}^3
\]

From Fig. E4-8 we obtain \(d_w = 0.85\) m and \(H = 0.5B \tan(45° + \phi/2) = 2.40\) m. Substituting into Eq. (4-4), we have

\[
\gamma_e = (2 \times 2.4 - 0.85) \frac{0.85 \times 18.10}{2.4^2} + \frac{20.12 - 9.807}{2.4^2}(2.40 - 0.85)^2
\]

\[
= 14.85 \text{ kN/m}^3
\]
Step 2. Obtain bearing-capacity factors for the Hansen equation using Tables 4-1 and 4-4. Do not compute $\phi_{pa}$, since footing is square. For $\phi = 35^\circ$ use program BEARING on your diskette and obtain

\[ N_q = 33 \quad N_r = 34 \quad 2\tan \phi \cdot \cdot \cdot = 0.255 \quad \text{(also in Table 4-4)} \]

\[ s_q = 1 + \frac{B'}{L'} \sin \phi = 1.57 \quad s_r = 1 - 0.4 \frac{B'}{L'} = 0.6 \]

\[ d_q = 1 + 2 \tan \cdot \cdot \cdot \frac{D}{B} \]

\[ d_r = 1 + 0.255 \cdot \cdot \cdot \frac{1.1}{2.5} = 1.11 \quad d_r = 1.10 \]

From Table 4-1 and dropping any terms that are not used or are 1.0, we have

\[ q_{ult} = \gamma DN_q s_q d_q + 0.5 \gamma_r B' N_r s_r d_r \]

Substituting values (note $\gamma = \text{soil above base}$), we see

\[ q_{ult} = 1.1(18.10)(33)(1.57)(1.11) + 0.5(14.86)(2.5)(34)(0.6)(1.0) \]
\[ = 1145 + 379 = 1524 \text{ kPa} \]

\[ q_a = \frac{1524}{2} = 762 \text{ kPa} \quad \text{(a very large bearing pressure)} \]

It is unlikely that this large a bearing pressure would be allowed—a possible maximum is 500 kPa (about 10 ksf). We might simply neglect the $\gamma_r B N_r$ term to obtain $q_a = 570 \text{ kPa}$ (still large). If the latter term is neglected, the computations are considerably simplified; and doing so has little effect on what would normally be recommended as $q_a$ (around 500 kPa in most cases).

### 4-8 BEARING CAPACITY FOR FOOTINGS ON LAYERED SOILS

It may be necessary to place footings on stratified deposits where the thickness of the top stratum from the base of the footing $d_1$ is less than the $H$ distance computed as in Fig. 4-2. In this case the rupture zone will extend into the lower layer(s) depending on their thickness and require some modification of $q_{ult}$. There are three general cases of the footing on a layered soil as follows:

**Case 1.** Footing on layered clays (all $\phi = 0$) as in Fig. 4-5a.

- a. Top layer weaker than lower layer ($c_1 < c_2$)
- b. Top layer stronger than lower layer ($c_1 > c_2$)

**Case 2.** Footing on layered $\phi-c$ soils with $a, b$ same as case 1.

**Case 3.** Footing on layered sand and clay soils as in Fig. 4-5b.

- a. Sand overlying clay
- b. Clay overlying sand

Experimental work to establish methods to obtain $q_{ult}$ for these three cases seems to be based mostly on models—often with $B < 75 \text{ mm}$. Several analytical methods exist as well, and apparently the first was that of Button (1953), who used a circular arc to search for an approximate minimum, which was found (for the trial circles all in the top layer) to give $N_c = 5.5 < 2\pi$ as was noted in Sec. 4-2.
The use of trial circular arcs can be readily programmed for a computer (see program B-1 on diskette) for two or three layers using $s_u$ for the layers. Note that in most cases the layer $s_u$ will be determined from $q_u$ tests, so the circle method will give reasonably reliable results. It is suggested that circular arcs be limited to cases where the strength ratio $C_R = c_2/c_1$ of the top two layers is on the order of

$$0.6 < C_R \leq 1.3$$

Where $C_R$ is much out of this range there is a large difference in the shear strengths of the two layers, and one might obtain $N_c$ using a method given by Brown and Meyerhof (1969) based on model tests as follows:

For $C_R \leq 1$

$$N_{c,s} = \frac{1.5d_1}{B} + 5.14C_R \leq 5.14 \quad \text{(for strip footing)} \quad (4-5)$$

For a circular base with $B = \text{diameter}$

$$N_{c,r} = \frac{3.0d_1}{B} + 6.05C_R \leq 6.05 \quad \text{(for round base)} \quad (4-6)$$

---

(a) Footing on layered clay soil. For very soft $c_1$ failure may occur along sliding block 1abc and not a circular arc and reduce $N_c$ to a value less than 5.14.
When \( C_R > 0.7 \) reduce the foregoing \( N_{c,i} \) by 10 percent.

For \( C_R > 1 \) compute:

\[
N_{1,s} = 4.14 + \frac{0.5B}{d_1} \quad \text{stripe) } \tag{4-7}
\]

\[
N_{2,s} = 4.14 + \frac{1.1B}{d_1} \tag{4-7a}
\]

\[
N_{1,r} = 5.05 + \frac{0.33B}{d_1} \quad \text{(round base) } \tag{4-8}
\]

\[
N_{2,r} = 5.05 + \frac{0.66B}{d_1} \tag{4-8a}
\]

In the case of \( C_R > 1 \) we compute both \( N_{1,i} \) and \( N_{2,i} \) depending on whether the base is rectangular or round and then compute an averaged value of \( N_{c,i} \) as

\[
N_{c,i} = \frac{N_{1,i} \cdot N_{2,i}}{N_{1,i} + N_{2,i}} \cdot 2 \tag{4-9}
\]

The preceding equations give the following typical values of \( N_{c,i} \), which are used in the bearing-capacity equations of Table 4-1 for \( N_c \).

<table>
<thead>
<tr>
<th>( d_1/B )</th>
<th>Strip</th>
<th>Round</th>
<th>( C_R = 0.4 )</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>2.50</td>
<td>3.32</td>
<td>( N_{1,s} )</td>
<td>( N_{2,s} )</td>
</tr>
<tr>
<td>0.7</td>
<td>3.10</td>
<td>4.52</td>
<td>4.85</td>
<td>5.71</td>
</tr>
<tr>
<td>1.0</td>
<td>3.55</td>
<td>5.42</td>
<td>4.64</td>
<td>5.24</td>
</tr>
</tbody>
</table>

When the top layer is very soft with a small \( d_1/B \) ratio, one should give consideration either to placing the footing deeper onto the stiff clay or to using some kind of soil improvement method. Model tests indicate that when the top layer is very soft it tends to squeeze out from beneath the base and when it is stiff it tends to “punch” into the lower softer layer [Meyerhof and Brown (1967)]. This result suggests that one should check this case using the procedure of Sec. 4-2 that gave the “lower-bound” solution—that is, if \( q_{ult} > 4c_1 + \bar{q} \) of Eq. (c) the soil may squeeze from beneath the footing.

Purushothamaraj et al. (1974) claim a solution for a two-layer system with \( \phi-c \) soils and give a number of charts for \( N_c \) factors; however, their values do not differ significantly from \( N_c \) in Table 4-4. From this observation it is suggested for \( \phi-c \) soils to obtain modified \( \phi \) and \( c \) values as follows:

1. Compute the depth \( H = 0.5B \tan(45 + \phi/2) \) using \( \phi \) for the top layer.
2. If \( H > d_1 \) compute the modified value of \( \phi \) for use as

\[
\phi' = \frac{d_1 \phi_1 + (H - d_1) \phi_2}{H} \tag{5}
\]

5This procedure can be extended to any number of layers as necessary, and “weighting” may be used.
3. Make a similar computation to obtain \( c' \).
4. Use the bearing-capacity equation (your choice) from Table 4-1 for \( q_{ult} \) with \( \phi' \) and \( c' \).

If the top layer is soft (low \( c \) and small \( \phi \)) you should check for any squeezing using Eq. (c) of Sec. 4-2.

For bases on sand overlying clay or clay overlying sand, first check if the distance \( H \) will penetrate into the lower stratum. If \( H > d_1 \) (refer to Fig. 4-5) you might estimate \( q_{ult} \) as follows:

1. Find \( q_{ult} \) based on top-stratum soil parameters using an equation from Table 4-1.
2. Assume a punching failure bounded by the base perimeter of dimensions \( B \times L \). Here include the \( \bar{q} \) contribution from \( d_1 \), and compute \( q'_{ult} \) of the lower stratum using the base dimension \( B \). You may increase \( q'_{ult} \) by a fraction \( k \) of the shear resistance on the punch perimeter \( (2B + 2L) \times kq_s \) if desired.
3. Compare \( q_{ult} \) to \( q'_{ult} \) and use the smaller value.

In equation form the preceding steps give the controlling \( q'_{ult} \) as

\[
q'_{ult} = q''_{ult} + \frac{pp_vK_s \tan \phi}{A_f} + \frac{pd_1c}{A_f} \leq q_{ult} \tag{4-10}
\]

where
- \( q_{ult} = \) bearing capacity of top layer from equations in Table 4-1
- \( q''_{ult} = \) bearing capacity of lower layer computed as for \( q_{ult} \) but also using \( B = \) footing dimension, \( \bar{q} = \gamma d_1; \phi \) of lower layer
- \( p = \) total perimeter for punching [may use \( 2(B + L) \) or \( \pi \times \) diameter]
- \( p_v = \) total vertical pressure from footing base to lower soil computed as \( \int_0^{d_1} \gamma h \, dh + \bar{q}d_1 \)
- \( K_s = \) lateral earth pressure coefficient, which may range from \( \tan^2(45 \pm \phi/2) \) or use \( K_o \) from Eq. (2-18a)
- \( \tan \phi = \) coefficient of friction between \( p_vK_s \) and perimeter shear zone wall
- \( pd_1c = \) cohesion on perimeter as a force
- \( A_f = \) area of footing (converts perimeter shear forces to a stress)

This equation is similar to that of Valsangkar and Meyerhof (1979) and applies to all soils.

Note that there will not be many cases of a two- (or three-) layer cohesive soil with clearly delineated strata. Usually the clay gradually transitions from a hard, overconsolidated surface layer to a softer one; however, exceptions may be found, primarily in glacial deposits. In these cases it is a common practice to treat the situation as a single layer with a worst-case \( s_u \) value. A layer of sand overlying clay or a layer of clay overlying sand is somewhat more common, and the stratification is usually better defined than for the two-layer clay.

A possible alternative for \( \phi-c \) soils with a number of thin layers is to use average values of \( c \) and \( \phi \) in the bearing-capacity equations of Table 4-1 obtained as

\[
c_{av} = \frac{c_1H_1 + c_2H_2 + c_3H_3 + \cdots + c_nH_n}{\sum H_i} \tag{a}
\]
\[ \phi_{av} = \tan^{-1} \frac{H_1 \tan \phi_1 + H_2 \tan \phi_2 + \cdots + H_n \tan \phi_n}{\sum H_i} \quad (b) \]

where \( c_i \) = cohesion in stratum of thickness \( H_i \); \( c \) may be 0
\( \phi_i \) = angle of internal friction in stratum of thickness \( H_i \); \( \phi \) may be zero

Any \( H_i \) may be multiplied by a weighting factor (1.0 is used in these equations) if desired. The effective shear depth of interest is limited to approximately \( 0.5B \tan(45^\circ + \phi/2) \). One or two iterations may be required to obtain the best average \( \phi-c \) values, since \( B \) is not usually fixed until the bearing capacity is established.

One can use a slope-stability program such as that written by Bowles (1974a) to obtain the bearing capacity for layered soils. The program given in that reference has been modified to allow the footing pressure as a surcharge (program B-22). An increase in shear strength with depth could be approximated by addition of “soils” with the same \( \phi \) and \( \gamma \) properties but increased cohesion strength. The ultimate bearing capacity is that value of \( q_o \) producing \( F = 1 \).

**Example 4-9.** A footing of \( B = 3 \times L = 6 \) m is to be placed on a two-layer clay deposit as in Fig. E4-9.

\[ H = 0.5B \tan \left( 45^\circ + \frac{\phi}{2} \right) \]
\[ = 0.5(3) \tan 45 = 1.5 \text{ m} \]

\[ C_R = \frac{c_2}{c_1} = \frac{115}{77} = 1.5 > 1.0 \]

\[ \frac{d_1}{B} = \frac{1.22}{3} = 0.4 \]

Using Eqs. (4-7), (4-6α), and (4-8), we obtain (similar to table)

\[ N_{1,s} = 5.39 \quad N_{2,s} = 6.89 \]

\[ N_c = 6.05 \text{ (some larger than 5.14 that would be used for a one-layer soil)} \]
Substituting values into Hansen's equation, we obtain

\[ t_{\text{ult}} = cN_c(1 + s'_q + d'_q) + qNs_qd_q \]

\[ = 77(6.05)(1 + 0.1 + 0.24) + 1.83(17.26)(1)(1) \]

\[ = 624.2 + 31.5 = 655.7 \text{ kPa} \]

Squeezing is not likely as \( d_1 \) is fairly large compared to \( H \) and we are not using an \( N_c \) value much larger than that for a one-layer soil.

Example 4-10. You are given the soil footing geometry shown in Fig. E4-10. Note that, with the GWT on clay, it would be preferable to keep the footing in sand if possible.

![Figure E4-10](image)

**Required.** What is ultimate bearing capacity and \( q_a \) if \( SF = 2 \) for sand and \( 3 \) for clay?

**Solution.** We will use Hansen's method. For the sand layer, we have

\[ N_q = 29.4 \quad N_\gamma = 28.7 \quad (\text{using Table 4-4}) \]

\[ s_q = 1 + \tan 34^\circ = 1.67 \quad s_\gamma = 0.6 \]

\[ d_q = 1 + 0.262 \left( \frac{1.5}{2} \right) = 1.2 \quad d_\gamma = 1 \]

Substituting into Hansen's equation and rounding the \( N \) factors (and using \( s_u = c = q_u/2 = 75 \text{ kPa} \)), we may write

\[ q_{\text{ult}} = 1.5(17.25)(29)(1.67)(1.2) + 0.5(17.25)(2)(29)(0.6)(1) \]

\[ = 1804 \rightarrow 1800 \text{ kPa} \]
For clay, we have

\[ N_c = 5.14 \quad \text{(using Table 4-4)} \]

\[ s'_c = 0.2 \left( \frac{B}{L} \right) = 0.2 \left( \frac{2}{2} \right) = 0.2 \quad s_q = d_q = 1 \]

\[ d'_c = 0.4 \tan^{-1} \frac{D}{B} = 0.4 \tan^{-1} \left( \frac{2.1}{2} \right) = 0.32 \quad \left( \frac{D}{B} > 1 \right) \]

\[ q''_{ult} = 5.14(75)(1 + 0.2 + 0.32) + 2.1(17.25)(1)(1) \]

\[ = 622 \text{ kPa} \]

Note: This \( s_u \) is common for the strength parameter for clay.

Now obtain the punching contribution. For the perimeter shear force on a strip 1 m wide, we write

\[ P_v = qd_1 + \int_0^{d_1} \gamma h dh \quad \text{(kN/m)} \]

\[ P_v = 1.5(17.25)(0.6) + 17.25 \frac{h^2}{2} \int_0^{0.6} \]

\[ = 15.5 + 3.1 = 18.6 \text{ kN/m} \]

Estimate \( K_s = K_d = 1 - \sin \phi \) [from Eq. (2-18a)] = 1 - \sin 34° = 0.44. By inserting values into Eq. (4-10), the revised maximum footing pressure based on the clay soil and including punching is

\[ q'_{ult} = q''_{ult} + \frac{pP_vK_s \tan 34^\circ}{A_f} + \frac{pd_1c}{A_f} \]

But cohesion is zero in sand and the perimeter is \( 2(2+2) = 8 \text{ m}, \) so

\[ q'_{ult} = 622 + \frac{8(18.6)(0.44) \tan 34^\circ}{2 \times 2} = 633 \text{ kPa} < q_{ult} \text{ of } 1800 \]

The maximum footing pressure is controlled by the clay layer, giving \( q_{ult} = 634 \text{ kPa}. \) The allowable footing contact soil pressure is

\[ q = \frac{633}{3} = 211 \quad \text{(say, 200 kPa)} \]

FOOTINGS ON ANISOTROPIC SOIL. This situation primarily occurs in cohesive soils where the undrained vertical shear strength \( s_{u,v} \) is different (usually larger) from the horizontal shear strength \( s_{u,h}. \) This is a frequent occurrence in cohesive field deposits but also is found in cohesionless deposits. To account for this situation \( (\phi = 0), \) Davis and Christian (1971) suggest the following:

When you measure both vertical and horizontal shear strength \( (c = s_u), \) compute the bearing capacity as

\[ q_{ult} = 0.9N_c \cdot \frac{s_{u,v} + s_{u,h}}{2} + \bar{q}' \]
When you only have $s_{u,v}$, compute the bearing capacity as

$$q_{ult} = 0.85 s_{u,v} N_c + q'$$

In these two equations take $N_c = 5.14$ (Hansen’s value). You may include Hansen’s $s_i$, $d_i$, and other factors at your own discretion, but they were not included by Davis and Christian (1971).

### 4-9 BEARING CAPACITY OF FOOTINGS ON SLOPES

A special problem that may be encountered occasionally is that of a footing located on or adjacent to a slope (Fig. 4-6). From the figures it can be seen that the lack of soil on the slope side of the footing will tend to reduce the stability of the footing.

The author developed Table 4-7 using program B-2 on your diskette to solve the footing on or adjacent to a slope as follows:

1. Develop the exit point $E$ for a footing as shown in Fig. 4-6. The angle of the exit is taken as $45^\circ - \phi/2$ since the slope line is a principal plane.
2. Compute a reduced $N_c$ based on the failure surface $ade = L_0$ of Fig. 4-3 and the failure surface $adE = L_1$ of Fig. 4-6a to obtain

$$N'_c = N_c \frac{L_1}{L_0}$$

3. Compute a reduced $N_q$ based on the ratio of area $ecfg$ (call it $A_0$) of Fig. 4-3 to the equivalent area $Efg = A_\phi$ of Fig. 4-6a, or the alternative $Efgh = A_\phi$ of Fig. 4-6b, to obtain the following:

$$N'_q = N_q \frac{A_1}{A_0}$$

**Figure 4-6** Footings on or adjacent to a slope.
Note that when the distance $b$ of Fig. 4-6b is such that $A_1 \geq A_0$ we have $N'_q = N_q$. This distance appears to be about $b/B > 1.5$ (or possibly 2).

4. The overall slope stability should be checked for the effect of the footing load using your favorite slope-stability program or program B-22. At least a few trial circles should touch point $c$ of Fig. 4-6a,b as well as other trial entrance points on top of and on the slope.

The ultimate bearing capacity may be computed by any of the equations of Table 4-1; however, the author suggests using the Hansen equation modified to read as follows:

$$q_{ult} = cN'_c s_c i_c + \bar{q}N'_q s_q i_q + \frac{1}{2} \gamma B N'_r s_y i_y$$

Obtain the $N'_c$ and $N'_q$ factors from Table 4-7 [or use the included computer program B-2 if interpolation is not desired]. The $d_i$ factors are not included in the foregoing equation since the depth effect is included in the computations of ratios of areas. It will be conservative to use shape factors $s_c = s_q = 1$ (but compute $s_y$).

The $N'_r$ factor probably should be adjusted to $N'_r$ to account for the reduction in passive pressure on the slope side of the wedge $caf$ of Fig. 4-6 when the base is either within the $b/B < 2$ zone on top of the slope or when $b/B = 0$. Saran et al. (1989) proposed an analytical solution to account for this reduction; however, the results do not seem adequately conservative and additionally there are too many algebraic manipulations for there to be great confidence in the end result. A simpler solution that compares reasonably well with test results (on models) is as follows:

1. Assume no reduction of $N'_r$ for $b/B \geq 2$ of Fig. 4-6b. Use computer program B-2 for $D/B$ and $b/B < 2$, for interpolation is not very accurate, especially for larger $\phi$ angles.

2. Use the Hansen $N'_r$ factor and adjust as follows:
   
a. Compute the Coulomb passive pressure coefficients for the slope angle $\beta$ using $\beta = (-)$ for one computation and $(+)$ for the other. See Chap. 11 (and use program FFACTOR on furnished diskette). Use the friction angle $\delta = \phi$ for both computations. When you use $\beta = (+ or 0)$ you are computing the passive pressure coefficient $K_p = K_{max}$ on the base side away from the slope and when $\beta = (-)$ you are computing $K_p = K_{min}$.
   
b. Now using $K_{max}$ and $K_{min}$ compute an $R$ ratio as
   $$R = K_{min}/K_{max}$$
   
c. Obtain the Hansen value of $N'_r$ from Table 4-4 (or compute it). Now divide by 2 (allow for a contribution of $\frac{1}{2}$ from either side of the wedge $caf$ of Fig. 4-6a or b). The side away from the wedge will contribute the full $\frac{1}{2}$ of $N'_r$, but the contribution from the slope side will be a fraction depending on the foregoing $R$ ratio and the distance $b/B$.
   
d. Now set up the following:
   $$N'_r = \frac{N'_r}{2} + \frac{N'_r}{2} \left[ R + \frac{b}{2B} (1 - R) \right]$$

This equation is easily checked:

At $b/2B = 0$: $N'_r = N_r/2 + N_rR/2$ (on slope)

At $b/2B = 2$: $N'_r = 2N_r/2 = N_r$ (top of slope and out of slope influence)
### TABLE 4-7

**Bearing capacity** \( N'_c, N'_q \) **for footings on or adjacent to a slope**

Refer to Fig. 4-4 for variable identification. Base values \( (\beta = 0) \) may be used when length or area ratios > 1 or when \( b/B > 1.5 \) to 2.0 (approximate). Values given should cover usual range of footing locations and depths of embedment.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \phi = 0 )</th>
<th>( 10 )</th>
<th>( 20 )</th>
<th>( 30 )</th>
<th>( 40 )</th>
<th>( D/B = 0 )</th>
<th>( b/B = 0 )</th>
<th>( D/B = 0.75 )</th>
<th>( b/B = 0 )</th>
<th>( D/B = 1.50 )</th>
<th>( b/B = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>( N'_c = )</td>
<td>5.14</td>
<td>8.35</td>
<td>14.83</td>
<td>30.14</td>
<td>75.31</td>
<td>5.14</td>
<td>8.35</td>
<td>14.83</td>
<td>30.14</td>
<td>75.31</td>
</tr>
<tr>
<td></td>
<td>( N'_q = )</td>
<td>1.03</td>
<td>2.47</td>
<td>6.40</td>
<td>18.40</td>
<td>64.20</td>
<td>1.03</td>
<td>2.47</td>
<td>6.40</td>
<td>18.40</td>
<td>64.20</td>
</tr>
<tr>
<td>10°</td>
<td></td>
<td>4.89</td>
<td>7.80</td>
<td>13.37</td>
<td>26.80</td>
<td>64.42</td>
<td>5.14</td>
<td>8.35</td>
<td>14.83</td>
<td>30.14</td>
<td>75.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.03</td>
<td>2.47</td>
<td>6.40</td>
<td>18.40</td>
<td>64.20</td>
<td>0.92</td>
<td>1.95</td>
<td>4.43</td>
<td>11.16</td>
<td>33.94</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
</tr>
<tr>
<td>20°</td>
<td></td>
<td>4.63</td>
<td>7.28</td>
<td>12.39</td>
<td>23.78</td>
<td>55.01</td>
<td>5.14</td>
<td>8.35</td>
<td>14.83</td>
<td>30.14</td>
<td>66.81</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.03</td>
<td>2.47</td>
<td>6.40</td>
<td>18.40</td>
<td>64.20</td>
<td>0.92</td>
<td>1.90</td>
<td>4.11</td>
<td>9.84</td>
<td>28.21</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
</tr>
<tr>
<td>25°</td>
<td></td>
<td>4.51</td>
<td>7.02</td>
<td>11.82</td>
<td>22.38</td>
<td>50.80</td>
<td>5.14</td>
<td>8.35</td>
<td>14.83</td>
<td>28.76</td>
<td>62.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.03</td>
<td>2.47</td>
<td>6.40</td>
<td>18.40</td>
<td>64.20</td>
<td>0.92</td>
<td>1.82</td>
<td>3.85</td>
<td>9.00</td>
<td>25.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
</tr>
<tr>
<td>30°</td>
<td></td>
<td>4.38</td>
<td>6.77</td>
<td>11.28</td>
<td>21.05</td>
<td>46.88</td>
<td>5.14</td>
<td>8.35</td>
<td>14.83</td>
<td>27.14</td>
<td>57.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.03</td>
<td>2.47</td>
<td>6.40</td>
<td>18.40</td>
<td>64.20</td>
<td>0.88</td>
<td>1.71</td>
<td>3.54</td>
<td>8.08</td>
<td>21.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
</tr>
<tr>
<td>60°</td>
<td></td>
<td>3.62</td>
<td>5.33</td>
<td>8.33</td>
<td>14.34</td>
<td>28.56</td>
<td>4.70</td>
<td>6.83</td>
<td>10.55</td>
<td>17.85</td>
<td>34.84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.03</td>
<td>2.47</td>
<td>6.40</td>
<td>18.40</td>
<td>64.20</td>
<td>0.37</td>
<td>0.63</td>
<td>1.17</td>
<td>2.36</td>
<td>5.52</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.62</td>
</tr>
<tr>
<td>$\beta$</td>
<td>(D/B = 0)</td>
<td>(b/B = 0.75)</td>
<td>(D/B = 0.75)</td>
<td>(b/B = 0.75)</td>
<td>(D/B = 1.50)</td>
<td>(b/B = 0.75)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10°</td>
<td>5.14</td>
<td>8.33</td>
<td>14.34</td>
<td>28.02</td>
<td>66.60</td>
<td>1.03</td>
<td>2.47</td>
<td>6.40</td>
<td>18.40</td>
<td>64.20</td>
<td>5.14</td>
</tr>
<tr>
<td>20°</td>
<td>5.14</td>
<td>8.31</td>
<td>13.90</td>
<td>26.19</td>
<td>59.31</td>
<td>1.03</td>
<td>2.47</td>
<td>6.40</td>
<td>18.40</td>
<td>64.20</td>
<td>5.14</td>
</tr>
<tr>
<td>25°</td>
<td>5.14</td>
<td>8.29</td>
<td>13.69</td>
<td>25.36</td>
<td>56.11</td>
<td>1.03</td>
<td>2.47</td>
<td>6.40</td>
<td>18.40</td>
<td>64.20</td>
<td>5.14</td>
</tr>
<tr>
<td>30°</td>
<td>5.14</td>
<td>8.27</td>
<td>13.49</td>
<td>24.57</td>
<td>53.16</td>
<td>1.03</td>
<td>2.47</td>
<td>6.40</td>
<td>18.40</td>
<td>64.20</td>
<td>5.14</td>
</tr>
<tr>
<td>60°</td>
<td>5.14</td>
<td>7.94</td>
<td>12.17</td>
<td>20.43</td>
<td>39.44</td>
<td>1.03</td>
<td>2.47</td>
<td>6.40</td>
<td>18.40</td>
<td>64.20</td>
<td>5.14</td>
</tr>
<tr>
<td>(D/B = 0)</td>
<td>(b/B = 1.50)</td>
<td>(D/B = 0.75)</td>
<td>(b/B = 1.50)</td>
<td>(D/B = 1.50)</td>
<td>(b/B = 1.50)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10°</td>
<td>5.14</td>
<td>8.35</td>
<td>14.83</td>
<td>29.24</td>
<td>68.78</td>
<td>5.14</td>
<td>8.35</td>
<td>14.83</td>
<td>30.14</td>
<td>75.31</td>
<td>1.03</td>
</tr>
<tr>
<td>20°</td>
<td>5.14</td>
<td>8.35</td>
<td>14.83</td>
<td>28.59</td>
<td>63.60</td>
<td>5.14</td>
<td>8.35</td>
<td>14.83</td>
<td>30.14</td>
<td>75.31</td>
<td>1.03</td>
</tr>
<tr>
<td>30°</td>
<td>5.14</td>
<td>8.35</td>
<td>14.83</td>
<td>28.09</td>
<td>59.44</td>
<td>5.14</td>
<td>8.35</td>
<td>14.83</td>
<td>30.14</td>
<td>70.32</td>
<td>1.03</td>
</tr>
<tr>
<td>60°</td>
<td>5.14</td>
<td>8.35</td>
<td>14.83</td>
<td>26.52</td>
<td>50.32</td>
<td>5.14</td>
<td>8.35</td>
<td>14.83</td>
<td>30.03</td>
<td>66.60</td>
<td>1.03</td>
</tr>
</tbody>
</table>
One should not adjust $\phi_r$ to $\phi_{ps}$, as there are considerable uncertainties in the stress state when there is loss of soil support on one side of the base, even for strip (or long) bases. The use of these factors and method will be illustrated in Example 4-11, which is based on (and compared with) load tests from the cited source.

**Example 4-11.**

**Given.** Data from a strip footing load test for a base located on the top of a slope [from Shields et al. (1977)]. Other data are as follows:

- Slope $\beta = 26.5^\circ$ (1 on 2) and “compact” sand
- $\phi_r = 36^\circ$ (estimated from the author’s interpretation of the reference figure of $\phi$ vs. $\sigma_3$ (the confining pressure)
- $c = 0$ (no cohesion)
- $\gamma = 14.85$ kN/m$^3$ (effective value and not very dense)

Consider two test cases:

- **Case I:** $b/B = 0.75$ \hspace{1em} $D/B = 1.50$
- **Case II:** $b/B = 1.50$ \hspace{1em} $D/B = 0.0$

**Required.** Compare the author’s suggested method with Shield’s test curves. Also for Case II compare the author’s method with Hansen’s method using the ground factor $g_1$.

**Solution.**

**Case I:**

(a) By Shields’ method.

\[
q_{ult} = cN_c + \frac{1}{2} \gamma BN_{\gamma q}
\]

From curves, obtain

\[
N_{\gamma q} \approx 120 \quad \text{[Fig. 11 of Shields et al. (1977)]}
\]

and

\[
q_{ult} = \frac{1}{2}(14.85)B(120) = 891B
\]

(b) By Table 4-7 and using Hansen’s $N'_{\gamma}$. We will not adjust $\phi_r$ to $\phi_{ps}$ for reasons stated earlier in this section. For a strip base all $s_i = 1.0$. Also here, since $H_i = 0$, all $i_i = 1.0$; because the base is horizontal, $b_i = 1$; and we take $g_i = 1$ since this method already accounts for the slope angle $\beta$.

From side computations of Chap. 11 (using program FFACTOR) obtain the Coulomb earth pressure coefficients (using $\phi = 36^\circ$, $\delta = 36^\circ$, vertical wall, $\alpha = 90^\circ$) as

\[
K_{max} = 128.2 \quad (\beta = 26.5^\circ) \quad K_{min} = 2.8 \quad (\beta = -26.5^\circ)
\]

\[
R = K_{min}/K_{max} = 2.8/128.2 = 0.022 \quad 1.000 - R = 0.978
\]

$N_{\gamma} = 40.0$ and (refer to step d given just before this Example)

\[
N'_{\gamma} = \frac{40}{2} + \frac{40}{2} \left[ 0.022 + \frac{b}{2B}(0.978) \right]
\]

\[
= 20 + 20 \left[ 0.022 + \frac{0.75}{2}(0.978) \right] = 20 + 20(0.388)
\]

\[
= 27.8 \rightarrow 28 \quad \text{(and is less than 40 as expected)}
\]
At $b/B = 1.5$ (which we will use for Case II), we compute

$$N'_y = 20 + 20(0.756) = 35 \quad \text{(rounded)}$$

**For Case I** Bowles' method gives

$$q_{ult} = \bar{q}N'_q + \frac{1}{2} \gamma BN'_y$$

Also $N'_q = 27$ \quad [rounded and using program B-2 (or Table 4-7)]

$$q_{ult} = 14.85(1.5B)(27) + \frac{1}{2}(14.85)(B)(28)$$

$$= 601B + 207B = 808B < 891B \text{ kPa}$$

This result compares reasonably well to (within 10 percent) the 8915 actually measured.

**Case II** Let $D/B = 0.0$ (base on surface; $\bar{q} = 0$) and $b/B = 1.5$ from edge of slope. From Shields et al. (1977) we obtain approximately

$$q_{ult} = 14.85(1.5B)(27) = 260B \text{ kPa}$$

By Bowles' method and noting $N'_q = 27$ as before and $N'_y = 35$, we obtain

$$q_{ult} = 14.85(1.5B)(27) + \frac{1}{2}(14.85)(B)(35) = 259.9B \rightarrow 260B \text{ kPa}$$

By Hansen's method only the $\frac{1}{2} \gamma B N_y g_y$ term applies (since $c = \bar{q} = 0$), so

$$g_y = (1 - 0.5 \tan \beta)^5 = (1 - 0.5 \tan 26.5^\circ)^5 = 0.238$$

Directly substituting, we find

$$q_{ult} = 0 + 0 + \frac{1}{2}(14.85)(B)(35)(1)(0.238) = 61.8 \text{ kPa}$$

Inspection of the Vesic computation for $g_y$ gives $g_y = 0.251 > 0.238$.

These computations indicate that Bowles' method appears to give the best solution based on the limited load-test data available. Both the Hansen and Vesic methods appear too conservative but were all that was available at the time they were proposed. Keep in mind that most real slopes exist in soils with both $c$ and $\phi$ and not just sand, as in the model test used here for confirmation of methodology. In any case the use of a sand model has severely tested the several methods.

---

### 4-10 BEARING CAPACITY FROM SPT

The SPT is widely used to obtain the bearing capacity of soils directly. One of the earliest published relationships was that of Terzaghi and Peck (1967). This has been widely used, but an accumulation of field observations has shown these curves to be overly conservative. Meyerhof (1956, 1974) published equations for computing the allowable bearing capacity for a 25-mm settlement. These could be used to produce curves similar to those of Terzaghi and Peck and thus were also very conservative. Considering the accumulation of field observations and the stated opinions of the authors and others, this author adjusted the Meyerhof equations for an approximate 50 percent increase in allowable bearing capacity to obtain the following:

$$q_a = \frac{N}{F_1}K_d \quad B \leq F_4$$  \hspace{1cm} (4-11)

$$q_a = \frac{N}{F_2}\left(\frac{B + F_3}{B}\right)^2K_d \quad B > F_4$$  \hspace{1cm} (4-12)
where \( q_a \) = allowable bearing pressure for \( \Delta H_o = 25 \)-mm or 1-in. settlement, kPa or ksf

\[
K_d = 1 + 0.33 \frac{D}{B} \leq 1.33 \text{ [as suggested by Meyerhof (1965)]}
\]

\( F \) factors as follows:

<table>
<thead>
<tr>
<th></th>
<th>( N_{55} )</th>
<th>( N_{70} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SI</td>
<td>Fps</td>
<td>SI</td>
</tr>
<tr>
<td>( F_1 )</td>
<td>0.05</td>
<td>2.5</td>
</tr>
<tr>
<td>( F_2 )</td>
<td>0.08</td>
<td>4.0</td>
</tr>
<tr>
<td>( F_3 )</td>
<td>0.30</td>
<td>1.0</td>
</tr>
<tr>
<td>( F_4 )</td>
<td>1.20</td>
<td>4.0</td>
</tr>
</tbody>
</table>

These equations have been in existence for quite some time and are based primarily on \( N \) values from the early 1960s back and, thus, \( E_r \) is likely on the order of 50 to 55 and not 70+ as suggested in Sec. 3-7. Since lower \( E_r \) produces higher blow counts \( N \) if the preceding equations are standardized to \( N'_{70} \), we must use revised values for factors \( F_1 \) and \( F_2 \) as shown in the table of \( F \) factors. Summarizing, use the left values under \( N_{55} \) and the given \( F \) factors, or standardize \( N \) to \( N'_{70} \) and use the right columns of \( F \) factors in Eqs. (4-11), (4-12), and (4-13). Figure 4-7 is a plot of Eqs. (4-11) and (4-12) based on \( \approx N_{55} \).

In these equations \( N \) is the statistical average value for the footing influence zone of about \( 0.5B \) above footing base to at least \( 2B \) below. If there are consistently low values of \( N \) below this zone, settlements may be troublesome if \( N \) is not reduced somewhat to reflect this event. Figure E4-12 is a method of presenting \( q_a \) versus \( N \) for design office use.

We note in these equations that footing width is a significant parameter. Obviously if the depth of influence is on the order of \( 2B \) a larger footing width will affect the soil to a greater depth and strains integrated over a greater depth will produce a larger settlement. This is taken into account somewhat for mats, which were considered also by Meyerhof (and adjusted by the author for a 50 percent increase) to obtain

\[
q_a = \frac{N}{F_2} K_d \quad (4-13)
\]

In these equations the allowable soil pressure is for an assumed 25-mm settlement. In general the allowable pressure for any settlement \( \Delta H_j \) is

\[
q'_a = \frac{\Delta H_j}{\Delta H_o} q_a \quad (4-14)
\]

where \( \Delta H_o = 25 \text{ mm for SI and 1 in. for Fps.} \Delta H_j \) is the actual settlement that can be tolerated, in millimeters or inches. On a large series of spread footings on sand D’Appolonia et al. (1968) found that use of the Meyerhof equations (4-11) and (4-12) when \( N_{55} \) was corrected using \( C_N \) of Eq. (3-3) predicted settlements very well. The sand involved, however, was either overconsolidated or compacted to a very dense state. This soil state should have produced somewhat higher blow counts (or \( N \)-values) than for a less dense state.

Parry (1977) proposed computing the allowable bearing capacity of cohesionless soils as

\[
q_a = 30N_{55} \quad (\text{kPa}) \quad (D \leq B) \quad (4-15)
\]
Example.

Use chart to find $q_a$

1. $N_{70} = 24$
2. Footing depth $D = 1$ m
3. Footing width $B = 3$ m

Solution. $F_3 = 0.3$

- $F_2 = 0.08$
- $N_{55} = 24 \times 70/55 \approx 30 > 24$

At ground surface:

(Refer to chart)

- $q_a = \frac{30}{0.08} \left(3 + 0.3\right)^2 \approx 450$ kPa

At $D = 1$ m:

- $K_d = 1 + 0.33(1/3) = 1.11$
- $q_a = 450 \times K_d = 450 \times 1.11 \approx 500$ kPa

Figure 4-7  Allowable bearing capacity for surface-loaded footings with settlement limited to approximately 25 mm. Equation used is shown on figure.

where $N_{55}$ is the average SPT value at a depth about $0.75B$ below the proposed base of the footing. The allowable bearing pressure $q_a$ is computed for settlement checking as

$$q_a = \frac{N_{55}}{15B} \text{ (kPa)} \quad \text{(for a } \Delta H_o = 20 \text{ mm)}$$  \hspace{1cm} (4-15a)

Use a linear ratio ($\Delta H/20$) to obtain $q_a$ for settlements $\Delta H \neq 20$ mm ($B$ is in meters, $q_a$ in kPa). Use the smaller of the computed values from Eqs. (4-15) and (4-15a) for design.

Equation (4.15) was based on back-computing $N_q$ and $N_\gamma$ using an angle of internal friction $\phi$ based on $N_{55}$ as

$$\phi = 25 + 28 \left(\frac{N_{55}}{\bar{q}}\right)^{1/2}$$  \hspace{1cm} (4-16)

Here $\bar{q}$ is the effective overburden pressure at the location of the average $N_{55}$ count. The footing depth $D$ must be such that there is an overburden ($\bar{q}N_q$) term.
Example 4-12

**Given.** The average \( N_{b0} \) blow count = 6 in the effective zone for a footing located at \( D = 1.6 \) m (blow count average in range from 1- to 4-m depth).

**Required.** What is the allowable bearing capacity for a 40-mm settlement? Present data as a curve of \( q_a \) versus \( B \).

**Solution.** From Table 3-4 we can see \( D_r \) is small, soil is “loose,” and settlement may be a problem. Should one put a footing on loose sand or should it be densified first?

Program Eqs. (4-12)-(4-14) with \( F_2 = 0.06 \) and \( F_3 = 0.30 \) (including \( K_d \)) on a programmable calculator or personal computer and obtain the following table, which is plotted as Fig. E4-12. Note \( q_a' = q_a(40/25) \).

<table>
<thead>
<tr>
<th>( B, \text{m} )</th>
<th>( q_a', \text{kPa} )</th>
<th>( q_a, \text{kPa} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 &gt; 1.2</td>
<td>192</td>
<td>310</td>
</tr>
<tr>
<td>2</td>
<td>167</td>
<td>267</td>
</tr>
<tr>
<td>3</td>
<td>142</td>
<td>228</td>
</tr>
<tr>
<td>4</td>
<td>131</td>
<td>209</td>
</tr>
<tr>
<td>5</td>
<td>124</td>
<td>199</td>
</tr>
<tr>
<td>6</td>
<td>120</td>
<td>192</td>
</tr>
<tr>
<td>10</td>
<td>112</td>
<td>179</td>
</tr>
</tbody>
</table>

For final design round \( q_a \) to multiples of 25 kPa.

### Figure E4-12

4-11 BEARING CAPACITY USING THE CONE PENETRATION TEST (CPT)

The bearing capacity factors for use in the Terzaghi bearing-capacity equation of Table 4-1 can be estimated [see Schmertmann (1978)] as

\[
0.8N_q \approx 0.8N_r = q_c
\]

where \( q_c \) is averaged over the depth interval from about \( B/2 \) above to \( 1.1B \) below the footing base. This approximation should be applicable for \( D/B \leq 1.5 \). For cohesionless soils one may use

\[
\text{Strip} \quad q_{ult} = 28 - 0.0052(300 - q_c)^{1.5} \quad (\text{kg/cm}^2) \quad (4-18)
\]

\[
\text{Square} \quad q_{ult} = 48 - 0.009(300 - q_c)^{1.5} \quad (\text{kg/cm}^2) \quad (4-18a)
\]

For clay one may use

\[
\text{Strip} \quad q_{ult} = 2 + 0.28q_c \quad (\text{kg/cm}^2) \quad (4-19)
\]

\[
\text{Square} \quad q_{ult} = 5 + 0.34q_c \quad (\text{kg/cm}^2) \quad (4-19a)
\]
Equations (4-18) through (4-19a) are based on charts given by Schmertmann (1978) credited to an unpublished reference by Awakti.

According to Meyerhof (1956) the allowable bearing capacity of sand can be computed using Eqs. (4-11) and (4-12), making a substitution for \( q_c \) as

\[
N_{55} = \frac{q_c}{4}
\]  

(4-20)

and with \( q_c \) in units of kg/cm\(^2\). If \( q_c \) is in units other than kg/cm\(^2\)(= tsf) you must convert to these units prior to using Eq. (4-20). Note also that making the foregoing conversion of \( q_c \) to \( N_{55} \) to use Eqs. (4-11) and (4-12) adjusts the original Meyerhof recommendations to a 50 percent increase of the allowable bearing capacity as similarly done for directly obtained SPT \( N \) values.

It is evident that one can use the CPT correlations of Sec. 3-11 to obtain \( f \) or \( s_u \) so that the bearing capacity equations of Table 4-1 can be used more directly.

4-12 BEARING CAPACITY FROM FIELD LOAD TESTS

Obviously the most reliable method of obtaining the ultimate bearing capacity at a site is to perform a load test. This would directly give the bearing capacity if the load test is on a full-size footing; however, this is not usually done since an enormous load would have to be applied. Such a load could be developed from two piles driven into the ground with a very large girder spanning between them so a hydraulic jack could be placed on the footing to jack against the girder for the footing load. This is very costly as one consideration; another factor is that the bearing capacity obtained is for that size only and if there is more than one size then additional tests would be required. For the test just described the cost could be very high.

4-12.1 Standard Method

The usual practice is to load-test small steel plates (although one could also pour small concrete footings, which would be troublesome to remove if that location were needed for other purposes) of diameters from 0.3 to 0.75 m or squares of side 0.3 \( \times \) 0.3 and perhaps 0.6 \( \times \) 0.6 m. These sizes are usually too small to extrapolate to full-size footings, which may be 1.5 to 4 or 5 m\(^2\). Several factors cause the extrapolation to be questionable:

1. The significant influence depth of approximately 4\( B \) is substantially different for the model-versus-prototype footing. Any stratification below the \( H \) depth of Fig. 4-2 or Fig. 4-3b has minimal effect on the model but may be a major influence on the full-size footing.
2. The soil at greater depths has more overburden pressure acting to confine the soil so it is effectively "stiffer" than the near-surface soils. This markedly affects the load-settlement response used to define \( q_{ult} \).
3. Previous discussion has noted that as \( B \) increases there is a tendency to a nonlinear increase in \( q_{ult} \). It develops that for small models of say, 0.3, 0.45, and 0.6 m, the plot of \( B \) versus \( q_{ult} \) is nearly linear (as it is for using two sizes of, say, 2 m and 2.5 m). It takes a larger range of sizes to develop the nonlinear curve for that soil deposit.
In spite of these major shortcomings, load tests are occasionally used. The procedure has been standardized as ASTM D 1194, which is essentially as follows:

1. Decide on the type of load application. If it is to be a reaction against piles, they should be driven or installed first to avoid excessive vibration and loosening of the soil in the excavation where the load test will be performed.

2. Excavate a pit to the depth the test is to be performed. The test pit should be at least four times as wide as the plate and to the depth the foundation is to be placed. If it is specified that three sizes of plates are to be used for the test, the pit should be large enough so that there is an available spacing between tests of 3D of the largest plate.

3. A load is placed on the plate, and settlements are recorded from a dial gauge accurate to 0.25 mm. Observations on a load increment should be taken until the rate of settlement is beyond the capacity of the dial gauge. Load increments should be approximately one-fifth of the estimated bearing capacity of the soil. Time intervals of loading should not be less than 1 h and should be approximately of the same duration for all the load increments.

4. The test should continue until a total settlement of 25 mm is obtained, or until the capacity of the testing apparatus is reached. After the load is released, the elastic rebound of the soil should be recorded for a period of time at least equal to the time duration of a load increment.

Figure 4-8 presents the essential features of the load test. Figure 4-9a is a typical semilog plot of time versus settlement (as for the consolidation test) so that when the slope is approximately horizontal the maximum settlement for that load can be obtained as a point on the load-versus-settlement curve of Fig. 4-9b. Where the load-versus-settlement approaches the vertical, one interpolates $q_{ult}$. Sometimes, however, $q_{ult}$ is obtained as that value corresponding to a specified displacement (as, say, 25 mm).

Extrapolating load-test results to full-size footings is not standard. For clay soils it is common to note that the $BN_r$ term is zero, so that one might say that $q_{ult}$ is independent of footing...
size, giving

\[ q_{\text{ult}} = q_{\text{ult, foundation}} = q_{\text{ult, load test}} \]

In cohesionless (and \( \phi-c \)) soils all three terms of the bearing-capacity equation apply and, noting that the \( N_y \) term includes the footing width, one might say

\[ q_{\text{ult, foundation}} = M + N \frac{B_{\text{foundation}}}{B_{\text{load test}}} \]

where \( M \) includes the \( N_c \) and \( N_q \) terms and \( N \) is the \( N_y \) term. By using several sizes of plates this equation can be solved graphically for \( q_{\text{ult}} \). Practically, for extrapolating plate-load tests for sands (which are often in a configuration so that the \( N_q \) term is negligible), use the following

\[ q_{\text{ult}} = q_{\text{plate}} \left( \frac{B_{\text{foundation}}}{B_{\text{plate}}} \right) \]  \hspace{1cm} (4-21)

The use of this equation is not recommended unless the \( B_{\text{foundation}}/B_{\text{plate}} \) is not much more than about 3. When the ratio is 6 to 15 or more the extrapolation from a plate-load test is little more than a guess that could be obtained at least as reliably using an SPT or CPT correlation.

### 4-12.2 Housel’s Method for Bearing Capacity from Plate-Load Tests

Housel (1929) and Williams (1929) both\(^6\) gave an equation for using at least two plate-load tests to obtain an allowable load \( P_s \) for some settlement as

\[ P_s = Aq_1 + pq_2 \quad \text{(kPa or ksf)} \]  \hspace{1cm} (4-22)

---

\(^6\)Housel is generally given credit for this equation; however, when Williams presented it no credit was given, so the equation may have been proposed simultaneously by both persons.
where \( A \) = area of plate used for the load test, m\(^2\) or ft\(^2\)

\( p \) = perimeter of load-test plate, m or ft

\( q_1 \) = bearing pressure of interior zone of plate, kPa or ksf

\( q_2 \) = edge shear of plate, kN/m or k/ft

Equation (4-22) is used as follows:

1. Perform two or more load tests using plates with different \( A \) and \( p \). Plot curves of either load \( P \) or bearing pressure \( q \) versus settlement \( \Delta H \).

2. At the desired settlement obtain from these plots the load \( P_s = q \cdot A \) if the plot is pressure \( q \) versus settlement \( \Delta H \). One possible set of values is at \( \frac{1}{2} P_{ult} \); however, values at plate settlements of 6, 10, or 15 mm might also be used.

3. Using \( P_s \), plate area, and perimeter solve Eq. (4-22) for \( q_1, q_2 \). For more than two tests make as many solutions as possible and average the results for \( q_1, q_2 \).

For example, consider these data:

<table>
<thead>
<tr>
<th>Test #</th>
<th>( B ), m</th>
<th>( A ), m(^2)</th>
<th>( p ), m</th>
<th>( P_s ), kN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.45</td>
<td>( 0.45^2 = 0.2025 )</td>
<td>( 4 \times 0.45 = 1.8 )</td>
<td>30.4</td>
</tr>
<tr>
<td>2</td>
<td>0.60</td>
<td>( 0.60^2 = 0.3600 )</td>
<td>( 4 \times 0.60 = 2.4 )</td>
<td>45.1</td>
</tr>
</tbody>
</table>

which give

\[
0.2025q_1 + 1.8q_2 = 30.4 \\
0.3600q_1 + 2.4q_2 = 45.1
\]

On solving, we obtain \( q_1 = 50.83 \) kPa and \( q_2 = 11.17 \) kN/m.

It is now necessary to solve by trial to find footing dimensions for a given design load. The following illustrates the approach.

The allowable \( P_a \) for a base that is 3\( \times \)3 m is

\[
P_a = (3 \times 3)50.83 + (4 \times 3)11.17 = 591.5 \text{ kN}
\]

If the design load \( P_d = 591.5 \) kN, use this trial \( B \). If the design load is less, use a smaller \( B \) and make another trial computation, etc., until the computed footing load has converged within reasonable limits. Remember that your selection of the \( P_s \) values has approximately set the settlement for that base of dimension \( B \).

This method is generally called Housel's method. It was widely used until the early 1950s even though Terzaghi (1929) did not approve of it and did not even mention it in Terzaghi (1943), where the Terzaghi bearing-capacity method was first introduced.

### 4-13 BEARING CAPACITY OF FOUNDATIONS WITH UPLIFT OR TENSION FORCES

Footings in industrial applications—such as for the legs of elevated water tanks, anchorages for the anchor cables of transmission towers, and bases for legs of power transmission...
Footings for tension loads.

Footings to develop tension resistance are idealized in Fig. 4-10. Balla (1961) considered this problem. He assumed a failure surface (the dashed line \( ab \) in Fig. 4-10) as circular and developed some highly complicated mathematical expressions that were verified on model tests in a small glass jar and by some larger tests of others. The only footings he considered were circular. Meyerhof and Adams (1968) also considered the problem and proposed the conditions of Fig. 4-10, namely, that footings should be considered as either shallow or deep since deep footings could develop only to some limiting pull-out force. Circular and rectangular footings were considered and in both cohesive and cohesionless soils. They compared the theory (following equations) with models as well as full-scale tests on circular footings and found considerable scatter; however, with a factor of safety of 2 to 2.5 these equations should be satisfactory.

The following equations are developed by neglecting the larger pull-out zone observed in the tests (as \( ab \) of Fig. 4-10) and using an approximation of shear resistance along line \( ab' \). Shape factors are used together with a limiting depth ratio \( D/B \) or \( H/B \) to make the simplified equations adequate for design use. In the general case we have for the ultimate tension

\[
T_u = \text{Perimeter resistance}, \ s_u p D + \text{Base weight } W
\]

with adjustments for depth and shape (whether perimeter is round or rectangular). This equation gives (only for footings in sands) the following:

**For shallow footings**

Round: \[
T_u = \pi B s_u D + s_f \pi B \gamma \left( \frac{D^2}{2} \right) K_u \tan \phi + W
\]

Rectangular: \[
T_u = 2 s_u D (B + L) + \gamma D^2 (2 s_f B + L - B) K_u \tan \phi + W
\]

where the side friction adjustment factor \( s_f = 1 + m D/B \).
For deep footings (base depth $D > H$)

Round: \[ T_u = \pi s_u BH + s_f \pi B \gamma (2D - H) \left( \frac{H}{2} \right) K_u \tan \phi + W \] (4-25)

Rectangular: \[ T_u = 2s_u H (B + L) + \gamma (2D - H) (2s_f B + L - B) H K_u \tan \phi + W \] (4-26)

where $s_f = 1 + mH/B$.

For footing shape

Round: \[ B = \text{diameter} \]

Square: \[ L = B \]

Rectangular: use $B$ and $L$

Obtain shape factor $s_f$, ratios $m$ and $H/B$ [all $f(\phi)$] from the following table—interpolate as necessary:

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>20°</th>
<th>25°</th>
<th>30°</th>
<th>35°</th>
<th>40°</th>
<th>45°</th>
<th>48°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limiting $H/B$</td>
<td>2.5</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td>$m$</td>
<td>0.05</td>
<td>0.10</td>
<td>0.15</td>
<td>0.25</td>
<td>0.35</td>
<td>0.50</td>
<td>0.60</td>
</tr>
<tr>
<td>Maximum $s_f$</td>
<td>1.12</td>
<td>1.30</td>
<td>1.60</td>
<td>2.25</td>
<td>4.45</td>
<td>5.50</td>
<td>7.60</td>
</tr>
</tbody>
</table>

For example: $\phi = 20^\circ$ so obtain $s_f = 1.12$, $m = 0.05$, and $H/B = 2.5$. Therefore, $H = 2.5B$, and total footing depth to be a "deep" footing $D > 2.5B$. If $B = 1$ m, $D$ of Fig. 4-10 must be greater than 2.5 m, or else use "shallow footing" equations [Eqs. (4-23) or (4-24)].

The lateral earth pressure coefficient $K_u$ can be taken as one of the following:

\[ K_u = \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) = K_p \]
\[ K_u = \tan \left( 45^\circ + \frac{\phi}{2} \right) = \sqrt{K_p} \]
\[ K_u = \tan^2 \left( 45^\circ - \frac{\phi}{2} \right) = K_a \]
\[ K_u = \tan \left( 45^\circ - \frac{\phi}{2} \right) = K_a \]
\[ K_u = 0.65 + 0.5\phi \quad (\phi \text{ in radians}) \]

With these several choices the user must make a judgment analysis for $K_u$. Using $K_o$ or an average of $K_p$, $K_o$, and $K_a$ may be reasonable.

The equations for a rectangular footing in tension are based on an assumption made by Meyerhof that the shape factor is acting on the end parts in a zone of $B/2$ along $L$ and the interior part of $(L - B)$ is similar to a long strip footing with $s_f = 1$. Most tension footings, however, are round (common) or square.

For footings founded in very poor soils, Robinson and Taylor (1969) found that a satisfactory design resistance for transmission tower anchorages could be obtained by using only the weight term $W$ in Eqs. (4-23) through (4-26) and with a safety factor slightly greater than one. Compute the footing weight $W$ based on the volume of the footing concrete plus the weight of any soil that will be uplifted when the base is pulled up. If the footing is a poured concrete shaft (with or without an enlarged base) in clay, use about 80 percent of the shaft length to
compute a perimeter area. The perimeter is based on $\pi B'$, where $B'$ is either the diameter of the shaft or that of the base if it is larger or belled. This perimeter area is used with adhesion defined as $k \cdot s_u$ between shaft perimeter zone and foundation soil. The use of $0.8D$ allows for soil damage or tension cracks in the upper zone of the embedment depth. The tension force is now computed as

$$T_u = W + \pi B'(0.8D)k \cdot s_u$$

In general, one reduces the ultimate tension resistance to the design value $T_a$ as

$$T_a = \frac{T_u}{SF}$$

where the safety factor may range from, say, 1.2 to 4 or 5 depending on the importance of the footing, reliability of the soil parameters, and the likelihood that quality backfill over the footing will produce a reliable $W$ term and a reasonably adequate shear zone along line $ab'$. 

**Example 4-13.** A footing $1.2 \times 1.2 \times 0.6$ m is placed at a depth of 1.80 m in a soil of $\gamma = 17.29$ kN/m$^3$; $\phi = 20^\circ$; $s_u = 20$ kPa.

**Required.** Estimate the allowable uplift force for a SF = 2.5.

**Solution.** $D/B = 1.8/1.2 = 1.5 < H/B = 2.5$ for $\phi = 20^\circ$; therefore, the footing is classed as shallow and we will use Eq. (4-24) to calculate $T_u$.

$$T_u = 2s_uD(B + L) + \gamma D^2(2s_fB + L - B)K_u \tan \phi + W$$

$$s_f = 1 + \frac{mD}{B} = 1 + 0.05(1.5) = 1.075 < 1.12$$ in table preceding this example

Several values of $K_u$ are as follows:

$$K_u = \tan^2\left(45^\circ + \frac{20^\circ}{2}\right) = 2.04 = K_p$$

$$K_u = \sqrt{K_p} = 1.43$$

$$K_u = 0.65 + 0.5\phi = 0.82$$

$$K_u = K_o = 1 - \sin 20^\circ = 0.658$$

Average $K_u = (2.04 + 1.43 + 0.82 + 0.66)/4 = 1.24$

$W$ = Weight of concrete + Weight of soil uplifted

$$W = 1.2(1.2)(0.6)(23.6) + 1.2(1.2)(1.8 - 0.6)(17.29) = 50.3 \text{ kN}$$

Substituting values into Eq. (4-23), we find

$$T_u = 2(20)(1.8)(1.2 + 1.2) + 17.29(1.8)^2[2(1.075)(1.2) + 1.2 - 1.2](1.24)\tan 20^\circ$$

$$+ 50.3$$

$$= 172.8 + 65.2 + 50.3 = 288.3 \text{ kN}$$

$$T_a = \frac{288.3}{2.5} = 115 \text{ kN}$$

The structural design of this anchor footing would be on the basis of $T_a \times$ some SF (or load factor).
4-14 BEARING CAPACITY BASED ON BUILDING CODES (PRESUMPTIVE PRESSURE)

In many cities the local building code stipulates values of allowable soil pressure to use when designing foundations. These values are usually based on years of experience, although in some cases they are simply used from the building code of another city. Values such as these are also found in engineering and building-construction handbooks. These arbitrary values of soil pressure are often termed presumptive pressures. Most building codes now stipulate that other soil pressures may be acceptable if laboratory testing and engineering considerations can justify the use of alternative values. Presumptive pressures are based on a visual soil classification.

Table 4-8 indicates representative values of building code pressures. These values are primarily for illustrative purposes, since it is generally conceded that in all but minor construction projects some soil exploration should be undertaken. Major drawbacks to the use of presumptive soil pressures are that they do not reflect the depth of footing, size of footing, location of water table, or potential settlements.

**TABLE 4-8**

Presumptive bearing capacities from indicated building codes, kPa

Soil descriptions vary widely between codes. The following represents author’s interpretations.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay, very soft</td>
<td>25</td>
<td></td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Clay, soft</td>
<td>75</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Clay, ordinary</td>
<td>125</td>
<td></td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Clay, medium stiff</td>
<td>175</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clay, stiff</td>
<td>210</td>
<td></td>
<td>140</td>
<td></td>
</tr>
<tr>
<td>Clay, hard</td>
<td>300</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sand, compact and clean</td>
<td>240</td>
<td></td>
<td>140</td>
<td>200</td>
</tr>
<tr>
<td>Sand, compact and silty</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inorganic silt, compact</td>
<td>125</td>
<td></td>
<td>140</td>
<td>210</td>
</tr>
<tr>
<td>Sand, loose and fine</td>
<td>140</td>
<td></td>
<td>240</td>
<td>300</td>
</tr>
<tr>
<td>Sand, loose and coarse, or sand-gravel mixture, or compact and fine</td>
<td>400</td>
<td>240</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>Gravel, loose and compact coarse sand</td>
<td>300</td>
<td></td>
<td>240</td>
<td>300</td>
</tr>
<tr>
<td>Sand-gravel, compact</td>
<td></td>
<td></td>
<td>240</td>
<td>300</td>
</tr>
<tr>
<td>Hardpan, cemented sand, cemented gravel</td>
<td></td>
<td></td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>Soft rock</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sedimentary layered rock (hard shale, sandstone, siltstone)</td>
<td>600</td>
<td>950</td>
<td>340</td>
<td></td>
</tr>
<tr>
<td>Bedrock</td>
<td>9600</td>
<td>9600</td>
<td>6000</td>
<td>1400</td>
</tr>
</tbody>
</table>

*Note: Values converted from psf to kPa and rounded.
*Building Officials and Code Administrators International, Inc.
†Author interpretation.
Buildings are designed on the basis of determining the service loads and obtaining a suitable ratio of material strength to these loads, termed either a safety or a load factor. None of the quantities in this factor is precisely known, so that codes or experience are relied upon to develop the ratio as, one hopes, a lower-bound value—the real value is this or something larger. Engineering materials such as steel and concrete are manufactured with strict quality control; nevertheless, in strength design for concrete the effective ultimate strength is taken as 85 percent of the unconfined compressive strength. The yield stress for steel and other metals is a lower-bound value—in the case of steel on the order of 10 to 20 percent less than the general range of measured yield strengths. Thus, a “safety factor” of sorts is already applied.

Code values used to develop live and other loads are a compromise between upper and near-upper bound. Building self-weight, or dead load, is reasonably identified (at least after the structure is designed). Either the service loads are multiplied by a suitable set of load factors and compared with the “ultimate strength,” or the structural material or the yield strength is divided by a suitable safety or load factor and compared with the loads. We note in passing that in concrete strength design the load factors for dead and live loads represent in a limited way the different degrees of uncertainty associated with each type of loading.

There are more uncertainties in determining the allowable strength of the soil than in the superstructure elements. A number of these uncertainties can be deduced from discussions in Chaps. 2 and 3. These may be summarized as follows:

- Complexity of soil behavior
- Lack of control over environmental changes after construction
- Incomplete knowledge of subsurface conditions
- Inability to develop a good mathematical model for the foundation
- Inability to determine the soil parameters accurately

These uncertainties and resulting approximations have to be evaluated for each site and a suitable safety factor directly (or indirectly) assigned that is not overly conservative but that takes into account at least the following:

1. Magnitude of damages (loss of life, property damage, and lawsuits) if a failure results
2. Relative cost of increasing or decreasing SF
3. Relative change in probability of failure by changing SF
4. Reliability of soil data
5. Changes in soil properties from construction operations, and later from any other causes
6. Accuracy of currently used design/analysis methods

It is customary to use overall safety factors on the order of those shown in Table 4-9. Shear should be interpreted as bearing capacity for footings. Although the SF values in Table 4-9

---

<sup>7</sup>At this writing (1995), the terms usually used are load factor for designing the superstructure elements and safety factor for estimating the allowable soil pressure.
Values of stability numbers (or safety factors) usually used

<table>
<thead>
<tr>
<th>Failure mode</th>
<th>Foundation type</th>
<th>SF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear</td>
<td>Earthworks</td>
<td>1.2–1.6</td>
</tr>
<tr>
<td></td>
<td>Dams, fills, etc.</td>
<td></td>
</tr>
<tr>
<td>Shear</td>
<td>Retaining structure</td>
<td>1.5–2.0</td>
</tr>
<tr>
<td></td>
<td>Walls</td>
<td></td>
</tr>
<tr>
<td>Shear</td>
<td>Sheeptiling cofferdams</td>
<td>1.2–1.6</td>
</tr>
<tr>
<td></td>
<td>Temporary braced excavations</td>
<td>1.2–1.5</td>
</tr>
<tr>
<td>Shear</td>
<td>Footings</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Spread</td>
<td>2–3</td>
</tr>
<tr>
<td></td>
<td>Mat</td>
<td>1.7–2.5</td>
</tr>
<tr>
<td></td>
<td>Uplift</td>
<td>1.7–2.5</td>
</tr>
<tr>
<td>Seepage</td>
<td>Uplift, heaving</td>
<td>1.5–2.5</td>
</tr>
<tr>
<td></td>
<td>Piping</td>
<td>3–5</td>
</tr>
</tbody>
</table>

do not appear larger than for, say, steel design, the uncertainties in developing the allowable shear stress (in most cases) produce larger real safety factors than shown. For example, as shown in Example 4-4 using $q_a = q_u$, the apparent SF $\approx 3^+$. But $q_u$ is obtained from very disturbed samples, so that the value may only be 50 to 60 percent of the in situ value resulting in the true SF being much larger. Further, where settlement controls, the allowable bearing capacity will be further reduced—which in turn further increases the real safety factor.

Some persons [Hansen (1967), Meyerhof (1970)] advocate consideration of partial safety factors for the soil parameters, e.g., using a value of, say, 1.2 to 1.3 on $\phi$ and 1.5 to 2.5 on cohesion. The latter are larger, since cohesion is somewhat more state-dependent.

The design load is obtained from the most critical of several possible cases. Using the load-term abbreviations of Table 4-10 and code-load factors $R_t$, the following might be investigated:

Design load = $R_D \text{DL} + R_L \text{LL} + R_S S + HS \quad (SF = 3.0)$

<table>
<thead>
<tr>
<th>Load</th>
<th>Includes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead load (DL)</td>
<td>Weight of structure and all permanently attached material</td>
</tr>
<tr>
<td>Live load (LL)</td>
<td>Any load not permanently attached to the structure, but to which the structure may be subjected</td>
</tr>
<tr>
<td>Snow load (S)</td>
<td>Acts on roofs; value to be used generally stipulated by codes</td>
</tr>
<tr>
<td>Wind load (W)</td>
<td>Acts on exposed parts of structure</td>
</tr>
<tr>
<td>Earthquake (E)</td>
<td>A lateral force (usually) that acts on the structure</td>
</tr>
<tr>
<td>Hydrostatic (HS)</td>
<td>Any loads due to water pressure; may be either (+) or (−)</td>
</tr>
<tr>
<td>Earth pressure (EP)</td>
<td>Any loads due to earth pressures—commonly lateral but may be in other directions</td>
</tr>
</tbody>
</table>
A number of other possible load combinations, including 0.5LL and DL, E and HS, etc. are commonly investigated. It is usual to use smaller safety factors for transitory loads such as wind and earthquake but this requirement is not absolute.

We should especially note that the geotechnical consultant will make a recommendation for an allowable strength (bearing capacity, etc.) that has the safety factor already included. The structural designer then factors this value or factors the loads to produce the design. In general the structural designer should not arbitrarily assume the geotechnical consultant used a specific value of SF as in Table 4-9. Rather the recommendation is what should be used. If the designer has a high load intensity from some transitory load combination the recommended bearing pressure should not be arbitrarily increased one-third, or whatever, without first discussing this with the geotechnical consultant.

4-16 BEARING CAPACITY OF ROCK

With the exception of a few porous limestone and volcanic rocks and some shales, the strength of bedrock in situ will be greater than the compressive strength of the foundation concrete. This statement may not be true if the rock is in a badly fractured, loose state where considerable relative slip between rock fragments can occur. The major problem is to identify the rock soundness and on occasion take cores for unconfined compression testing of the intact fragments. On very important projects and where it is economically feasible, one may make in situ strength tests.

Settlement is more often of concern than is the bearing capacity, and most test effort is undertaken to determine the in situ deformation modulus $E$ and Poisson’s ratio so that some type of settlement analysis can be made. This comment is made since most rock loads are from piles or drilled piers with the points embedded to some depth into the rock mass. Thus, one must make an analysis based on a load on the interior of a semi-infinite elastic body. The finite-element method FEM is sometimes used, but if the rock is fractured results are speculative unless one has measured data that can be used to revise the model. Even if the rock is not fractured the FEM seldom provides good results because uncertain elastic parameters are used.

It is common to use building code values for the allowable bearing capacity of rock; however, geology, rock type, and quality (as RQD) are significant parameters, which should be used together with the recommended code value. It is common to use large safety factors in rock capacity. The SF should be somewhat dependent on RQD defined in Sec. 3-17; i.e., an RQD of 0.80 would not require as high an SF as for RQD = 0.40. It is common to use SF from 6 to 10 with the higher values for RQD less than about 0.75 unless RQD is used to reduce the ultimate bearing capacity (as shown following). Table 4-11 may be used as a guide to estimate bearing capacity from code values or to obtain trial elastic parameters for preliminary FEM analyses.

One may use bearing-capacity equations of the form given by Terzaghi in Table 4-1 to obtain the bearing capacity of rocks using the angle of internal friction and cohesion of the rock from high-pressure triaxial tests. According to Stagg and Zienkiewicz (1968, p. 151)
TABLE 4-11  
Range of properties for selected rock groups; data from several sources

<table>
<thead>
<tr>
<th>Type of rock</th>
<th>Typical unit wt., kN/m$^3$</th>
<th>Modulus of elasticity $E$, MPa $\times 10^3$</th>
<th>Poisson's ratio, $\mu$</th>
<th>Compressive strength, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basalt</td>
<td>28</td>
<td>17–103</td>
<td>0.27–0.32</td>
<td>170–415</td>
</tr>
<tr>
<td>Granite</td>
<td>26.4</td>
<td>14–83</td>
<td>0.26–0.30</td>
<td>70–276</td>
</tr>
<tr>
<td>Schist</td>
<td>26</td>
<td>7–83</td>
<td>0.18–0.22</td>
<td>35–105</td>
</tr>
<tr>
<td>Limestone</td>
<td>26</td>
<td>21–103</td>
<td>0.24–0.45</td>
<td>35–170</td>
</tr>
<tr>
<td>Porous limestone</td>
<td></td>
<td>3–83</td>
<td>0.35–0.45</td>
<td>7–35</td>
</tr>
<tr>
<td>Sandstone</td>
<td>22.8–23.6</td>
<td>3–42</td>
<td>0.20–0.45</td>
<td>28–138</td>
</tr>
<tr>
<td>Shale</td>
<td>15.7–22</td>
<td>3–21</td>
<td>0.25–0.45</td>
<td>7–40</td>
</tr>
<tr>
<td>Concrete</td>
<td>15.7–23.6</td>
<td>Variable</td>
<td>0.15</td>
<td>15–40</td>
</tr>
</tbody>
</table>

*Depends heavily on confining pressure and how determined; $E =$ tangent modulus at approximately 50 percent of ultimate compression strength.

Use the Terzaghi shape factors of Table 4-1 with these bearing-capacity factors. The rock angle of internal friction is seldom less than 40° (often 45° to 55°) and rock cohesion ranges from about 3.5 to 17.5 MPa (500 to 2500 psi). It is evident from Eq. (4-27) that very high values of ultimate bearing capacity can be computed. The upper limit on allowable bearing capacity is, as previously stated, taken as $f'_c$ of the base concrete or not more than the allowable bearing pressure of metal piles.

The angle of internal friction of rock is pressure-dependent, similar to soil. Also, inspection of rock parameters from a number of sources indicates that, similar to sand, we could estimate $\phi = 45^\circ$ for most rock except limestone or shale where values between 38° and 45° should be used. Similarly we could in most cases estimate $s_u = 5$ MPa as a conservative value. Finally we may reduce the ultimate bearing capacity based on RQD as

$$q'_{ul} = q_{ult}(RQD)^2$$

In many cases the allowable rock-bearing pressure is taken in the range of one-third to one-tenth the unconfined compression strength obtained from intact rock samples and using RQD as a guide, for example, as one-tenth for a small RQD. Others simply use an allowable bearing pressure from the local building code (as in Table 4-8) based on rock type from a visual inspection of the rock cores.

Few building foundations such as mats or spread bases are placed directly on rock. Most situations involving rock-bearing capacity require large-diameter drilled shafts (termed drilled piers as in Chap. 19), which are socketed 2 to 3 shaft diameters into the rock. Recent load tests on this type of foundation [see Rowe and Armitage (1987)] indicate the allowable bearing pressure is on the order of

$$q_a = q_u \text{ to } 2.5q_u$$

where $q_u =$ unconfined compression strength of intact rock core samples. This value is substantially larger than the values of one-third and one-tenth previously cited. The large increase
in allowable pressure can be at least partially attributed to the triaxial confining effect developing at the pier base from the embedment depth. The lower values previously suggested are applicable for foundations located at the rock surface.

When rock coring produces no intact pieces of consequence (RQD → 0) one should treat as a soil mass and obtain the bearing capacity using equations from Table 4-1 and best estimates of the soil parameters φ and c.

Example 4-14. We have a drilled pier with a diam. = 1 m to be founded at a depth of 3.5 m into a rock mass to get through the surface irregularities and the weathered rock zone as determined by coring to a depth of 6.5 m into the rock. From the cores the average RQD = 0.50 (or 50 percent) below the pier point.

Required. Estimate the allowable bearing capacity for the pier base. For the pier concrete we will use $f_c' = 28$ MPa (allowable $f_c$ is, of course, somewhat less).

Solution. Assume from inspection of the rock cores that $\phi = 45^\circ$ and take $c = 3.5$ MPa (both reasonably conservative—cohesion may be overly so).

The Terzaghi shape factors are $s_c = 1.3$ and $s_\gamma = 0.6$. Assume the unit weight of the dense rock $\gamma_{\text{rock}} = 25.15$ kN/m$^3$. Compute the following:

$$N_c = 5 \tan^4 \left(45^\circ + \frac{45^\circ}{2}\right) = 170$$

$$N_q = \tan^6 \left(45^\circ + \frac{45^\circ}{2}\right) = 198$$

$$N_\gamma = N_q + 1 = 199$$

We will omit the soil overburden pressure to the soil-rock interface. Substituting in, and dividing by 1000 where necessary to convert to MPa, we have

$$q_{\text{ult}} = cN_c s_c + qN_q + 0.5\gamma B N_\gamma s_\gamma$$

$$= (3.5)(170)1.3 + \frac{3.5(25.15)(198)}{1000} + 0.5(25.15)(1)(199)(0.6)$$

$$= 773.5 + 17.4 + 1.5 = 792.4 \text{ MPa}$$

Use a SF = 3 and RQD = 0.5 to obtain the reduced allowable bearing pressure as

$$q_a = \frac{q_{\text{ult}}(0.5)^2}{3} = \frac{792.4(0.25)}{3} = 66 \text{ MPa}$$

This appears O.K., because 66 ≈ $2.4 \times f_c'$

Recommend $q_a = 30$ MPa as this is approximately $f_c'$.

Comments. Since $f_c'$ is seldom over 40 MPa for drilled piers we see bearing capacity of rock is seldom a controlling factor. It may be more critical for steel HP piles, however—depending on whether one uses the actual or projected area for bearing.

We might question in the previous example why unconfined or triaxial compression tests were not performed to obtain the strength parameters. These could have been done since the cores are available; however, the following are major considerations:
1. For either type of test, several rock samples with an $L/d > 2$ would have to be cut with the ends accurately flat and perpendicular to the longitudinal axis. This is costly.

2. Tests on intact rock samples where the RQD = 0.5 can give an incorrect strength for the mass.

3. Testing an intact sample for $q_u$ would give $c = q_u/2$ but no $\phi$ angle, so the $N_c$ term of Eq. (4-27) could not be obtained ($\tan^6 45^\circ = 1$ is not a good estimate). A $q_u$ strength is too low for intact rock.

4. Testing a triaxial sample requires access to high-pressure cell capabilities or else the results are little better than $q_u$ values. This still requires making an estimate of lateral cell pressure to duplicate in situ confinement. Using an estimate for cell pressure makes it difficult to justify the test expense.

As a final note, what can one do if the bearing pressure is inadequate? In this case we have options. We can go deeper into the rock or we can utilize skin resistance of the shaft-to-rock interface (considered in more detail in Chap. 19). We can abandon the site, or we can treat the rock. Rock treatment usually involves drilling a number of small holes and pressure-injecting cement grout to fill the cracks to provide mass continuity after the grout hardens. The latter requires further coring to see if the joints have been adequately grouted.

**PROBLEMS**

4-1. What is the allowable bearing capacity using the Hansen, Vesic, Meyerhof, and Terzaghi methods for the assigned problem in the following data set?

Other data: Use $B = 1.83$ m or 6.0 ft and $D = 0.75B$. The average unit weight in the zone of interest is 17.3 kN/m$^3$ or 110 pcf and the water table is not a problem.

<table>
<thead>
<tr>
<th>$\phi_{tu}$</th>
<th>Cohesion $c$</th>
<th>$H$</th>
<th>$V$</th>
<th>$M$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 20</td>
<td>15 kPa</td>
<td>232/5</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>(b) 25</td>
<td>10</td>
<td>—</td>
<td>323</td>
<td>—</td>
<td>250</td>
</tr>
<tr>
<td>(c) 30</td>
<td>5</td>
<td>9</td>
<td>—</td>
<td>436</td>
<td>341</td>
</tr>
<tr>
<td>(d) 34</td>
<td>0</td>
<td>—</td>
<td>—</td>
<td>919/19</td>
<td>667</td>
</tr>
<tr>
<td>(e) 38</td>
<td>0</td>
<td>1366</td>
<td>1582</td>
<td>1781</td>
<td>1194</td>
</tr>
</tbody>
</table>

*Note: Answer rounded to nearest integer.

4-2. Redo the assigned problem from the data of Prob. 4-1 if $L/B = 2.5$. Note that answers are from computer output and rounded only sightly for checking.

<table>
<thead>
<tr>
<th>Problem</th>
<th>$H$</th>
<th>$V$</th>
<th>$M$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>4/198</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>(b)</td>
<td>—</td>
<td>280</td>
<td>—</td>
<td>250</td>
</tr>
<tr>
<td>(c)</td>
<td>7.5</td>
<td>—</td>
<td>367</td>
<td>341</td>
</tr>
<tr>
<td>(d)</td>
<td>—</td>
<td>—</td>
<td>775/16</td>
<td>667</td>
</tr>
<tr>
<td>(e)</td>
<td>1221</td>
<td>1411</td>
<td>1464</td>
<td>1194/25</td>
</tr>
</tbody>
</table>
4-3. Find the required size of square footing using the soil data of Prob. 4-1 if \( D = 1.3 \) m and the footing load is as given below. Use \( SF = 3 \) for soil with cohesion and 2 for cohesionless soil.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Load, kN</th>
<th>( H )</th>
<th>( V )</th>
<th>( M )</th>
<th>( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>1200</td>
<td>2.40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>1200</td>
<td></td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>1200</td>
<td></td>
<td></td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td>2500</td>
<td></td>
<td></td>
<td></td>
<td>1.95</td>
</tr>
<tr>
<td>(e)</td>
<td>4000</td>
<td></td>
<td>1.45</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4-4. Referring to Fig. P4-4, find the size of square footing to carry the inclined load (with \( V \) and \( H \) components shown). Use Meyerhof’s, Vesić’s, or Hansen’s method as assigned and a \( SF = 5.0 \) on \( q_\text{ult} \). Column is square of size shown. Use \( \alpha_1 = 2.5 \) and \( \alpha_2 = 3.5 \) in Hansen’s method.

Partial answer: \( H = 2.95 \) m; \( V = 2.95 \) m; \( M = 3.05 \) m.

4-5. Redo Prob. 4-4 if there is also a moment of 600 kN \( \cdot \) m. Use \( SF = 5.0 \) as previously. Use the Meyerhof, Hansen, or Vesić equation as assigned.

Answer (all 3 methods):

\( H = 2.20 \times 3.00 \) m \hspace{1cm} \( V = 2.80 \times 3.65 \) m \hspace{1cm} \( M = 3.00 \times 3.80 \) m

4-6. Redo Example 4-6 using \( \phi_{ps} = 44^\circ \) and 46°. Comment on the effect of small changes in \( \phi \) on the computed bearing capacity.

Answer: \( \phi = 46 \rightarrow 1442 \) kPa; \( \phi = 44 \rightarrow 1035 \) kPa

4-7. Redo Example 4-6 using \( \phi_{ps} = 47^\circ \) but vary \( \alpha_1 > 2.5 \) and vary \( \alpha_2 > 3.5 \) (values of 2.5 and 3.5 used in example). Comment on the effect of these two parameters on allowable bearing pressure \( q_a \).

Answer: Using \( \alpha_1 = 4 \) and \( \alpha_2 = 5 \rightarrow q_\text{ult} = 807 \) kPa

4-8. Redo Example 4-7 if the force \( H \) is reversed (acts from right to left). Estimate ground slope \( \beta = -80^\circ \). Also use the Vesić method if it is assigned by your instructor for a comparison of methods.

4-9. A footing is located in the slope shown in Fig. P4-9. What is the allowable bearing capacity using Table 4-7 and the Hansen or Vesić bearing-capacity equations? What value of \( q_a \) do you recommend? Why?

4-10. Redo Example 4-7. Let the depth to the water table be 1.4 m instead of the 1.95 m shown in the example. Can you draw any conclusions about the effect of the water table location on the basis of this \( q_a \) and that from Example 4-7?
4-11. For the square footing on the layered soil of Fig. P4-11 find $B$ to carry the 1000 kN load using a $SF = 3$.

4-12. Redo Prob. 4-11 if the layers are reversed, i.e., the upper layer is the "stiff" clay with a 2 m thickness and the footing is at $D = 1$ m.

4-13. Prepare a set of design charts of $q_a/N_{70}$ versus $B$ for the maximum range of $D/B$ using Eqs. (4-11) and (4-12). Should you use an arithmetic or semilog plot?

4-14. Prepare a set of design charts of $q_a/q_c$ versus $B$ for the maximum range of $D/B$ using appropriate equations. Hint: Take $q_c = 4N$.

4-15. For the SPT data shown in Fig. 3-34, estimate the allowable bearing pressure at $-6.0$ ft. Will the GWT be a problem?

4-16. For the boring log shown in Fig. P3-10 what do you recommend as $q_a$ for footings located in the vicinity of the 2-meter depth? What does Table 4-8 suggest for $q_a$ using the BOCA code?

4-17. A portion of a cone-penetration test is in Fig. P4-17. Estimate the allowable bearing pressure at the 2- and 5-m depths.

Answer: About 425 kPa at the 5-m depth using $SF = 6$.

4-18. For the portion of the CPT test shown in Fig. 3-14c, estimate the allowable bearing pressure at the 2-m depth. Will water be a problem?

4-19. Using the CPT data of Table P3-11, estimate the allowable bearing pressure at the 2-m and 15-m depths.

4-20. The following load-test data are obtained from Brand et al. (1972). The footings are all square with the given dimensions and located approximately 1.5 m below the ground surface. Plot the assigned load test and estimate the failure or "ultimate" load. Compare this estimated load with $q_{ult}$ computed using the Meyerhof equations. Comment on your assumptions and results. See Example 4-3 for a computation of $q_{ult}$ for the 1.05-m footing and additional comments. The
TABLE P4-20
Displacements, inches

<table>
<thead>
<tr>
<th>Load, tons</th>
<th>Square plate size, m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.05</td>
</tr>
<tr>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.030</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.075</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.134</td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.212</td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.331</td>
</tr>
<tr>
<td>11</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.537</td>
</tr>
<tr>
<td>13</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>0.706</td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

displacements in Table P4-20 are in inch units (for example, 0.030 inches, 0.043 inches, etc.). Use $s_u = 1.5$ tsf.

4-21. What is the required footing dimension of the Housel method of Sec. 4-12.1 if the design load $P_d = 500$ kN?
   Answer: $2.75 \times 2.75$ m

4-22. What would you use for $q_a$ in Example 4-14 if $c = 0.8$ ksi? What does your local building code suggest?

4-23. What is the fraction of $q_u$ used in Example 4-14 to obtain $q_a$, assuming the cohesion parameter was obtained from an unconfined compression test?